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EFFECTS OF HEAT TRANSFER ON PERISTALTIC PUMPING OF A NEWTONIAN FLUID IN ASYMMETRIC CHANNEL WITH RADIATION

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ABSTRACT

In this paper, the influence of radiation on peristaltic flow of a viscous fluid in an asymmetric channel has received little interest. Hence, an attempt is made to study the peristaltic flow of a Newtonian fluid in an asymmetric channel under the assumptions of long wavelength and low Reynolds number. Expressions for the velocity, temperature and pressure gradient are obtained analytically. The effects of various emerging parameters on the pumping characteristics and temperature field are studied in detail.

Keywords: *Asymmetric channel; Heat transfer; Radiation.*

1. INTRODUCTION

Past five decades researchers have extensively paying attention on the peristaltic pumping of Newtonian and non-Newtonian fluids. Especially, peristaltic pumping occurs in many practical applications involving biomechanical systems such as roller and finger pumps. In particular, the peristaltic pumping of corrosive fluids and slurries could be useful as it is desirable to prevent their contact with mechanical parts of the pump. In these investigations, solutions for peristaltic flow of the fluid, the geometry of the channel and the propagating waves were obtained for various degrees of approximation.

A lot attention had been restricted to symmetric channels or tubes, but there exist also flows which may not be symmetric. An interesting study was made by Eytan and Elad [3] whose results have been used to analyze the fluid flow pattern in a non-pregnant uterus. In another paper, Eytan *et al.* [4] discussed the characterization of non-pregnant women uterine contractions as they are composed of variable amplitudes and a range of different wave lengths. Mishra and Ramachandra Rao [5] studied the peristaltic flow of a Newtonian fluid in an asymmetric channel. In another attempt, Ramachandra Rao and Mishra [8] investigated the non-linear and curvature effects on peristaltic transport of a Newtonian fluid in an asymmetric channel when the ratio of channel width to the wave length is small. An example for a peristaltic type motion is the intra-uterine fluid flow due to momentarily contraction, where the myometrial contractions may occur in both symmetric and asymmetric directions. Elshewey *et al.* [2] analyzed peristaltic transport of a Newtonian fluid through a porous medium in an asymmetric. Peristaltic transport of a power law fluid in an asymmetric channel was investigated by Subba Reddy *et al.* [9]. Prasanth Reddy and Subba Reddy [6] studied the peristaltic pumping of third grade fluid in an asymmetric channel under the effect of magnetic fluid.

The study of heat transfer analysis is another important area in connection with peristaltic motion, which has industrial applications like sanitary fluid transport, blood pumps in heart lungs machine and transport of corrosive fluids where the contact of fluid with the machinery parts are prohibited. There are only a limited number of research available in literature in which peristaltic phenomenon has discussed in the presence of heat transfer [7, 10, 11].

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However, the influence of radiation on peristaltic flow of a viscous fluid in an asymmetric channel has received little interest. Hence, an attempt is made to study the peristaltic flow of a Newtonian fluid in an asymmetric channel under the assumptions of long wavelength and low Reynolds number. Expressions for the velocity, temperature and pressure gradient are obtained analytically. The effects of various emerging parameters on the pumping characteristics and temperature field are studied in detail.

2. MATHEMATICAL FORMULATION

We consider the flow of an incompressible viscous fluid in a two-dimensional asymmetric channel induced by sinusoidal wave trains propagating with constant speed c along the channel walls. A rectangular co-ordinate system (X,Y) is chosen such that X -axis lies along the centre line of the channel in the direction of wave propagation and Y -axis transverse to it, as shown in Fig. 1. The lower and upper walls of the channel are maintained at constant temperatures T_1 and T_0, respectively.

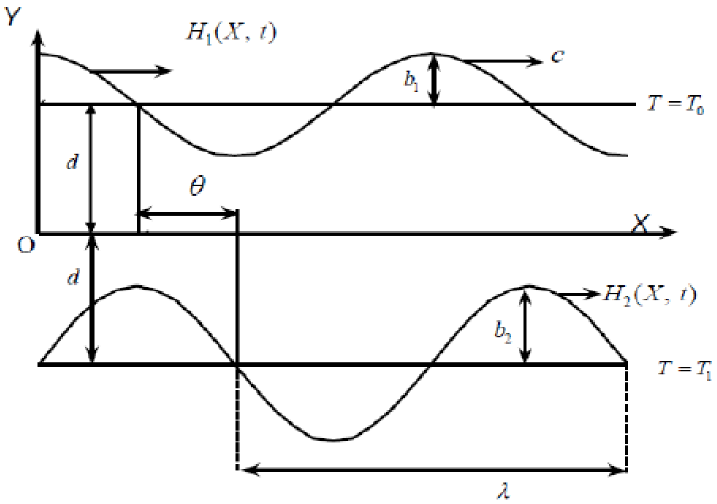

Fig. 1: The Physical Model

The channel walls are characterized by

$$Y = H_1(X,t) = d + b_1 \cos\frac{2\pi}{\lambda}(X - ct) \qquad \text{(Upper wall)}$$

$$Y = H_1(X,t) = -d - b_2 \cos\left(\frac{2\pi}{\lambda}(X - ct) + \theta\right) \qquad \text{(Lower wall)} \tag{2.1}$$

where b_1, b_2 are the amplitudes of the waves, λ is the wavelength, $2d$ is the width of the channel, θ is the phase difference which varies in the range $0 \le \theta \le \pi$, $\theta = 0$ corresponds to a symmetric channel with waves out of phase and $\theta = \pi$.

In fixed frame (X,Y), the flow is unsteady but if we choose moving frame (x, y), which travel in the X -direction with the same speed as the peristaltic wave, then the flow can be treated as steady.

The transformation between two frames are related by

$$x = X - ct, y = Y, u = U - c, v = V \text{ and } p(x) = P(X,t) \tag{2.2}$$

where (u,v) and (U,V) are the velocity components, p and P are the pressures in wave and fixed frames of reference respectively.

The pressure p remains a constant across any axial station of the channel, under the assumption that the wavelength is large and the curvature effects are negligible.

In the absence of an input electric field, the equations governing the flow field in a wave frame, are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.3}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+v\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (2.4)$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \quad (2.5)$$

$$\rho c_p\left[u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=k\nabla^2 T+\mu\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right\}-\frac{\partial q}{\partial y} \quad (2.6)$$

where ρ is the density, μ is the co-efficient of viscosity of the fluid, c_p is the specific heat at constant pressure, v is kinematic viscosity of the fluid, k is thermal conductivity of the fluid and T is temperature of the fluid. Following Cogley et al. [1], it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q}{\partial y}=4\alpha^2\left(T_0-T\right) \quad (2.7)$$

here α is the mean radiation absorption coefficient.

In order to write the governing equations and the boundary conditions in dimensionless form the following non - dimensional quantities are introduced.

$$\bar{x}=\frac{x}{\lambda};\bar{y}=\frac{y}{d};\bar{u}=\frac{U}{c},\bar{v}=\frac{V}{c\delta},\delta=\frac{d}{\lambda},\bar{p}=\frac{a_1^2 p}{\mu c\lambda},\bar{t}=\frac{ct}{\lambda},h_1=\frac{H_1}{d},$$

$$h_2=\frac{H_2}{d},a=\frac{b_1}{d},a=\frac{b_2}{d},\ \Theta=\frac{T-T_0}{T_1-T_0},\Pr=\frac{\rho v c_p}{k},Ec=\frac{c^2}{c_p(T_1-T_0)} \quad (2.8)$$

where δ is the wave number and a and b are amplitude ratios of upper and lower waves respectively.

In view of (2.8), the Equations (2.3) – (2.6), after dropping bars, reduce to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (2.9)$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} \quad (2.10)$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial x}+\delta^2\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \quad (2.11)$$

$$\mathrm{Re}\,\delta\left[u\frac{\partial \Theta}{\partial x}+v\frac{\partial \Theta}{\partial y}\right]=\frac{1}{\Pr}\left(\delta^2\frac{\partial^2 \Theta}{\partial x^2}+\frac{\partial^2 \Theta}{\partial y^2}\right)+E\left\{\begin{matrix}4\delta^2\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\\ +\delta^4\left(\frac{\partial v}{\partial x}\right)^2+2\delta^2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\end{matrix}\right\}-N^2\Theta \quad (2.12)$$

where $\mathrm{Re}=\frac{\rho dc}{\mu}$ is the Reynolds number, $N^2=\frac{4\alpha^2 d^2}{k}$ is the radiation parameter.

Under the assumption of long wave length $(\delta \ll 1)$ the Equations (2.10) - (2.12) become

$$\frac{\partial p}{\partial x}=\frac{\partial^2 u}{\partial y^2} \quad (2.13)$$

$$\frac{\partial p}{\partial y}=0 \quad (2.14)$$

$$\frac{1}{Pr}\left[\frac{\partial^2\Theta}{\partial y^2}\right]+Ec\left[\frac{\partial u}{\partial y}\right]^2-N^2\Theta=0 \tag{2.15}$$

The non-dimensional boundary conditions are

$$u=-1 \text{ at } y=h_1(x),h_2(x) \tag{2.16}$$

$$\Theta=0 \text{ at } y=h_1(x) \tag{2.17}$$

$$\Theta=1 \text{ at } y=h_2(x) \tag{2.18}$$

here $h_1(x)=1+a\cos 2\pi x$ and $h_2(x)=-1-b\cos(2\pi x+\theta)$.

Equation (2.14) implies that $p\neq p(y)$, hence p is only function of x. Therefore, the Equation (2.13) can be rewritten as

$$\frac{dp}{dx}=\frac{\partial^2 u}{\partial y^2} \tag{2.19}$$

The rate of volume flow rate through each section in a wave frame, is calculated as

$$q=\int_{h_2}^{h_1}udy \tag{2.20}$$

The flux at any axial station in the laboratory frame is

$$Q(x,t)=\int_{h_2}^{h_1}(u+1)dy=q+h_1-h_2 \tag{2.21}$$

The average volume flow rate over one period (T= λ/c) of the peristaltic wave is defined as

$$\overline{Q}=\frac{1}{T}\int_0^T Qdt=q+2 \tag{2.22}$$

The dimensionless pressure rise per one wavelength in the wave frame is defined as

$$\Delta p=\int_0^1\frac{dp}{dx}dx \tag{2.23}$$

3. SOLUTION: Solving Equation (2.19) using boundary conditions (2.16), we get

$$u=\frac{1}{2}\frac{dp}{dx}\left(y^2-\left(h_1+h_2\right)y+h_1h_2\right)-1 \tag{3.1}$$

Substituting Equation (3.1) in the Equation (2.15) and Solving Equation (2.15) using the boundary conditions (2.17) and (2.18), we obtain

$$\Theta=c_1\cos N_1y+c_2\sin N_1y-\frac{\Pr Ec}{4N_1^2}\left(4y^2-4\left(h_1+h_2\right)y+\left(h_1+h_2\right)^2-\frac{8}{N_1^2}\right) \tag{3.2}$$

here $c_1=\left(A_1\sin Nh_2-A_2\cos Nh_1\right)/\sin N\left(h_2-h_1\right)$,

$c_2=\left(A_2\cos Nh_1-A_1\cos Nh_2\right)/\sin N\left(h_2-h_1\right)$,

$A_1=\frac{\Pr Ec}{4N_1^2}\left(\frac{dp_0}{dx}\right)^2\left[\left(h_1-h_2\right)^2-\frac{8}{N_1^2}\right]$ and

$A_2=\frac{\Pr Ec}{4N_1^2}\left(\frac{dp_0}{dx}\right)^2\left[\left(h_1-h_2\right)^2-\frac{8}{N_1^2}\right]-1.$

The volume flow rate q in the wave frame of reference is given by

$$q=\frac{1}{12}\frac{dp}{dx}\left[h_1^2+h_2^2+6h_1h_2\right]\left(h_1-h_2\right)-\left(h_1-h_2\right) \tag{3.3}$$

From (3.3), we have

$$\frac{dp}{dx}=\frac{12\left[q+\left(h_1-h_2\right)\right]}{\left[h_1^2+h_2^2+6h_1h_2\right]\left(h_1-h_2\right)} \tag{3.4}$$

4. DISCUSSION OF THE RESULTS

In order to get a the physical insight of the problem, pumping characteristics and temperature field are computed numerically for different values of various emerging parameters, viz., phase shift θ, amplitude ratios a,b, $\Pr, Ec$ and N and are presented in figures 2-11.

The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of phase shift θ with $a=0.5$ and $b=0.5$ is shown in Fig. 2. It is found that, the $\overline{Q}$ decreases with increasing phase shift θ in all the three regions, vz., pumping region $(\Delta p>0)$, free pumping region $(\Delta p=0)$ and co-pumping region $(\Delta p<0)$. Moreover, the $\overline{Q}$ increases with increasing θ for appropriately chosen $\Delta p(<0)$.

The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of a with $b=0.5$ and $\theta=\frac{\pi}{4}$, is shown in Fig. 3. It is found that, the $\overline{Q}$ increases with an increase in a in both pumping and free pumping regions. But in the co-pumping region, the $\overline{Q}$ decreases with increasing a, for an appropriately chosen $\Delta p(<0)$.

Fig. 4 represents the variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of b with $a=0.5$ and $\theta=\frac{\pi}{4}$. It is noted that, the $\overline{Q}$ increases on increasing b in the both pumping and free pumping regions, while in co-pumping region, the $\overline{Q}$ decreases as b increases, for an appropriately chosen $\Delta p(<0)$.

Fig. 5 shows the variation of temperature Θ with y for different values of phase shift θ with $a=0.5, b=0.5$, $x=0.5$, $\Pr=0.7$, $Ec=0.1$, $N=1$ and $\overline{Q}=-1$. It is found that, the temperature Θ decreases with an increase in θ.

The variation of temperature Θ with y for different values of upper wave amplitude a with $\theta=\frac{\pi}{4}, b=0.5$, $x=0.5$, $\Pr=0.7$, $Ec=0.1$, $N=1$ and $\overline{Q}=-1$ is shown in Fig. 6. It is observed that, the temperature Θ initially increases and then decreases with increasing a.

Fig. 7 depicts the variation of temperature Θ with y for different values of lower wave amplitude b with $a=0.5$, $\theta=\frac{\pi}{4}$, $x=0.5$, $\Pr=0.7$, $Ec=0.1$, $N=1$ and $\overline{Q}=-1$. It is found that, the temperature Θ increases with an increase in b.

The variation of temperature Θ with y for different values of Prandtl number $\Pr$ with $a=0.5$, $b=0.5$, $x=0.5$, $\theta=\frac{\pi}{4}$, $Ec=0.1$, $N=1$ and $\overline{Q}=-1$ is depicted in Fig. 8. It is noted that, the temperature Θ increases with increasing $\Pr$.

Fig. 9 shows the variation of temperature Θ with y for different values of radiation parameter N with $a=0.5$, $b=0.5$, $x=0.5$, $\Pr=0.7$, $Ec=0.1$, $\theta=\frac{\pi}{4}$ and $\overline{Q}=-1$. It is found that, the temperature Θ increases with increasing N.

The variation of temperature Θ with y for different values of Eckert number Ec with $a=0.5$, $b=0.5$, $x=0.5$, $\Pr=0.7$, $\theta=\frac{\pi}{4}$, $N=1$ and $\overline{Q}=-1$ is shown in Fig. 10. It is observed that, the temperature Θ increases with an increase in Ec.

Fig. 11 illustrates the variation of temperature Θ with y for different values of time averaged flux $\overline{Q}$ with $a=0.5$, $b=0.5$, $x=0.5$, $\Pr=0.7$, $Ec=0.1$, $N=1$ and $\theta=\frac{\pi}{4}$. It is noted that, the temperature Θ decreases with increasing $\overline{Q}$. **5.**

CONCLUSIONS

In this chapter, we investigated the effects of Heat transfer and radiation on the peristaltic flow of a Newtonian fluid in an asymmetric channel under longwavelength approximation. The expressions for the velocity filed and temperature field are obtained. It is found that, in the pumping region the time averaged flux $\overline{Q}$ increases with increasing a and b while it decrease with increasing θ. It is observed that the temperature field Θ increases with increasing b, $\Pr, N$ and Ec, while it decreases with increasing a and θ.

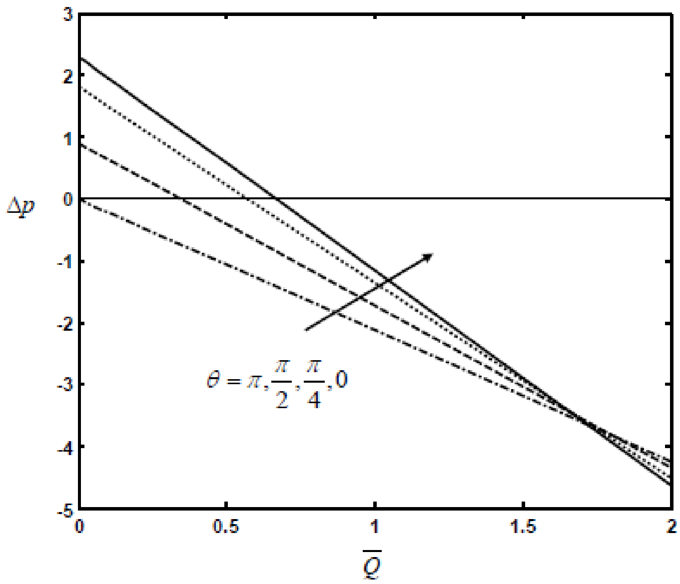

Fig. 2: The variation of pressure rise Δp with $\overline{Q}$ for different values of phase shift θ with $a=0.5$ and $b=0.5$.

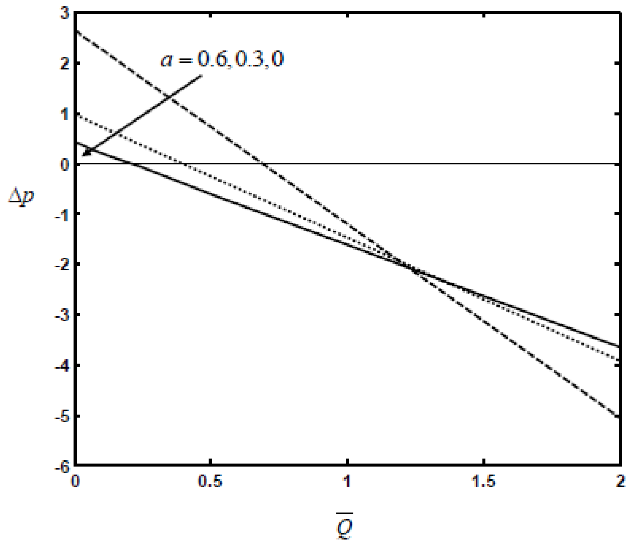

Fig. 3: The variation of pressure rise Δp with $\overline{Q}$ for different values of a with $\theta = \dfrac{\pi}{4}$ and $b = 0.5$.

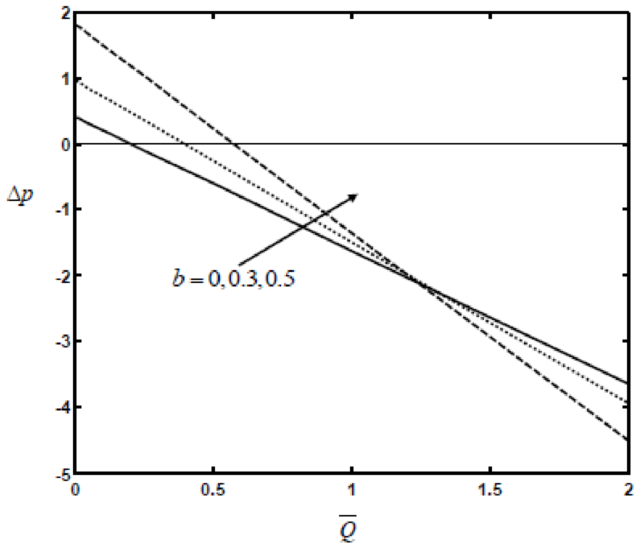

Fig. 4: The variation of pressure rise Δp with $\overline{Q}$ for different values of b with $a = 0.5$ and $\theta = \dfrac{\pi}{4}$.

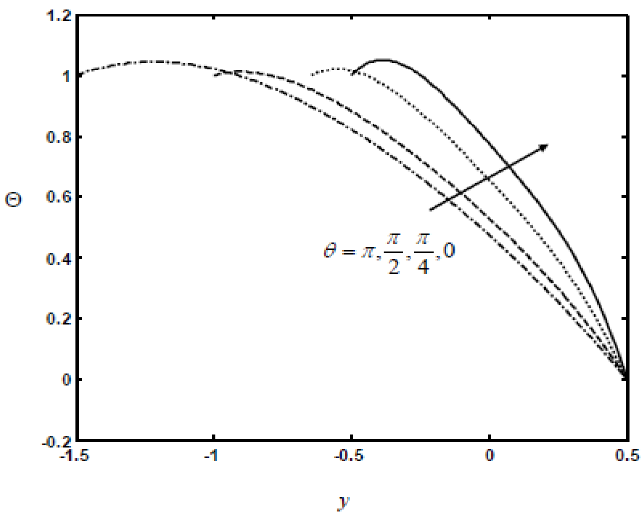

Fig. 5: The variation of temperature Θ with y for different values of phase shift θ with $a = 0.5, b = 0.5$, $x = 0.5, \Pr = 0.7$, $Ec = 0.1$, $N = 1$ and $\overline{Q} = -1$.

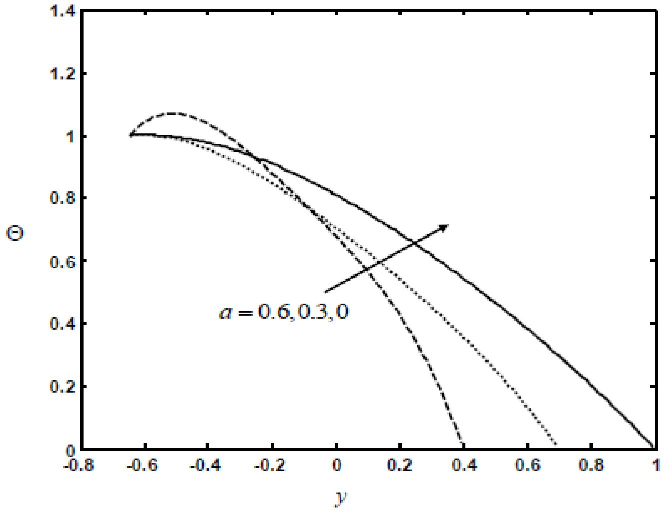

Fig. 6: The variation of temperature Θ with y for different values of upper wave amplitude a with $\theta = \dfrac{\pi}{4}$, $b = 0.5$, $x = 0.5$, $\Pr = 0.7$, $Ec = 0.1$, $N = 1$ and $\overline{Q} = -1$.

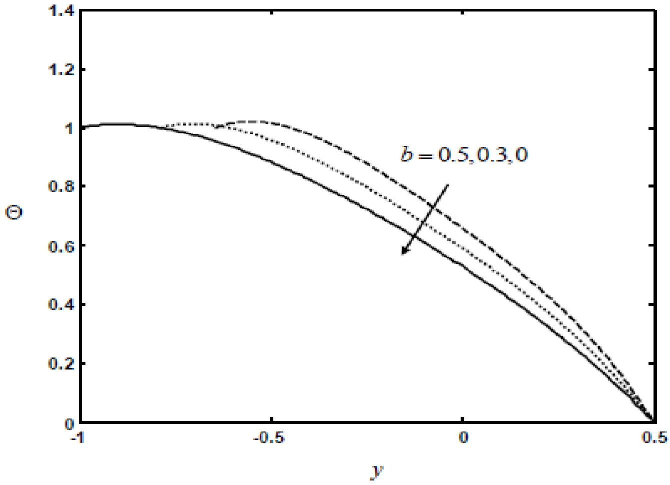

Fig. 7: The variation of temperature Θ with y for different values of lower wave amplitude b with $a = 0.5$, $\theta = \dfrac{\pi}{4}$, $x = 0.5$, $\Pr = 0.7$, $Ec = 0.1$, $N = 1$ and $\overline{Q} = -1$.

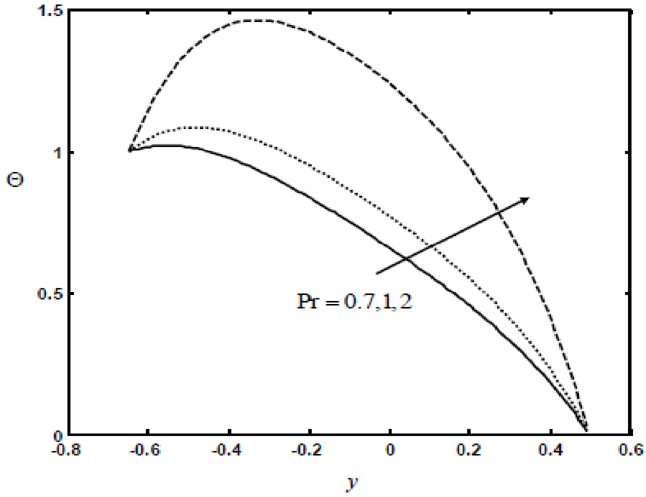

Fig. 8: The variation of temperature Θ with y for different values of Prandtl number $\Pr$ with $a = 0.5$, $b = 0.5$, $x = 0.5$, $\theta = \dfrac{\pi}{4}$, $Ec = 0.1$, $N = 1$ and $\overline{Q} = -1$.

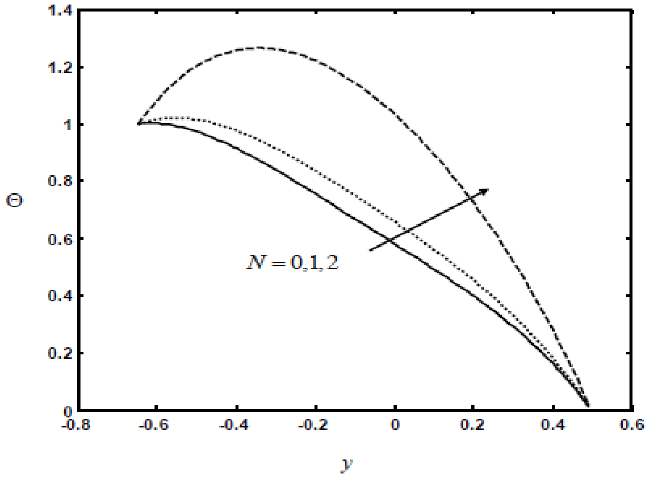

Fig. 9: The variation of temperature Θ with y for different values of radiation parameter N with $a = 0.5$, $b = 0.5$, $x = 0.5$, $\Pr = 0.7$, $Ec = 0.1$, $\theta = \frac{\pi}{4}$ and $\overline{Q} = -1$.

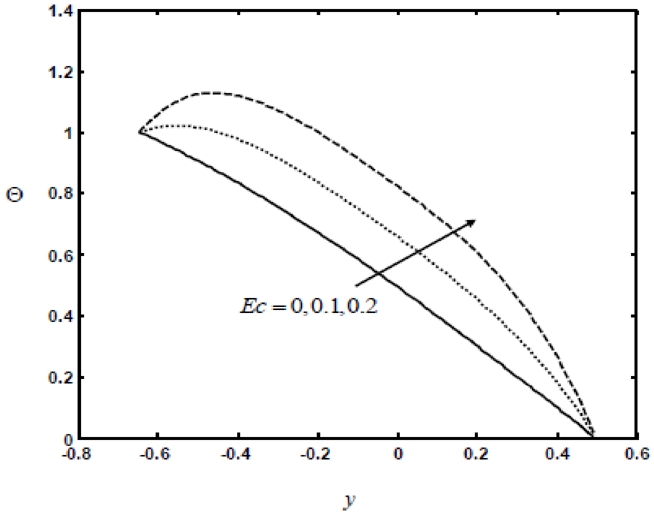

Fig. 10: The variation of temperature Θ with y for different values of Eckert number Ec with $a = 0.5$, $b = 0.5$, $x = 0.5$, $\Pr = 0.7$, $\theta = \frac{\pi}{4}$, $N = 1$ and $\overline{Q} = -1$.

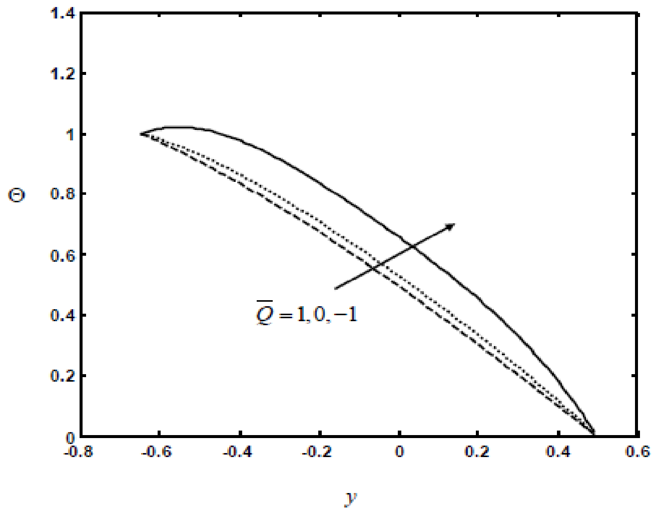

Fig. 11: The variation of temperature Θ with y for different values of time averaged flux $\overline{Q}$ with $a = 0.5$, $b = 0.5$, $x = 0.5$, $\Pr = 0.7$, $Ec = 0.1$, $N = 1$ and $\theta = \frac{\pi}{4}$.

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