

ON T-FUZZY BI-IDEALS AND  $(\lambda, \mu)$ -FUZZY BI-IDEALS IN NEAR-RING  
WITH RESPECT TO t-NORM

M. HIMAYA JALEELA BEGUM\*<sup>1</sup>, S. JEYALAKSHMI<sup>2</sup>

<sup>1</sup>Department of Mathematics,  
Sadakathullah Appa College, Tirunelveli-11, Tamil Nadu, India.

<sup>2</sup>Department of Mathematics,  
Sri Parasakthi College for Women, Courtallam, Tamil Nadu, India.

(Received On: 15-02-16; Revised & Accepted On: 17-03-16)

---

ABSTRACT

In this paper we introduce the concept of T-fuzzy bi-ideals using t-norm and t-norm  $(\lambda, \mu)$ -fuzzy bi-ideals in near-ring and investigate some of their properties.

---

1. INTRODUCTION

The theory of fuzzy set was first inspired by Zadeh [5]. Triangular norms were introduced by Schweizer and Sklar[3,4] to model the distances in probabilistic metric spaces. P.Dheena, G.Mohanraj [2] and M.Akram [1] have studied several properties of T-fuzzy ideals of rings and T-fuzzy ideals of near-rings. In this paper we introduce the notion of T-fuzzy bi-ideals using t-norm and t-norm  $(\lambda, \mu)$ -fuzzy bi-ideals in near-ring and investigate some related properties.

2. PRELIMINARIES

**Definition 2.1:** A fuzzy subset  $\mu$  of a near-ring  $N$  is called T- fuzzy bi-ideal if

- (1)  $\mu(x-y) \geq T(\mu(x), \mu(y))$
- (2)  $\mu(xyz) \geq T(\mu(x), \mu(z))$  for all  $x, y, z \in N$ .

**Definition 2.2:** A mapping  $f: N \rightarrow N'$  is called a near-ring homomorphism if  $f(x+y) = f(x) + f(y)$  and  $f(xy) = f(x)f(y)$  for all  $x, y \in N$ .

**Definition 2.3:** a quotient nearring is a near –ring that is the quotient of a near-ring and one of its bi-ideal  $I$ , denoted  $N/I$ . If  $I$  is a bi-ideal of a near-ring  $N$  and  $a \in N$ , then a coset of  $I$  is a set of the form  $a+I = \{a+s/s \in I\}$ . The set of all coset is denoted by  $N/I$ .

**Theorem 2.4:** If  $I$  is a bi-ideal of a near-ring  $N$ , the set  $N/I$  is a near-ring under the operations  $(a+I) + (b+I) = (a+b) + I$  and  $(a+I) \cdot (b+I) = (a \cdot b) + I$ .

**Lemma 2.5:** If  $\mu$  is a fuzzy bi-ideal of  $N$ , then  $\mu(0) \geq \mu(x)$  for all  $x \in N$ .

**Definition 2.6.[3]:** A t-norm is a function  $T: [0,1] \times [0,1] \rightarrow [0,1]$  that satisfies the following conditions for all  $x, y, z \in [0,1]$ ,

- (1)  $T(x, 1) = x$ ,
- (2)  $T(x, y) = T(y, x)$ ,
- (3)  $T(x, T(y, z)) = T(T(x, y), z)$ ,
- (4)  $T(x, y) \leq T(x, z)$  whenever  $y \leq z$ .

A simple example of such defined t-norm is a function  $T(x, y) = \min(x, y)$ . In general case,  $T(x, y) \leq \min(x, y)$  and  $T(x, 0) = 0$  for all  $x, y \in [0,1]$ .

---

**Corresponding Author: M. Himaya Jaleela Begum\*<sup>1</sup>, <sup>1</sup>Department of Mathematics,  
Sadakathullah Appa College, Tirunelveli-11, Tamil Nadu, India.**

**Definition 2.7:** A subgroup B of N is said to be bi-ideal if  $BNB \subseteq B$ .

**Definition 2.8:** A fuzzy subset  $\mu$  of a near-ring N is called fuzzy bi-ideal if

- (1)  $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$
- (2)  $\mu(xyz) \geq \min\{\mu(x), \mu(z)\}$  for all  $x, y, z \in N$ .

**Definition 2.9:** A fuzzy bi-ideal  $\mu$  of a near-ring N is said to be normal if  $\mu(0)=1$ .

**Definition 2.10:** Let N and N' be two near-rings and 'f' a function of N into N'.

- (i) If  $\lambda$  is a fuzzy set in N', then the preimage of  $\lambda$  under 'f' is the fuzzy set in N defined by  $\mu(x) = (\lambda \circ f)(x) = \lambda(f(x))$  for each  $x \in N$ ,
- (ii) If  $\mu$  is a fuzzy set of N, then the image of  $\mu$  under f is the fuzzy set in N' defined by

$$f(\mu)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu(x) & \text{if } f^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise for each } y \in N'. \end{cases}$$

**Definition 2.11:** Let  $\mu$  and  $\lambda$  be a T-fuzzy bi-ideal of a Near-ring N. Then the direct product of T-fuzzy bi-ideal is defined by  $(\mu \times \lambda)(x, y) = T(\mu(x), \lambda(y))$  for all  $x, y \in N$ .

### 3. THE QUOTIENT NEAR-RINGS VIA T-FUZZY BI-IDEALS

**Theorem 3.1:** Let I be a bi-ideal of a near-ring N. If  $\mu$  is a T-fuzzy bi-ideal of N, then the fuzzy set  $\bar{\mu}$  of N/I defined by  $\bar{\mu}(a+I) = \sup_{x \in I} \mu(a+x)$  is a T-fuzzy bi-ideal of the quotient near-ring N/I with respect to I.

**Proof:** Let N be a near-ring and  $\mu$  be a T-fuzzy bi-ideal of N.

Let  $a, b \in N$  such that  $a+I = b+I$ . Then  $b = a+y$  for some  $y \in I$ .

Thus  $\bar{\mu}(b+I) = \sup_{x \in I} \mu(b+x) = \sup_{x \in I} \mu(a+y+x) = \sup_{z = x+y \in I} \mu(a+z) = \bar{\mu}(a+I)$ . Hence  $\bar{\mu}$  is well defined.

Let  $x+I, y+I \in N/I$ ,

$$\begin{aligned} \bar{\mu}((x+I)-(y+I)) &= \bar{\mu}((x-y)+I) \\ &= \sup_{z \in I} \mu((x-y)+z) \\ &= \sup_{z = u-v \in I} \mu((x-y)+(u-v)) \\ &= \sup_{z = u-v \in I} \mu((x+u)-(y+v)) \\ &\geq \sup_{u, v \in I} T(\mu(x+u), \mu(y+v)) \\ &= T(\sup_{u \in I} \mu(x+u), \sup_{v \in I} \mu(y+v)) \\ &= T(\bar{\mu}(x+I), \bar{\mu}(y+I)) \end{aligned}$$

Therefore  $\bar{\mu}((x+I)-(y+I)) \geq T(\bar{\mu}(x+I), \bar{\mu}(y+I))$ .

For  $x+I, y+I, z+I \in N/I$ ,

$$\begin{aligned} \bar{\mu}((x+I)(y+I)(z+I)) &= \bar{\mu}((xyz)+I) \\ &= \sup_{u \in I} \mu(xyz+u) \\ &= \sup_{u \in I} \mu((x+u)(y+u)(z+u)) \\ &\geq \sup_{u \in I} T(\mu(x+u), \mu(y+u), \mu(z+u)) \\ &= T(\sup_{u \in I} \mu(x+u), \sup_{u \in I} \mu(y+u), \sup_{u \in I} \mu(z+u)) \\ &= T(\bar{\mu}(x+I), \bar{\mu}(y+I), \bar{\mu}(z+I)). \end{aligned}$$

Therefore  $\bar{\mu}((x+I)(y+I)(z+I)) \geq T(\bar{\mu}(x+I), \bar{\mu}(y+I), \bar{\mu}(z+I))$ .

Thus  $\bar{\mu}$  is a T-fuzzy bi-ideal of N/I.

**Theorem 3.2:** Let I be a bi-ideal of near-ring N. Then there is one-to-one correspondence between the set of T-fuzzy bi-ideals  $\mu$  of N such that  $\mu(0) = \mu(s)$  for all  $s \in I$  and the set of all T-fuzzy bi-ideals  $\bar{\mu}$  of N/I.

**Proof:** Let I be a bi-ideal of a near-ring N. Let  $\mu$  be T-fuzzy bi-ideal of a near-ring N. Using theorem 3.1, we prove that  $\bar{\mu}$  defined by  $\bar{\mu}(a+I) = \sup_{x \in I} \mu(a+x)$  is a T-fuzzy bi-ideal of N/I. Since  $\mu(0) = \mu(s)$  for all  $s \in I$ ,  $\mu(a+s) \geq T(\mu(a), \mu(s)) = \mu(a)$ . Again,  $\mu(a) = \mu(a+s-s) \geq T(\mu(a+s), \mu(s)) = \mu(a+s)$ . Thus  $\mu(a+s) = \mu(a)$  for all  $s \in I$ , that is,  $\bar{\mu}(a+I) = \mu(a)$ . Hence the correspondence  $\mu \mapsto \bar{\mu}$  is one-to-one. Let  $\bar{\mu}$  be a T-fuzzy bi-ideal of N/I and defined fuzzy set  $\mu$  in N by  $\mu(a) = \bar{\mu}(a+I)$  for all  $a \in I$ .

For any  $x, y \in N$ ,

$$\begin{aligned} \mu(x-y) &= \bar{\mu}((x-y)+I) = \bar{\mu}((x+I)-(y+I)) \\ &\geq T(\bar{\mu}(x+I), \bar{\mu}(y+I)) \\ &= T(\mu(x), \mu(y)) \end{aligned}$$

Therefore  $\mu(x-y) \geq T(\mu(x), \mu(y))$ .

For any  $x, y, z \in N$ , we have

$$\begin{aligned} \mu(xyz) &= \bar{\mu}((xyz)+I) = \bar{\mu}((x+I)(y+I)(z+I)) \\ &\geq T(\bar{\mu}(x+I), \bar{\mu}(z+I)) \\ &= T(\mu(x), \mu(z)). \end{aligned}$$

Therefore  $\mu(xyz) \geq T(\mu(x), \mu(z))$ .

Thus  $\mu$  is a T-fuzzy bi-ideal of N. Note that  $\mu(z) = \bar{\mu}(z+I) = \bar{\mu}(I)$  for all  $z \in I$ , which shows that  $\mu(z) = \mu(0)$  for all  $z \in I$ .

**Theorem 3.3:** Let T be t-norm and I be a bi-ideal of a near-ring N. Then for all  $\alpha \in [0,1]$ , there exists a T-fuzzy bi-ideal  $\mu$  of N such that  $\mu(0) = \alpha$  and  $U(\mu; \alpha) = I$ .

**Proof:** Let  $\mu : N \rightarrow [0,1]$  be a fuzzy subset of N defined by  $\mu(x) = \begin{cases} \alpha & \text{if } x \in I \\ 0 & \text{otherwise} \end{cases}$  Where  $\alpha$  is a fixed number in  $[0,1]$ . clearly  $U(\mu; \alpha) = I$  and  $\mu(0) = \alpha$ . Let  $x, y \in N$ , then a routine calculation shows that  $\mu$  is a T-fuzzy bi-ideal of N.

**Theorem 3.4:** Let  $\mu$  be a T-fuzzy bi-ideal of a near-ring N and let  $\mu(0) = \alpha$ . Then the fuzzy subset  $\mu^*$  of the quotient near-ring  $N/U(\mu; \alpha)$  defined by  $\mu^*(x+U(\mu; \alpha)) = \mu(x)$  for all  $x \in N$  is a T-fuzzy bi-ideal of  $N/U(\mu; \alpha)$ .

**Proof:** The proof is straightforward.

**Theorem 3.5:** Let I be a bi-ideal of a near-ring N and  $\phi$  be a T-fuzzy bi-ideal of N/I such that  $\phi(x+I) = \phi(x)$  only if  $x \in I$ . Then there exists a T-fuzzy bi-ideal of N such that  $U(\mu; \alpha) = I$ ,  $\alpha = \mu(0)$ , and  $\phi = \mu^*$ .

**Proof:** Let  $\phi$  be a T-fuzzy bi-ideal of N/I. Define a T-fuzzy bi-ideal  $\mu$  of N by  $\mu(x) = \phi(x+I)$  for all  $x \in N$ .

For  $x, y \in N$ ,  $\mu(x-y) = \phi((x-y)+I) \geq T(\phi(x+I), \phi(y+I)) = T(\mu(x), \mu(y))$

For  $x, y, z \in N$ ,  $\mu(xyz) = \phi(xyz+I) \geq T(\phi(x+I), \phi(z+I)) = T(\mu(x), \mu(z))$ . Therefore  $\mu$  is a T-fuzzy bi-ideal of N.

Next, we prove that

$$U(\mu; \alpha) = I. \text{ Let } x \in U(\mu; \alpha) \Leftrightarrow \mu(x) \geq \alpha = \mu(0) \Leftrightarrow \mu(x) \geq \mu(0) \Leftrightarrow \mu(x) = \mu(0) \Leftrightarrow \phi(x+I) = \phi(0+I) \Leftrightarrow \phi(x+I) = \phi(I) \Leftrightarrow x \in I.$$

Hence  $U(\mu; \alpha) = I$ . Finally, we prove that  $\mu^* = \phi$ . Now,  $\mu^*(x+I) = \mu^*(x+U(\mu; \alpha)) = \mu(x) = \phi(x+I)$ . Therefore  $\mu^* = \phi$ .

**Theorem 3.6:** An onto homomorphic image of a T-fuzzy bi-ideal with sup property is a T-fuzzy bi-ideal.

**Proof:** Let N and N' be two near-rings. Let  $f: N \rightarrow N'$  be an epimorphism and  $\mu$  be a T-fuzzy bi-ideal of N with sup property and  $\lambda$  be the image of  $\mu$  under f.

Let  $f(x), f(y), f(z) \in f(N)$  and let  $x_0 \in f^{-1}(f(x)), y_0 \in f^{-1}(f(y)), z_0 \in f^{-1}(f(z))$  such that  $\mu(x_0) = \sup_{t \in f^{-1}(f(x))} \mu(t)$ ,

$$\mu(y_0) = \sup_{t \in f^{-1}(f(y))} \mu(t), \mu(z_0) = \sup_{t \in f^{-1}(f(z))} \mu(t).$$

Now,

$$\begin{aligned} \text{(i)} \quad \lambda(f(x)-f(y)) &= \sup_{t \in f^{-1}(f(x) - f(y))} \mu(t) \\ &\geq \mu(x_0 - y_0) \\ &\geq T\{\mu(x_0), \mu(y_0)\} \\ &= T\left\{ \sup_{t \in f^{-1}(f(x))} \mu(t), \sup_{t \in f^{-1}(f(y))} \mu(t) \right\} \\ &= T\{\lambda(f(x)), \lambda(f(y))\} \end{aligned}$$

Therefore  $\lambda(f(x)-f(y)) \geq T\{\lambda(f(x)), \lambda(f(y))\}$ .

$$\begin{aligned} \text{(ii)} \quad \lambda(f(x)f(y)f(z)) &= \lambda(f(xyz)) \\ &= \sup_{t \in f^{-1}(f(xyz))} \mu(t) \\ &\geq \mu(x_0 y_0 z_0) \\ &\geq T\{\mu(x_0), \mu(z_0)\} \\ &= T\left\{ \sup_{t \in f^{-1}(f(x))} \mu(t), \sup_{t \in f^{-1}(f(z))} \mu(t) \right\} \\ &= T\{\lambda(f(x)), \lambda(f(z))\} \end{aligned}$$

Therefore  $\lambda(f(x)f(y)f(z)) \geq T\{\lambda(f(x)), \lambda(f(z))\}$ . Hence  $\lambda$  is a T-fuzzy bi-ideal of  $N'$ .

**Theorem 3.7:** An onto homomorphic pre-image of a T-fuzzy bi-ideal of near-ring is a T-fuzzy bi-ideal.

**Proof:** Let  $N$  and  $N'$  be two near-rings. Let  $f: N \rightarrow N'$  be an epimorphism.

Let  $x, y \in N$ ,

$$\begin{aligned} \text{(i)} \quad \mu(x-y) &= (\lambda \circ f)(x-y) \\ &= \lambda(f(x-y)) \\ &= \lambda((f(x)-f(y))) \\ &\geq T\{\lambda(f(x)), \lambda(f(y))\} \\ &= T\{(\lambda \circ f)(x), (\lambda \circ f)(y)\} \\ &= T\{\mu(x), \mu(y)\} \end{aligned}$$

Therefore  $\mu(x-y) \geq T(\mu(x), \mu(y))$ .

Similarly we can prove that  $\mu(xyz) \geq T(\mu(x), \mu(z))$  for all  $x, y, z \in N$ . Hence  $\mu$  is a T-fuzzy bi-ideal of  $N$ .

**Theorem 3.8:** Let  $N$  and  $N'$  be two near-rings. If  $\mu$  and  $\lambda$  are T-fuzzy bi-ideals of  $N$  and  $N'$  respectively, then  $\mu \times \lambda$  is a T-fuzzy bi-ideal of the direct product  $N \times N'$ .

**Proof:** Let  $(x_1, y_1), (x_2, y_2) \in N \times N'$ .

$$\begin{aligned} \text{Now, } (\mu \times \lambda)((x_1, y_1) - (x_2, y_2)) &= (\mu \times \lambda)(x_1 - x_2, y_1 - y_2) \\ &= T[\mu(x_1 - x_2), \lambda(y_1 - y_2)] \\ &\geq T[T(\mu(x_1), \mu(x_2)), T(\lambda(y_1), \lambda(y_2))] \\ &= T(T[T(\mu(x_1), \mu(x_2)), \lambda(y_1)], \lambda(y_2)) \\ &= T(T(\lambda(y_1), T(\mu(x_1), \mu(x_2))), \lambda(y_2)) \\ &= T(T[T(\lambda(y_1), (\mu(x_1), \mu(x_2))), \lambda(y_2)) \\ &= T[T(\mu(x_1), (\lambda(y_1)), T(\mu(x_2), \lambda(y_2))] \\ &= T[(\mu \times \lambda)(x_1, y_1), (\mu \times \lambda)(x_2, y_2)] \end{aligned}$$

Therefore  $(\mu \times \lambda)((x_1, y_1) - (x_2, y_2)) \geq T((\mu \times \lambda)(x_1, y_1), (\mu \times \lambda)(x_2, y_2))$ .

Let  $(x_1, y_1), (x_2, y_2), (x_3, y_3) \in N \times N'$ ,

$$\begin{aligned} \text{Now, } (\mu \times \lambda)((x_1, y_1)(x_2, y_2)(x_3, y_3)) &= (\mu \times \lambda)(x_1 x_2 x_3, y_1 y_2 y_3) \\ &= T(\mu(x_1 x_2 x_3), \lambda(y_1 y_2 y_3)) \\ &\geq T[T(\mu(x_1), \mu(x_3)), T(\lambda(y_1), \lambda(y_3))] \\ &= T(T[T(\mu(x_1), \mu(x_3)), \lambda(y_1)], \lambda(y_3)) \\ &= T(T(\lambda(y_1), T(\mu(x_1), \mu(x_3))), \lambda(y_3)) \\ &= T(T[T(\lambda(y_1), (\mu(x_1), \mu(x_3))), \lambda(y_3)) \\ &= T[T(\mu(x_1), (\lambda(y_1)), T(\mu(x_3), \lambda(y_3))] \\ &= T[(\mu \times \lambda)(x_1, y_1), (\mu \times \lambda)(x_3, y_3)] \end{aligned}$$

Therefore  $(\mu \times \lambda)((x_1, y_1)(x_2, y_2)(x_3, y_3)) \geq T((\mu \times \lambda)(x_1, y_1), (\mu \times \lambda)(x_3, y_3))$ .

Hence  $\mu \times \lambda$  is a T-fuzzy bi-ideal of the direct product  $N \times N'$ .

**Theorem 3.9:** Let  $\mu$  be a T-fuzzy bi-ideal of  $N$ . If  $\mu(x+y) = \mu(0)$ , then  $\mu(x) = \mu(y)$ .

**Proof:** Assume that  $\mu(x+y) = \mu(0)$  for all  $x, y \in N$ .

$$\begin{aligned} \text{Then } \mu(x) &= \mu((x+y)-y) \\ &\geq T(\mu(x+y), \mu(y)) \\ &= T(\mu(0), \mu(y)) \\ &= \mu(y) \end{aligned}$$

Therefore  $\mu(x) \geq \mu(y)$ .

$$\begin{aligned} \text{Now, } \mu(y) &= \mu((x+y)-x) \\ &\geq T(\mu(x+y), \mu(x)) \\ &= T(\mu(0), \mu(x)) \\ &= \mu(x) \end{aligned}$$

Therefore  $\mu(y) \geq \mu(x)$ . Hence  $\mu(x) = \mu(y)$ .

**Theorem 3.10.:** Let  $N$  be a near-ring and  $\mu$  be a T-fuzzy bi-ideal of  $N$ . Then the set  $N_\mu = \{x \in N / \mu(0) = \mu(x)\}$  is a bi-ideal of  $N$ .

**Proof:** Let  $\mu$  be a T-fuzzy bi-ideal of  $N$ . Let  $x, y \in N_\mu \Rightarrow \mu(x) = \mu(y) = \mu(0)$ . Consider  $\mu(x-y) \geq T(\mu(x), \mu(y)) \geq T(\mu(0), \mu(0)) \geq \mu(0)$  and so  $\mu(x-y) = \mu(0) \Rightarrow x-y \in N_\mu$  and hence  $N_\mu$  is a subgroup of  $N$ . Let  $x, z \in N_\mu$  and  $y \in N$ . Then  $\mu(x) = \mu(z) = \mu(0)$ . Now  $\mu(xyz) \geq T(\mu(x), \mu(z)) \geq T(\mu(0), \mu(0)) = \mu(0)$  and so  $\mu(xyz) = \mu(0) \Rightarrow xyz \in N_\mu$ . Hence  $N_\mu$  is a bi-ideal of  $N$ .

**Theorem 3.11:** If  $\{\mu_i / i \in \Lambda\}$  is a family of T-fuzzy bi-ideals of a near-ring  $N$ , then  $\bigvee_{i \in \Lambda} \mu_i$  is also a T-fuzzy bi-ideal of  $N$  where  $\bigvee_{i \in \Lambda} \mu_i$  is defined by  $(\bigvee_{i \in \Lambda} \mu_i)(x) = \sup\{\mu_i(x) / i \in \Lambda\}$  for all  $x \in N$ .

**Proof:** Let  $\{\mu_i / i \in \Lambda\}$  be a family of T-fuzzy bi-ideals of  $N$  and  $x, y, z \in N$ . Then we have that

$$\begin{aligned} (\bigvee_{i \in \Lambda} \mu_i)(x-y) &= \sup\{\mu_i(x-y) / i \in \Lambda\} \\ &\geq \sup\{T(\mu_i(x), \mu_i(y)) / i \in \Lambda\} \\ &= T(\{\sup\{\mu_i(x) / i \in \Lambda\}, \sup\{\mu_i(y) / i \in \Lambda\}\}) \\ &= T((\bigvee_{i \in \Lambda} \mu_i)(x), (\bigvee_{i \in \Lambda} \mu_i)(y)) \end{aligned}$$

Therefore  $(\bigvee_{i \in \Lambda} \mu_i)(x-y) \geq T((\bigvee_{i \in \Lambda} \mu_i)(x), (\bigvee_{i \in \Lambda} \mu_i)(y))$ .

$$\begin{aligned} \text{Now, } (\bigvee_{i \in \Lambda} \mu_i)(xyz) &= \sup\{\mu_i(xyz) / i \in \Lambda\} \\ &\geq \sup\{T(\mu_i(x), \mu_i(z)) / i \in \Lambda\} \\ &= T(\sup\{\mu_i(x) / i \in \Lambda\}, \sup\{\mu_i(z) / i \in \Lambda\}) \\ &= T((\bigvee_{i \in \Lambda} \mu_i)(x), (\bigvee_{i \in \Lambda} \mu_i)(z)) \end{aligned}$$

Therefore  $(\bigvee_{i \in \Lambda} \mu_i)(xyz) \geq T((\bigvee_{i \in \Lambda} \mu_i)(x), (\bigvee_{i \in \Lambda} \mu_i)(z))$ . Hence  $\bigvee_{i \in \Lambda} \mu_i$  is a T-fuzzy bi-ideal of  $N$ .

**Theorem 3.12:** Let  $\mu$  be a T-fuzzy bi-ideal of a near-ring  $N$  and  $\hat{\mu}$  be a fuzzy subset in  $N$  defined by  $\hat{\mu}(x) = \frac{1}{\mu(0)}\mu(x)$  for all  $x \in N$ . Then  $\hat{\mu}$  is a normal T-fuzzy bi-ideal of  $N$ .

**Proof:** Let  $\mu$  be a T-fuzzy bi-ideal of a near-ring  $N$ . Clearly,  $\hat{\mu}(0) = \frac{1}{\mu(0)}\mu(0) = 1$ ,  $\hat{\mu}$  is normal.

For any  $x, y \in N$ ,

$$\begin{aligned} \text{Then } \hat{\mu}(x-y) &= \frac{1}{\mu(0)}\mu(x-y) \\ &\geq \frac{1}{\mu(0)}\min\{\mu(x), \mu(y)\} \\ &= \min\left\{\frac{\mu(x)}{\mu(0)}, \frac{\mu(y)}{\mu(0)}\right\} \\ &= \min\{\hat{\mu}(x), \hat{\mu}(y)\} \end{aligned}$$

Therefore  $\hat{\mu}(x-y) \geq \min\{\hat{\mu}(x), \hat{\mu}(y)\}$ . Similarly  $\hat{\mu}(xyz) \geq \min\{\hat{\mu}(x), \hat{\mu}(z)\}$  for all  $x, y, z \in N$ . Hence  $\hat{\mu}$  is a normal T-fuzzy bi-ideal of  $N$  and obviously  $\mu \subseteq \hat{\mu}$ .

**Corollary 3.13:** If  $\mu$  is a T-fuzzy bi-ideal of a near-ring  $N$  satisfying  $\hat{\mu}(x) = 0$  for some  $x \in N$ , then  $\mu(x) = 0$  also.

#### 4. t-NORM $(\lambda, \mu)$ -FUZZY bi-IDEALS

**Definition 4.1:** Let  $A$  be a fuzzy subset of  $N$ . Then  $A$  is called a t-norm  $(\lambda, \mu)$ -fuzzy bi-ideal of  $N$  if for all  $x, y, z \in N$ ,

- (i)  $A(x-y) \vee \lambda \geq t(t(A(x), A(y)), \mu)$
- (ii)  $A(xyz) \vee \lambda \geq t(t(A(x), A(z)), \mu)$ .

**Theorem 4.2:** Let  $A$  be a fuzzy subset of  $N$ . Then  $A$  is a t-norm  $(\lambda, \mu)$ -fuzzy bi-ideal of  $N$  iff  $A_\alpha$  is a bi-ideal of  $N$  for all  $\alpha \in (\lambda, \mu)$ .

**Proof:** Let  $A$  be a t-norm  $(\lambda, \mu)$ -fuzzy bi-ideal of  $N$ . Let  $x, y \in A_\alpha \Rightarrow A(x) \geq \alpha, A(y) \geq \alpha$ . Consider  $A(x-y) \vee \lambda \geq t(t(A(x), A(y)), \mu) \geq t(t(\alpha, \alpha), \mu) = \alpha$  (since  $\alpha > \lambda$ ). That is  $A(x-y) \vee \lambda \geq \alpha \Rightarrow A(x-y) \geq \alpha \Rightarrow x-y \in A_\alpha$ . Hence  $A_\alpha$  is a subgroup of  $N$ . Let  $x, z \in A_\alpha \Rightarrow A(x) \geq \alpha, A(z) \geq \alpha$  and  $y \in N$ . Then  $A(xyz) \vee \lambda \geq t(t(A(x), A(z)), \mu) \geq t(t(\alpha, \alpha), \mu) = \alpha$  (since  $\alpha > \lambda$ ). That is  $A(xyz) \vee \lambda \geq \alpha \Rightarrow A(xyz) \geq \alpha \Rightarrow xyz \in A_\alpha$ . Hence  $A_\alpha$  is a bi-ideal of  $N$ .

Conversely suppose that  $A_\alpha$  is a bi-ideal of  $N$  for all  $\alpha \in (\lambda, \mu)$ . Suppose  $A(x-y) \vee \lambda < t(t(A(x), A(y)), \mu) = \alpha$ . Then  $A(x-y) \vee \lambda < \alpha \Rightarrow A(x-y) < \alpha$  (since  $\alpha > \lambda$ )  $\Rightarrow x-y \notin A_\alpha$ , a contradiction to  $A_\alpha$  is a bi-ideal of  $N$ . Hence  $A(x-y) \vee \lambda \geq t(t(A(x), A(y)), \mu)$ . Similarly we can prove that  $A(xyz) \vee \lambda \geq t(t(A(x), A(z)), \mu)$ . Hence  $A$  is a t-norm  $(\lambda, \mu)$ -fuzzy bi-ideal of  $N$ .

**Theorem 4.3:** Let  $f: N_1 \rightarrow N_2$  be an onto homomorphism of a near-ring  $N$  and let  $A$  be a t-norm  $(\lambda, \mu)$ -fuzzy bi-ideal of  $N_1$ . Then  $f(A)$  is a t-norm  $(\lambda, \mu)$ -fuzzy bi-ideal of  $N_2$  where  $f(A)(y) = \sup\{A(x) / f(x) = y\}$  for all  $y \in N_2$ .

**Proof:** Let  $f: N_1 \rightarrow N_2$  be an onto homomorphism of a near-ring  $N$  and let  $A$  be a t-norm  $(\lambda, \mu)$ -fuzzy bi-ideal of  $N_1$ .

For any  $y_1, y_2 \in N_2$ , we have

$$\begin{aligned} \text{(i) } f(A)(y_1-y_2) \vee \lambda &= \sup\{A(x_1-x_2) / f(x_1-x_2) = y_1-y_2\} \vee \lambda \\ &= \sup\{A(x_1-x_2) \vee \lambda / f(x_1-x_2) = y_1-y_2\} \\ &\geq \sup\{t(t(A(x_1), A(x_2)), \mu) / f(x_1) = y_1, f(x_2) = y_2\} \\ &= t(t(\sup\{A(x_1) / f(x_1) = y_1\}, \sup\{A(x_2) / f(x_2) = y_2\}), \mu) \\ &= t(t(f(A)(y_1), f(A)(y_2)), \mu) \end{aligned}$$

Therefore  $f(A)(y_1-y_2) \vee \lambda \geq t(t(f(A)(y_1), f(A)(y_2)), \mu)$ .

Similarly we can prove that  $f(A)(y_1y_2y_3) \vee \lambda \geq t(t(f(A)(y_1), f(A)(y_3)), \mu)$ . Hence  $f(A)$  is a t-norm  $(\lambda, \mu)$ -fuzzy bi-ideal of  $N_2$ .

**Theorem 4.4:** Let  $f: N_1 \rightarrow N_2$  be an onto homomorphism of a near-ring  $N$  and let  $B$  be a t-norm  $(\lambda, \mu)$ -fuzzy bi-ideal of  $N_2$ . Then  $f^{-1}(B)$  is a t-norm  $(\lambda, \mu)$ -fuzzy bi-ideal of  $N_1$ , where  $f^{-1}(B)(x) = B(f(x))$  for all  $x \in N_1$ .

**Proof:** For any  $x_1, x_2 \in N_1$ ,

$$\begin{aligned} f^{-1}(B)(x_1-x_2) \vee \lambda &= B(f(x_1-x_2)) \vee \lambda \\ &= B(f(x_1)-f(x_2)) \vee \lambda \\ &\geq t(t(B(f(x_1)), B(f(x_2))), \mu) \\ &= t(t(f^{-1}(B)(x_1), f^{-1}(B)(x_2)), \mu) \end{aligned}$$

Therefore  $f^{-1}(B)(x_1-x_2) \vee \lambda \geq t(t(f^{-1}(B)(x_1), f^{-1}(B)(x_2)), \mu)$ . Similarly we can prove that  $f^{-1}(B)(x_1x_2x_3) \vee \lambda \geq t(t(f^{-1}(B)(x_1), f^{-1}(B)(x_3)), \mu)$ . Hence  $f^{-1}(B)$  is a t-norm  $(\lambda, \mu)$ -fuzzy bi-ideal of  $N_1$ .

#### 5. ACKNOWLEDGMENT

The first author expresses her deep sense of gratitude to the UGC- SERO, Hyderabad, No: F. MRP-5366/14 for financial assistance.

## REFERENCE

1. Akram.M, On T-fuzzy ideals in near-rings, Int.J. Math. Sci, vol 2007, Article ID 73514, 14 pages.
2. Dheena.P and Mohanraaj.G, T-fuzzy ideal in rings, International Journal of Computational Cognition, vol 9 (June 2011), NO.2.
3. Schweizer.B and Sklar.A, Statistical metric spaces, Pacific Journal of Mathematics, vol. 10(1960), no. 1, pp.313-334.
4. Schweizer.B and Sklar.A, Associative functions and abstract semigroups, Publications Mathematicae Debrecen, vol. 10(1963), pp. 69-81.
5. Zadeh.L.A , Fuzzy sets, Information and Control, 8(1965), 338-353.

**Source of support: UGC- SERO, Hyderabad, India, Conflict of interest: None Declared**

***[Copy right © 2016. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]***