International Journal of Mathematical Archive-7(3), 2016, 88-90
Available online through www.ijma.info ISSN 2229 – 5046

ON OUT AND IN BINARY NEIGHBORHOOD GRAPHS

VAISAKH VENU*

Asst. professor, Dept. of Mathematics,
Christ College of Engineering, Irinjalakkuda, Kerala, India-680125.

(Received On: 27-02-16; Revised & Accepted On: 16-03-16)

ABSTRACT

*The out binary neighborhood graph $B^{+}_{nd}(H)$ of a directed graph $H=(V,E)$ is the undirected graph with vertex set $V\bigcup S^{+}$ where S^{+} is the set of all open out neighborhood sets of **H** in which two vertex u,v are adjacent if $u,v\in S^{+}$ and $u\bigcap v\neq\phi$ or $u\in V$ and v is the out open neighborhood set of u .Similarly we can define the in open binary neighborhood graph. In this paper some properties of these new graphs are discussed.*

***Keywords**: Neighborhood set, In and out binary neighborhood graph, connected graph, Eulerian.*

1. INTRODUCTION

We are considering only finite, simple, directed or undirected graphs. Consider a graph $G=(V,E)$ and a vertex v in V . Then the *open neighborhood set* of v is the set $N(v)=\{u\in V:uv\in E\}$. The idea of *neighborhood graph* was introduced by Kulli in [1] and according to him for a graph $G=(V,E)$ the *neighborhood graph* of G is the graph with vertex set $V\bigcup S$ where S is the set of all *open neighborhood* sets of vertices of G in which two vertex u,v are adjacent if $u,v\in S$ and $u\bigcap v\neq\phi$ or $u\in V$ and v is an *open neighborhood set containing* u .

Let $H=(V,E)$ be a directed graph and v be a vertex in V . Then the *out open neighborhood* set of v is the set $N^{-}(v)=\{u\in V:vu\in E\}$ and the *in open neighborhood* set of v is $N^{-}(v)=\{u\in V:uv\in E\}$. We devote this paper to introduce a new type of graph using these definitions.

2. OUT AND IN BINARY NEIGHBORHOOD GRAPHS

Definition 2.1 Out binary neighborhood graph: The out binary neighborhood graph $B^{-}_{nd}(H)$ of a directed graph $H=(V,E)$ is the undirected graph with vertex set $V\bigcup S^{-}$,where S^{+} is the set of all non-empty *out open neighborhood sets* of vertices of H , in which two vertex u,v are adjacent if $u,v\in S^{-}$ and $u\bigcap v\neq\phi$ or $u\in V$ and v is the out open neighborhood set of u .

Definition 2.2 In binary neighborhood graph: The in binary neighborhood graph $B^{+}_{nd}(H)$ of a directed graph $H=(V,E)$ is the undirected graph with vertex set $V\bigcup S^{+}$,where S^{+} is the set of all non-empty *in open neighborhood sets* of vertices of H , in which two vertex u,v are adjacent $u,v\in S^{+}$ and $u\bigcap v\neq\phi$ or $u\in V$ and the *in open neighborhood* set of u .

Corresponding Author: Vaisakh Venu, Asst. professor, Dept. of Mathematics,*
Christ College of Engineering, Irinjalakkuda, Kerala, India-680125.

Example 2.3: In figure 1, 2 and 3 a directed graph H and its $B_{nd}^{-}(H)$ and $B_{nd}^{+}(H)$ are given. For H in figure 1 the *out open neighborhood* sets of H are $N^{-}(A)=\{C\}, N^{-}(B)=\{C,D\}, N^{-}(C)=\{D\}, N^{-}(D)=\{\varphi\}$ and the in *open neighborhood* sets of H are $N^{+}(A)=\{B]$, $N^{+}(B)=\varphi, N^{+}(C)=\{A,B\}, N^{+}(D)=\{B,C\}$.

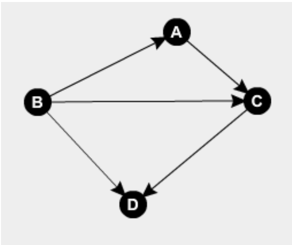

Figure-1: H

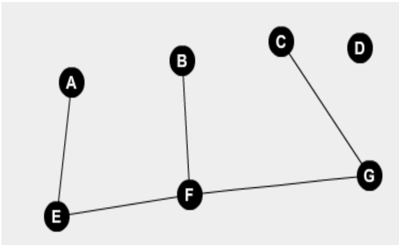

Figure-2: $B_{nd}^{-}(H)$

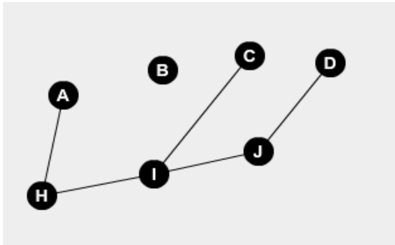

Figure-3: $B_{nd}^{+}(H)$

Where the vertices E, F, G, H, I & J represents,
$N^{-}(A), N^{-}(B), N^{-}(C), N^{-}(D), N^{+}(A), N^{+}(B), N^{+}(C) \,\&\, N^{+}(D)$ respectively.

Result 2.4: For any directed graph H, if a vertex v in H *isolated* then it remains *isolated* in both $B_{nd}^{+}(H)$ and $B_{nd}^{-}(H)$.

Definition 2.5 Source, sink vertex [2]: A vertex v of a directed graph H is said to be a *sink* vertex if $N^{-}(v)=\varphi$ and is said to be a *source* vertex if $N^{+}(v)=\varphi$.

Theorem 2.6:

a. A vertex v in H is isolated in $B_{nd}^{-}(H)$ iff it is *sink vertex* in H

b. A vertex v in H is isolated in $B_{nd}^{+}(H)$ iff it is *source* in H

Proof:

a. A vertex v in H is isolated in $B_{nd}^{-}(H)$ iff $N^{-}(v)=\varphi$ iff v is *sink vertex* in H.

b. A vertex v in H is isolated in $B_{nd}^{+}(H)$ iff $N^{+}(v)=\varphi$ iff it is a *source* in H.

Result 2.7

a. If v is a vertex of the directed graph H, then the degree of v in $B_{nd}^{-}(H)$ is equal to 0 if v is a *sink* in H and equal to 1 if v is not a *sink* in H.

b. If v is a vertex of the directed graph H, then the degree of v in $B_{nd}^{+}(H)$ is equal to 0 if v is a *source* in H and equal to 1 if v is not a *source* in H.

Remark 2.8: Result 2.7 is the reason behind the name "*in / out binary neighborhood graph*".

Definition 2.9 I-sequence collection: Let $\Im=\{S_1,S_2,\ldots,S_n\}$ be a collection of non-empty sets. Then we call $\Im$ an *I-sequence collection* if there exist a sequential arrangement $S_{k_1},S_{k_2},\ldots,S_{k_m}$ of elements of $\Im$ such that it contains all elements of $\Im$ at least once and for all t, $1\le t\le n$, there exist at least one $d<t$ with $S_{k_t}\bigcap S_{k_d}\neq\phi$.

Lemma 2.10: For any directed graph $H=(V,E)$, the *out binary neighborhood graph* $B_{nd}^{-}(H)$ {or $B_{nd}^{+}(H)$} is connected then degree each *out open neighborhood* { respectively *in open neighborhood*} of vertices of H has degree at least 2 in $B_{nd}^{-}(H)$ {or $B_{nd}^{+}(H)$}.

Proof: follows from the connectedness of the graph and by result 2.7.

Theorem 2.11:

a. For any directed graph $H = (V, E)$, if the *out binary neighborhood graph* $B_{nd}^{-}(H)$ is connected then S^{-}, the set of all non-empty *out open neighborhood sets* of vertices of H, is an *I-sequence collection* and $N^{-}(v) \neq \varphi$ for all v in V.

b. For any directed graph $H = (V, E)$, if the in binary *neighborhood graph* $B_{nd}^{+}(H)$ is connected then S^{+}, the set of all non-empty *in open neighborhood sets* of vertices of H, is an *I-sequence collection* and $N^{+}(v) \neq \varphi$ for all v in V.

Proof:

a. Let $B_{nd}^{-}(H)$ is connected. Then clearly $N^{-}(v) \neq \varphi$ for all v in **V**. We need only to prove S^{-} is an *I-sequence collection*. For let $v_1, v_2, \ldots v_n$ be the vertices of H and $N^{-}(v_{k_1})$ be the *out open neighborhood set* of some vertex v_{k_1} of **V.** Then by lemma 2.10, there must exist another *out open neighborhood set* $N^{-}(v_{k_2})$ of some vertex $v_{k_2} \neq v_{k_1}$ of **V,** such that $N^{-}(v_{k_1})$ and $N^{-}(v_{k_2})$ are *adjacent* in $B_{nd}^{+}(H)$ .Then by definition $N^{-}(v_{k_1}) \cap N^{-}(v_{k_2}) \neq \varphi$.

   Now suppose we have an *I-sequence collection* $\{N^{-}(v_{k_1}), N^{-}(v_{k_2}), \ldots, N^{-}(v_{k_{t\}}})\}$ for $1 \leq t < n$ .Our aim is find a $N^{-}(v_{k_{t+1}})$ such that $\{N^{-}(v_{k_1}), N^{-}(v_{k_2}), \ldots, N^{-}(v_{k_{t\}}}), N^{-}(v_{k_{t+1}})\}$ is *I-sequence collection* .If possible no such $N^{-}(v_{k_{t+1}})$ exist. But that means none of the remaining *out open neighborhood sets* is adjacent to any of the sets in $\{N^{-}(v_{k_1}), N^{-}(v_{k_2}), \ldots, N^{-}(v_{k_{t\}}})\}$. But then by *connectedness* of $B_{nd}^{-}(H)$ at least one vertex in **V** has degree greater than 1. This is a contradiction. Hence by mathematical induction, the result follows.

b. Similar proof follows

The following theorem is useful,

Theorem 2.12: A connected graph **G** is *Eulerian*[3] if and only if every vertex of **G** has even degree.

Theorem 2.13: For any directed graph $H = (V, E)$, the *out binary neighborhood graph* $B_{nd}^{+}(H)$ or *the in binary neighborhood graph* $B_{nd}^{-}(H)$ are never *Eulerian*

Proof: follows from theorem 2.12 and result 2.7.

REFERENCES

1. V.R.Kulli, The neighborhood graph of a graph (2015, International Journal of Fuzzy Mathematical Archive
2. Bang-jensen, Jorgen; Gutin Greory, (2000),directed graphs: Theory, algorithms and Applications, Springer, ISBN-185233-268-9.
3. Chartrand and Gary *(1985), Introductory Graph Theory, Dover, ISBN 0-486-24775-9.*

Source of support: Nil, Conflict of interest: None Declared