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EFFECT OF POROUS MEDIUM ON MHD VISCOUS INCOMPRESSIBLE FLUID DOWN BETWEEN TWO INCLINED PARELLEL FLAT PLATES UNDER GRAVITY

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ABSTRACT

In the present paper, the effects of porous medium on MHD unsteady flow of viscous fluid between two inclined parallel flat plates have been studied. The velocity distribution for unsteady flow is obtained .The particular case for steady flow had also been deduced.

Key words: *Porous medium, MHD, Incompressible fluid, Viscous, Gravity*

INTRODUCTION:

Here we suppose uniform magnetic field applied perpendicular to the flow of viscous fluid down between two inclined parallel flat plates. Snedden (1951) had studied the flow of viscous incompressible fluid down an inclined plate; Sharma et. al. (1995) worked on the steady laminar free convective flow of an electrically conducting fluid along a porous hot vertical plate in the presence of heat source/sink; Ahmed et. al. (1997) discussed on the three dimensional free convective flow and heat transfer through a porous medium; Biswal et. al (2002) have also studied the MHD flow between two infinite parallel plates, one oscillating while other is uniform in motion; Rajeshwara et. al. (2003) studied the unsteady flow through a porous medium in a vertical channel under a transverse magnetic field. Recently Kumar and Singh (2008) studied the magnetic field on the motion under gravity of a viscous fluid down between two inclined parallel flat plates.

In this present paper we extended the research work Kumar and Singh (2008). We investigated the effect of porous medium on velocity distribution for unsteady flow of viscous fluid by using of Fourier Transform. For steady flow of viscous liquid had also deduced.

FORMULATION AND SOLUTION OF THE PROBLEM:

Consider the motion of a viscous incompressible fluid down in an inclined plate in the presence of magnetic field with the assumption that the velocity of the fluid at the free surface is known and a uniform magnetic field applied perpendicular to the parallel to the flat plates. For the slow motion of viscous liquid the governing equations of motion is

$$\frac{\partial \bar{v}}{\partial t} - \bar{F} + \frac{1}{\rho}\,\text{grad}(p) = -\nu \nabla^2 \bar{u} - \frac{\sigma B_0^2}{\rho}\bar{v} - \frac{1}{K}\bar{v} \tag{1}$$

Consider a viscous liquid of density ρ filling the surface between the two parallel plates with boundaries y = 0, y = h and moving under gravity, then we may take

$$\bar{F} = g(\sin\alpha - \cos\alpha, 0) \tag{2}$$

Since motion may be assumed to be the same in all planes parallel to the plane-xy. Vector $\bar{V}$ will be of the form

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(u, 0,0) where u is the function of x, y and t. Let us suppose that fluid is homogenous.

Equation of continuity is

$$\frac{\partial u}{\partial x} = 0 \tag{3}$$

$\Rightarrow$ u is the a function only of y and t.

Under these circumstance equation (1) becomes

$$\frac{\partial u}{\partial t} = g\sin\alpha - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u - \frac{1}{K}u \tag{4}$$

$$0 = g\cos\alpha - \frac{1}{\rho}\frac{\partial p}{\partial y} \tag{5}$$

Clearly $\left(g\sin\alpha - \frac{1}{\rho}\frac{\partial p}{\partial y}\right)$ is a function of y alone, and hence from equation (5), we have

$$p = \rho g\left(x\sin\alpha - y\cos\alpha\right) + x\rho X \tag{6}$$

Where X is constant

Therefore equation (4) becomes

$$\frac{\partial u}{\partial t} = -X + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u - \frac{1}{K}u \tag{7}$$

Now introduce the Finite Fourier Sine Transform

$$\bar{u}_s\left(n,t\right) = \int_0^h u\left(y,t\right)\sin\left(\frac{n\pi y}{h}\right)dy \tag{8}$$

Subject to the boundary conditions:

$u = U\left(t\right)$, on y = h

$u = 0$ on y = 0

i.e. $u\left(0,t\right) = 0$ at y = 0

and $u\left(h,t\right) = 0$ at y = h (9)

Multiply (7) by $\sin\left(\frac{n\pi y}{h}\right)$ and integrating with respect to y, from 0 → h, and using equation (9) we get

$$\int_0^h \frac{\partial u}{\partial t}\sin\frac{n\pi y}{h}dy = -X + \nu\int_0^h \frac{\partial^2 u}{\partial y^2}\sin\frac{n\pi y}{h}dy - \left(\frac{\sigma B_0^2}{\rho} + \frac{1}{K}\right)\int_0^h u\sin\frac{n\pi y}{h}dy$$

or $$\frac{du_s}{dt}+\left(\nu\frac{n^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho}+\frac{1}{K}\right)u_s=-X+\left(-1\right)^{n+1}\nu\frac{n\pi}{h}U(t) \tag{10}$$

This is ordinary linear equation. Solving equation (10), we get

$$\bar{u}_s=-X\frac{1}{\left(\nu\frac{n^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho}+\frac{1}{K}\right)}+(-1)^{n+1}\nu\frac{n\pi}{h}\int_{t_0}^{t}U(\tau)e^{-\left(\frac{\nu n^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho}+\frac{1}{K}\right)(t-\tau)}d\tau \tag{11}$$

Where t_0 is an arbitrary time defined by the initial conditions.

Using inverse formula for Finite Fourier Sine form, we get

$$u(y,t)=\frac{2}{h}\sum_{n=1}^{\infty}\left[-X\frac{1}{\left(\nu\frac{n^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho}+\frac{1}{K}\right)}+(-1)^{n+1}\nu\frac{n\pi}{h}\int_{t_0}^{t}U(\tau)e^{-\left(\frac{\nu n^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho}+\frac{1}{K}\right)(t-\tau)}d\tau\right]\times\sin\frac{n\pi y}{h} \tag{12}$$

If flow is steady, then U (t) is constant, therefore equation (9) becomes

$$\left(\nu\frac{n^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho}+\frac{1}{K}\right)\bar{u}_s=-X+(-1)^{n+1}\nu\frac{n\pi}{h}U$$

or $$\bar{u}_s=-X\frac{1}{\left(\nu\frac{n^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho}+\frac{1}{K}\right)}+(-1)^{n+1}\frac{\nu n\pi U}{\left(\nu\frac{n^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho}+\frac{1}{K}\right)} \tag{13}$$

Now using inverse formula for the Finite Fourier Sine Transform, we get

$$u(y,t)=\frac{2}{h}\sum_{n=1}^{\infty}\left[-X\frac{1}{\left(\nu\frac{n^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho}+\frac{1}{K}\right)}+(-1)^{n+1}\frac{\nu n\pi U}{\left(\nu\frac{n^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho}+\frac{1}{K}\right)}\right]\times\sin\frac{n\pi y}{h}$$

DISCUSSION:

The expression for velocity distribution of unsteady flow is given by equation (12) and the velocity distribution for steady flow is given by equation (14) respectively.

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