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ON OUT AND IN BINARY NEIGHBORHOOD GRAPHS

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ABSTRACT

The out binary neighborhood graph $B_{nd}^{+}(H)$ of a directed graph $H=(V,E)$ is the undirected graph with vertex set $V\bigcup S^{+}$ where S^{+} is the set of all open out neighborhood sets of ***H*** *in which two vertex u,v are adjacent if $u,v\in S^{+}$ and $u\bigcap v\neq\phi$ or $u\in V$ and v is the out open neighborhood set of u .Similarly we can define the in open binary neighborhood graph. In this paper some properties of these new graphs are discussed.*

***Keywords**: Neighborhood set, In and out binary neighborhood graph, connected graph, Eulerian.*

1. INTRODUCTION

We are considering only finite, simple, directed or undirected graphs. Consider a graph $G=(V,E)$ and a vertex v in V . Then the *open neighborhood set* of v is the set $N(v)=\{u\in V:uv\in E\}$. The idea of *neighborhood graph* was introduced by Kulli in [1] and according to him for a graph $G=(V,E)$ the *neighborhood graph* of G is the graph with vertex set $V\bigcup S$ where S is the set of all *open neighborhood* sets of vertices of G in which two vertex u,v are adjacent if $u,v\in S$ and $u\bigcap v\neq\phi$ or $u\in V$ and v is an *open neighborhood set containing* u .

Let $H=(V,E)$ be a directed graph and v be a vertex in V . Then the *out open neighborhood* set of v is the set $N^{-}(v)=\{u\in V:vu\in E\}$ and the *in open neighborhood* set of v is $N^{-}(v)=\{u\in V:uv\in E\}$. We devote this paper to introduce a new type of graph using these definitions.

2. OUT AND IN BINARY NEIGHBORHOOD GRAPHS

Definition 2.1 Out binary neighborhood graph: The out binary neighborhood graph $B_{nd}^{-}(H)$ of a directed graph $H=(V,E)$ is the undirected graph with vertex set $V\bigcup S^{-}$,where S^{+} is the set of all non-empty *out open neighborhood sets* of vertices of H, in which two vertex u,v are adjacent if $u,v\in S^{-}$ and $u\bigcap v\neq\phi$ or $u\in V$ and v is the out open neighborhood set of u .

Definition 2.2 In binary neighborhood graph: The in binary neighborhood graph $B_{nd}^{+}(H)$ of a directed graph $H=(V,E)$ is the undirected graph with vertex set $V\bigcup S^{+}$,where S^{+} is the set of all non-empty *in open neighborhood sets* of vertices of H, in which two vertex u,v are adjacent $u,v\in S^{+}$ and $u\bigcap v\neq\phi$ or $u\in V$ and the *in open neighborhood* set of u .

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Example 2.3: In figure 1, 2 and 3 a directed graph H and its $B_{nd}^{-}\left(H\right)$ and $B_{nd}^{+}\left(H\right)$ are given. For H in figure 1 the *out open neighborhood* sets of H are $N^{-}\left(A\right)=\{C\}, N^{-}\left(B\right)=\{C,D\}, N^{-}\left(C\right)=\{D\}, N^{-}\left(D\right)=\{\varphi\}$ and the in *open neighborhood* sets of H are $N^{+}\left(A\right)=\{B]$, $N^{+}\left(B\right)=\varphi, N^{+}\left(C\right)=\{A,B\}, N^{+}\left(D\right)=\{B,C\}$.

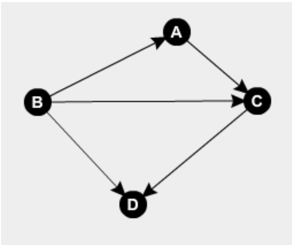

Figure-1: H

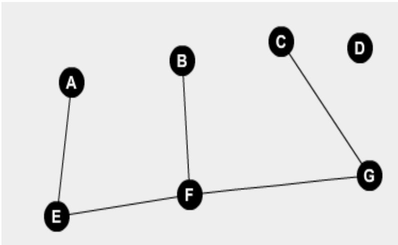

Figure-2: $B_{nd}^{-}\left(H\right)$

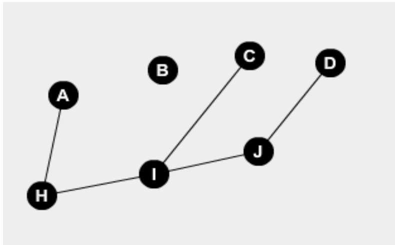

Figure-3: $B_{nd}^{+}\left(H\right)$

Where the vertices E, F, G, H, I & J represents,
$N^{-}\left(A\right), N^{-}\left(B\right), N^{-}\left(C\right), N^{-}\left(D\right), N^{+}\left(A\right), N^{+}\left(B\right), N^{+}\left(C\right) \& N^{+}\left(D\right)$ respectively.

Result 2.4: For any directed graph H, if a vertex v in H *isolated* then it remains *isolated* in both $B_{nd}^{+}\left(H\right)$ and $B_{nd}^{-}\left(H\right)$.

Definition 2.5 Source, sink vertex [2]: A vertex v of a directed graph H is said to be a *sink* vertex if $N^{-}\left(v\right)=\varphi$ and is said to be a *source* vertex if $N^{+}\left(v\right)=\varphi$.

Theorem 2.6:

a. A vertex v in H is isolated in $B_{nd}^{-}\left(H\right)$ iff it is *sink vertex* in H
b. A vertex v in H is isolated in $B_{nd}^{+}\left(H\right)$ iff it is *source* in H

Proof:

a. A vertex v in H is isolated in $B_{nd}^{-}\left(H\right)$ iff $N^{-}\left(v\right)=\varphi$ iff v is *sink vertex* in H.
b. A vertex v in H is isolated in $B_{nd}^{+}\left(H\right)$ iff $N^{+}\left(v\right)=\varphi$ iff it is a *source* in H.

Result 2.7

a. If v is a vertex of the directed graph H, then the degree of v in $B_{nd}^{-}\left(H\right)$ is equal to 0 if v is a *sink* in H and equal to 1 if v is not a *sink* in H.
b. If v is a vertex of the directed graph H, then the degree of v in $B_{nd}^{+}\left(H\right)$ is equal to 0 if v is a *source* in H and equal to 1 if v is not a *source* in H.

Remark 2.8: Result 2.7 is the reason behind the name "*in / out binary neighborhood graph*".

Definition 2.9 I-sequence collection: Let $\Im=\{S_1, S_2, ..., S_n\}$ be a collection of non-empty sets. Then we call $\Im$ an *I-sequence collection* if there exist a sequential arrangement $S_{k_1}, S_{k_2}, ..., S_{k_m}$ of elements of $\Im$ such that it contains all elements of $\Im$ at least once and for all t, $1 \le t \le n$, there exist at least one $d<t$ with $S_{k_t} \bigcap S_{k_d} \neq \phi$.

Lemma 2.10: For any directed graph $H=\left(V,E\right)$, the *out binary neighborhood graph* $B_{nd}^{-}\left(H\right)$ {or $B_{nd}^{+}\left(H\right)$} is connected then degree each *out open neighborhood* { respectively *in open neighborhood*} of vertices of H has degree at least 2 in $B_{nd}^{-}\left(H\right)$ {or $B_{nd}^{+}\left(H\right)$}.

Proof: follows from the connectedness of the graph and by result 2.7.

Theorem 2.11:

a. For any directed graph $H=(V,E)$, if the *out binary neighborhood graph* $B_{nd}^{-}(H)$ is connected then S^{-}, the set of all non-empty *out open neighborhood sets* of vertices of H, is an *I-sequence collection* and $N^{-}(v)\neq\varphi$ for all v in V.

b. For any directed graph $H=(V,E)$, if the in binary *neighborhood graph* $B_{nd}^{+}(H)$ is connected then S^{+},the set of all non-empty *in open neighborhood sets* of vertices of H, is an *I-sequence collection* and $N^{+}(v)\neq\varphi$ for all v in V.

Proof:

a. Let $B_{nd}^{-}(H)$ is connected. Then clearly $N^{-}(v)\neq\varphi$ for all v in **V**. We need only to prove S^{-} is an *I-sequence collection*. For let $v_1,v_2,\dots v_n$ be the vertices of H and $N^{-}(v_{k_1})$ be the *out open neighborhood set* of some vertex v_{k_1} of **V.** Then by lemma 2.10, there must exist another *out open neighborhood set* $N^{-}(v_{k_2})$ of some vertex $v_{k_2}\neq v_{k_1}$ of **V,** such that $N^{-}(v_{k_1})$ and $N^{-}(v_{k_2})$ are *adjacent* in $B_{nd}^{+}(H)$ .Then by definition $N^{-}(v_{k_1})\cap N^{-}(v_{k_2})\neq\varphi$.

   Now suppose we have an *I-sequence collection* $\{N^{-}(v_{k_1}),N^{-}(v_{k_2}),\dots,N^{-}(v_{k_t})\}$ for $1\le t<n$ .Our aim is find a $N^{-}(v_{k_{t+1}})$ such that $\{N^{-}(v_{k_1}),N^{-}(v_{k_2}),\dots,N^{-}(v_{k_t}),N^{-}(v_{k_{t+1}})\}$ is *I-sequence collection* .If possible no such $N^{-}(v_{k_{t+1}})$ exist. But that means none of the remaining *out open neighborhood sets* is adjacent to any of the sets in $\{N^{-}(v_{k_1}),N^{-}(v_{k_2}),\dots,N^{-}(v_{k_t})\}$. But then by *connectedness* of $B_{nd}^{-}(H)$ at least one vertex in **V** has degree greater than 1. This is a contradiction. Hence by mathematical induction, the result follows.

b. Similar proof follows

The following theorem is useful,

Theorem 2.12: A connected graph **G** is *Eulerian*[3] if and only if every vertex of **G** has even degree.

Theorem 2.13: For any directed graph $H=(V,E)$, the *out binary neighborhood graph* $B_{nd}^{+}(H)$ or *the in binary neighborhood graph* $B_{nd}^{-}(H)$ are never *Eulerian*

Proof: follows from theorem 2.12 and result 2.7.

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