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NOTES ON BIPOLAR VALUED FUZZY RW-CLOSED AND BIPOLAR VALUED FUZZY RW-OPEN SETS IN BIPOLAR VALUED FUZZY TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce some properties of bipolar valued fuzzy rw-closed and bipolar valued fuzzy rw-open sets in bipolar valued fuzzy topological spaces and study some important properties on these.

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Key Words: *Bipolar valued fuzzy subset, bipolar valued fuzzy topological spaces, bipolar valued fuzzy rw-closed set, bipolar valued fuzzy rw-open set.*

INTRODUCTION

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [20] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers are useful to us to work on this paper. C.L.Chang [4] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Tapas kumar mondal and S.K.Samanta [17] have introduced the topology of interval valued fuzzy sets. We are motivated to introduce the concept of bipolar valued fuzzy rw-closed sets and bipolar valued fuzzy rw-open sets in bipolar valued fuzzy topological spaces and established some results.

1. PRELIMINARIES

1.1 Definition: A bipolar valued fuzzy set A in X is defined as an object of the form $A = \{< x, A^+(x), A^-(x) >/ x\in X\}$, where $A^+ : X\to [0, 1]$ and $A^- : X\to [-1, 0]$. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A. If $A^+(x) \neq 0$ and $A^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for A and if $A^+(x) = 0$ and $A^-(x) \neq 0$, it is the situation that x does not satisfy the property of A, but somewhat satisfies the counter property of A. It is possible for an element x to be such that $A^+(x) \neq 0$ and $A^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X.

1.2 Example: $A = \{< a, 0.6, -0.4 >, < b, 0.8, -0.3 >, < c, 0.5, -0.5 >\}$ is a bipolar valued fuzzy subset of $X =\{a, b, c\}$.

1.3 Definition: Let A and B be two bipolar valued fuzzy subsets of a set X. We define the following relations and operations:

(i) $A \subset B$ if and only if $A^+(x) \leq B^+(x)$ and $A^-(x) \geq B^-(x)$, for all $x\in X$.
(ii) $A = B$ if and only if $A^+(x) = B^+(x)$ and $A^-(x) = B^-(x)$, for all $x\in X$.
(iii) $A\cup B = \{< x, \max (A^+(x), B^+(x)), \min (A^-(x), B^-(x)) > / x\in X\}$.
(iv) $A\cap B = \{< x, \min (A^+(x), B^+(x)), \max (A^-(x), B^-(x)) > / x\in X\}$.
(v) (v) $A^C = \{< x, 1 - A^+(x), -1 - A^-(x) > / x\in X\}$.

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1.4 Example: Let X = {a, b, c} be a set. Let A = {< a, 0.5, −0.4 >, < b, 0.1, −0.6 >, < c, 0.6, −0.2 >} and B = {< a, 0.4, −0.4 >, < b, 0.6, −0.3 >, < c, 0.8, −0.5 >} be any two bipolar valued fuzzy subsets of X. Then

(i) $A \cup B$ = {< a, 0.5, −0.4 >, < b, 0.6, −0.6 >, < c, 0.8, −0.5 >}.

(ii) $A \cap B$ = {< a, 0.4, −0.4 >, < b, 0.1, −0.3 >, < c, 0.6, −0.2 >}.

(iii) A^C = {< a, 0.5, −0.6 >, < b, 0.9, −0.4 >, < c, 0.4, −0.8 >}.

1.5 Definition: Let X be any set. Let 0_X and 1_X be the bipolar valued fuzzy sets on X defined as follows 0_X = {< x, 0, 0 >/ for all $x \in X$} and 1_X = {< x, 1, − 1 >/ for all $x \in X$}.

1.6 Definition: Let X be a set and $\Im$ be a family of bipolar valued fuzzy subsets of X. The family $\Im$ is called bipolar valued fuzzy topology on X if and only if $\Im$ satisfies the following axioms

(i) $0_X, 1_X \in \Im$,

(ii) If $\{A_i ; i \in I\} \subseteq \Im$, then $\bigcup_{i \in I} A_i \in \Im$,

(iii) If $A_1, A_2, A_3 \ldots\ldots A_n \in \Im$, then $\bigcap_{i=1}^{i=n} A_i \in \Im$.

The pair (X, $\Im$) is called bipolar valued fuzzy topological space. The members of $\Im$ are called bipolar valued fuzzy open sets in X. Bipolar valued fuzzy set A in X is said to be bipolar valued fuzzy closed set in X if and only if A^C is bipolar valued fuzzy open set in X.

1.7 Definition: Let (X,$\Im$) be bipolar valued fuzzy topological space and A be bipolar valued fuzzy set in X. Then $\cap\{B : B^c \in \Im \text{ and } B \supseteq A\}$ is called bipolar valued fuzzy closure of A and is denoted by bcl(A).

1.8 Theorem: Let A and B be two bipolar valued fuzzy sets in bipolar valued fuzzy topological space (X, $\Im$). Then the following results are trivial,

(i) bcl(A) is bipolar valued fuzzy closed set in X.

(ii) bcl(A) is the smallest bipolar valued fuzzy closed set containing A.

(iii) A is bipolar valued fuzzy closed if and only if A = bcl(A).

(iv) bcl(0_X) = 0_X, 0_X is the empty bipolar valued fuzzy set.

(v) bcl(bcl(A)) = bcl(A). (vi) bcl($A \cup B$) = bcl(A) $\cup$ bcl(B). (vii) bcl(A) $\cap$ bcl(B) $\supseteq$ bcl($A \cap B$).

1.9 Definition: Let (X, $\Im$) be bipolar valued fuzzy topological space and A be a bipolar valued fuzzy set in X. Then $\cup\{B: B \in \Im \text{ and } B \subseteq A\}$ is called bipolar valued fuzzy interior of A and is denoted by bint(A).

1.10 Theorem: Let (X, $\Im$) be bipolar valued fuzzy topological space, A and B be two bipolar valued fuzzy sets in X. The following results hold good,

(i) bint (A) is bipolar valued fuzzy open set in X.

(ii) bint(A) is the largest bipolar valued fuzzy open set in X which is contained in A.

(iii) A is bipolar valued fuzzy open set if and only if A = bint (A). (iv) $A \subseteq B$ implies bint (A) $\subseteq$ bint(B).

(iv) bint (bint (A)) = bint(A).

(v) bint($A \cap B$) = bint(A) $\cap$ bint(B).

(vi) bint(A) $\cup$ bint(B) $\subseteq$ bint($A \cup B$).

(vii) bint(A^C) = $(\text{bcl}(A))^C$. (ix) bcl(A^C) = $(\text{bint}(A))^C$.

1.11 Definition: Let (X, $\Im$) be bipolar valued fuzzy topological space and A be bipolar valued fuzzy set in X. Then A is said to be (i) bipolar valued fuzzy semi open if and only if there exists bipolar valued fuzzy open set V in X such that $V \subseteq A \subseteq \text{bcl}(V)$. (ii) bipolar valued fuzzy semiclosed if and only if there exists bipolar valued fuzzy closed set V in X such that $\text{bint}(V) \subseteq A \subseteq V$. (iii) bipolar valued fuzzy regular open set of X if bint(bcl(A)) = A. (iv) bipolar valued fuzzy regular closed set of X if bcl(bint(A)) = A. (v) bipolar valued fuzzy regular semi open set of X if there exists bipolar valued fuzzy regular open set V in X such that $V \subseteq A \subseteq \text{bcl}(V)$. We denote the class of bipolar valued fuzzy regular semi open sets in bipolar valued fuzzy topological space X by BVFRSO(X). (vi) bipolar valued fuzzy generalized closed if bcl(A) $\subseteq$ V whenever $A \subseteq V$ and V is bipolar valued fuzzy open set and A is bipolar valued fuzzy generalized open if A^C is bipolar valued fuzzy generalized closed.

1.12Theorem: The following are equivalent:

(i) A is bipolar valued fuzzy semi closed set. (ii) A^C is bipolar valued fuzzy semi open set.

(ii) bint(bcl (A)) $\subseteq$ A. (iv) bcl(bint (A^C)) $\supseteq$ A.

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1.13 Theorem: Any union of bipolar valued fuzzy semi open sets is bipolar valued fuzzy semi open set and any intersection of bipolar valued fuzzy semi closed sets is bipolar valued fuzzy semi closed.

Remark:
(i) Every bipolar valued fuzzy open set is bipolar valued fuzzy semi open but not conversely.
(ii) Every bipolar valued fuzzy closed set is bipolar valued fuzzy semi-closed set but not conversely.
(iii) The closure of bipolar valued fuzzy open set is bipolar valued fuzzy semi open set.
(iv) The interior of bipolar valued fuzzy closed set is bipolar valued fuzzy semi-closed set.

1.14 Theorem: Bipolar valued fuzzy set A of bipolar valued fuzzy topological space X is bipolar valued fuzzy regular open if and only if A^C is bipolar valued fuzzy regular closed set.

Remark:
(i) Every bipolar valued fuzzy regular open set is bipolar valued fuzzy open set but not conversely.
(ii) Every bipolar valued fuzzy regular closed set is bipolar valued fuzzy closed set but not conversely.

1.15 Theorem:
(i) The closure of bipolar valued fuzzy open set is bipolar valued fuzzy regular closed.
(ii) The interior of bipolar valued fuzzy closed set is bipolar valued fuzzy regular open set.

1.16 Theorem:
(i) Every bipolar valued fuzzy regular semi open set is bipolar valued fuzzy semi open set but not conversely.
(ii) Every bipolar valued fuzzy regular closed set is bipolar valued fuzzy regular semi open set but not conversely.
(iii) Every bipolar valued fuzzy regular open set is bipolar valued fuzzy regular semi open set but not conversely.

1.17 Theorem: Let $(X, \Im)$ be bipolar valued fuzzy topological space and A be bipolar valued fuzzy set in X. Then the following conditions are equivalent:
(i) A is bipolar valued fuzzy regular semi open.
(ii) A is both bipolar valued fuzzy semi open and bipolar valued fuzzy semi-closed.
(iii) A^c is bipolar valued fuzzy regular semi open in X.

1.18 Definition: Bipolar valued fuzzy set A of bipolar valued fuzzy topological space$(X, \Im)$ is called (i) bipolar valued fuzzy g-closed if $bcl(A) \subseteq V$ whenever $A \subseteq V$ and V is bipolar valued fuzzy open set in X. (ii) bipolar valued fuzzy g-open if its complement A^C is bipolar valued fuzzy g-closed set in X. (iii) bipolar valued fuzzy rg-closed if $bcl(A) \subseteq V$ whenever $A \subseteq V$ and V is bipolar valued fuzzy regular open set in X. (iv) bipolar valued fuzzy rg-open if its complement A^C is bipolar valued fuzzy rg-closed set in X. (v) bipolar valued fuzzy w-closed if $bcl(A) \subseteq V$ whenever $A \subseteq V$ and V is bipolar valued fuzzy semi open set in X. (vi) bipolar valued fuzzy w-open if its complement A^c is bipolar valued fuzzy w- closed set in X. (vii)bipolar valued fuzzy gpr-closed if $pbcl(A) \subseteq V$ whenever $A \subseteq V$ and V is bipolar valued fuzzy regular open set in X. (viii) bipolar valued fuzzy gpr -open if its complement A^c is bipolar valued fuzzy gpr-closed set in X.

1.19 Theorem: If A and B are bipolar valued fuzzy rw-closed sets in bipolar valued fuzzy topological space X, then the intersection of A and B need not be bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space X.

1.20 Theorem: If A and B are bipolar valued fuzzy rw-closed sets in bipolar valued fuzzy topological space X, then union of A and B is bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space X.

1.21 Theorem: Every bipolar valued fuzzy closed set is bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space X.

2. SOME PROPERTIES

2.1 Definition: Let $(X, \Im)$ be bipolar valued fuzzy topological space. Bipolar valued fuzzy set A of X is called bipolar valued fuzzy regular w-closed (briefly, bipolar valued fuzzy rw-closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is bipolar valued fuzzy regular semi open in bipolar valued fuzzy topological space X.

Note: We denote the family of all bipolar valued fuzzy regular w-closed sets in bipolar valued fuzzy topological space X by BVFRWC(X).

2.2 Definition: Bipolar valued fuzzy set A of bipolar valued fuzzy topological space X is called bipolar valued fuzzy regular w-open (briefly, bipolar valued fuzzy rw-open) set if its complement A^C is bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space X.

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Note: We denote the family of all bipolar valued fuzzy rw-open sets in bipolar valued fuzzy topological space X by BVFRWO(X).

2.3 Theorem: If bipolar valued fuzzy subset A of bipolar valued fuzzy topological space X is both bipolar valued fuzzy regular open and bipolar valued fuzzy rw-closed, then A is bipolar valued fuzzy regular closed set in bipolar valued fuzzy topological space X.

Proof: Suppose bipolar valued fuzzy subset A of bipolar valued fuzzy topological space X is both bipolar valued fuzzy regular open and bipolar valued fuzzy rw-closed. As every bipolar valued fuzzy regular open set is bipolar valued fuzzy regular semi open set and $A\subseteq A$, we have $bcl(A) \subseteq A$. Also $A\subseteq bcl(A)$. Therefore bcl(A) = A. That is A is bipolar valued fuzzy closed. Since A is bipolar valued fuzzy regular open, int (A) = A. Now bcl(bint(A)) = bcl(A) = A. Therefore A is a bipolar valued fuzzy regular closed set in bipolar valued fuzzy topological space X.

2.4 Theorem: If bipolar valued fuzzy subset A of bipolar valued fuzzy topological space X is both bipolar valued fuzzy regular semi open and bipolar valued fuzzy rw-closed, then A is bipolar valued fuzzy closed set in bipolar valued fuzzy topological space X.

Proof: Suppose bipolar valued fuzzy subset A of bipolar valued fuzzy topological space X is both bipolar valued fuzzy regular semi open and bipolar valued fuzzy rw-closed. Now $A\subseteq A$, we have $bcl(A) \subseteq A$. Also $A \subseteq bcl(A)$. Therefore bcl(A) = A and hence A is bipolar valued fuzzy closed set in bipolar valued fuzzy topological space X.

2.5 Corollary: If A is bipolar valued fuzzy regular semi open and bipolar valued fuzzy rw-closed in bipolar valued fuzzy topological space $(X, \mathfrak{I})$. Suppose that F is bipolar valued fuzzy closed in X then $A\cap F$ is bipolar valued fuzzy rw-closed in X.

Proof: Suppose A is both bipolar valued fuzzy regular semi open and bipolar valued fuzzy rw-closed set in X and F is bipolar valued fuzzy closed in X. By Theorem 2.4**,** A is bipolar valued fuzzy closed in X. So $A\cap F$ is bipolar valued fuzzy closed in X. Hence $A\cap F$ is bipolar valued fuzzy rw-closed in X.

2.6 Theorem: If bipolar valued fuzzy subset A of bipolar valued fuzzy topological space X is both bipolar valued fuzzy open and bipolar valued fuzzy generalized closed, then A is bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space X.

Proof: Suppose bipolar valued fuzzy subset A of bipolar valued fuzzy topological space X is both bipolar valued fuzzy open and bipolar valued fuzzy generalized closed. Now $A\subseteq A$, by hypothesis we have $bcl(A) \subseteq A$. Also $A\subseteq bcl(A)$. Therefore bcl(A) = A. That is A is bipolar valued fuzzy closed set and hence A is bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space X, as every bipolar valued fuzzy closed set is bipolar valued fuzzy rw-closed set.

2.7 Theorem: If bipolar valued fuzzy subset C is both bipolar valued fuzzy regular open and bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space X, then C need not be bipolar valued fuzzy generalized closed set in bipolar valued fuzzy topological space X.

Proof: Consider the example, let X = {1, 2, 3} and the bipolar valued fuzzy sets A, B, C be defined as A = {< 1, 1, -1 >, < 2, 0, 0 >, < 3, 0, 0 > }, B = {< 1, 0, 0 >, < 2, 1, - 1 >, < 3, 0, 0 >} and C = {< 1, 1, -1 >, < 2, 1, -1 >, < 3, 0, 0 >}. Consider $\mathfrak{I} = \{0_X, 1_X, A, B, C\}$. Then $(X, \mathfrak{I})$ is bipolar valued fuzzy topological space. In this bipolar valued fuzzy topological space X, C is both bipolar valued fuzzy open and bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space X but it is not bipolar valued fuzzy generalized closed.

2.8 Theorem: Let A be bipolar valued fuzzy rw-closed set of bipolar valued fuzzy topological space X and suppose $A \subseteq B \subseteq bcl(A)$. Then B is also bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space X.

Proof: Let $A \subseteq B \subseteq bcl(A)$ and A be bipolar valued fuzzy rw-closed set of bipolar valued fuzzy topological space X. Let E be any bipolar valued fuzzy regular semi open set such that $B \subseteq E$. Then $A \subseteq E$ and A is bipolar valued fuzzy rw-closed, we have $bcl(A) \subseteq E$. But $bcl(B) \subseteq bcl(A)$ and thus $bcl(B) \subseteq E$. Hence B is bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space X.

2.9 Theorem: In bipolar valued fuzzy topological space X if BVFRSO(X) = $\{0_X, 1_X\}$, where BVFRSO(X) is the family of all bipolar valued fuzzy regular semi open sets then every bipolar valued fuzzy subset of X is bipolar valued fuzzy rw-closed.

Proof: Let X be bipolar valued fuzzy topological space and BVFRSO(X) = $\{0_X, 1_X\}$. Let A be any bipolar valued fuzzy subset of X. Suppose A = 0_X. Then 0_X is bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space X. Suppose A $\neq 0_X$. Then1_X is the only bipolar valued fuzzy regular semi open set containing A and so bcl(A) $\subseteq 1_X$. Hence A is bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space X.

Remark: The converse of the above Theorem 2.9 need not be true in general.

2.10 Example: Let X = {1, 2, 3} and the bipolar valued fuzzy sets A, B be defined as A= {< 1, 1, -1>, < 2, 0, 0 >, < 3, 0, 0 >} and B = {< 1, 0, 0 >, < 2, 1, -1 >, < 3, 1, -1 >}. Consider $\mathfrak{I} = \{0_X, 1_X, A, B\}$. Then (X, $\mathfrak{I}$) is bipolar valued fuzzy topological space. In this bipolar valued fuzzy topological space X, every bipolar valued fuzzy subset of X is bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space X, but BVFRSO = $\{0_X, 1_X, A, B\}$.

2.11 Theorem: If A is bipolar valued fuzzy rw-closed set of bipolar valued fuzzy topological space X and bcl(A) $\cap$ $(\text{bcl(A)})^C = 0_X$, then bcl(A)–A does not contain any non-zero bipolar valued fuzzy regular semi open set in bipolar valued fuzzy topological space X.

Proof: Suppose A is bipolar valued fuzzy rw-closed set of bipolar valued fuzzy topological space X and bcl(A)$\cap$$(\text{bcl(A)})^C = 0_X$. We prove the result by contradiction. Let B be bipolar valued fuzzy regular semi open set such that bcl(A)–A $\supseteq$ B and B $\neq 0_X$. Now B $\subseteq$ bcl(A)– A, i.e B $\subseteq 1_X$ –A which implies A $\subseteq 1_X$ – B. Since B is bipolar valued fuzzy regular semi open set, by Theorem 1.17**,** 1_X –B is also bipolar valued fuzzy regular semi open set in bipolar valued fuzzy topological space X. Since A is bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space X, by definition bcl(A) $\subseteq 1_X$ – B. So B $\subseteq 1_X$– bcl(A). Therefore B $\subseteq$ bcl(A) $\cap$ (1_X– bcl(A))= 0_X, by hypothesis. This shows that B = 0_X which is a contradiction. Hence bcl(A) –A does not contain any non-zero bipolar valued fuzzy regular semi open set in bipolar valued fuzzy topological space X.

2.12 Corollary: If A is bipolar valued fuzzy rw-closed set of bipolar valued fuzzy topological space X and bcl(A) $\cap$ (1_X – bcl(A)) = 0_X, then bcl(A) –A does not contain any non-zero bipolar valued fuzzy regular open set in bipolar valued fuzzy topological space X.

Proof: Follows form the Theorem 2.11 and the fact that every bipolar valued fuzzy regular open set is bipolar valued fuzzy regular semi open set in bipolar valued fuzzy topological space X.

2.13 Corollary: If A is bipolar valued fuzzy rw-closed set of bipolar valued fuzzy topological space X and bcl(A) $\cap$ (1_X – bcl(A)) = 0_X, then bcl(A) –A does not contain any non-zero bipolar valued fuzzy regular closed set in bipolar valued fuzzy topological space X.

Proof: Follows form the Theorem 2.11 and the fact that every bipolar valued fuzzy regular closed set is bipolar valued fuzzy regular semi open set in bipolar valued fuzzy topological space X.

2.14 Theorem: Let A be bipolar valued fuzzy rw-closed set of bipolar valued fuzzy topological space X and bcl(A)$\cap$ (1_X – bcl(A)) = $0_{X.}$ Then A is bipolar valued fuzzy closed set if and only if bcl(A)–A is bipolar valued fuzzy regular semi open set in bipolar valued fuzzy topological space X.

Proof: Suppose A is bipolar valued fuzzy closed set in bipolar valued fuzzy topological space X. Then bcl(A) = A and so bcl(A) –A = 0_X, which is bipolar valued fuzzy regular semi open set in bipolar valued fuzzy topological space X.

Conversely, suppose bcl(A) –A is bipolar valued fuzzy regular semi open set in bipolar valued fuzzy topological space X. Since A is bipolar valued fuzzy rw-closed, by Theorem 2.11 bcl(A) –A does not contain any-non zero bipolar valued fuzzy regular open set in bipolar valued fuzzy topological space X. Then bcl(A) –A= 0_X. That is bcl(A) = A and hence A is a bipolar valued fuzzy closed set in bipolar valued fuzzy topological space X.

2.15 Theorem: If bipolar valued fuzzy subset A of bipolar valued fuzzy topological space X is bipolar valued fuzzy open, then it is bipolar valued fuzzy rw-open but not conversely.

Proof: Let A be bipolar valued fuzzy open set of bipolar valued fuzzy topological space X. Then A^C is bipolar valued fuzzy closed. Now by Theorem 1.21, A^C is bipolar valued fuzzy rw-closed. Therefore A is bipolar valued fuzzy rw-open set in bipolar valued fuzzy topological space X.

Remark: The converse of the above Theorem need not be true in general.

2.16 Example: Let X = {1, 2, 3}. Define bipolar valued fuzzy subset A in X by A= {< 1, 1, - 1 >, < 2, 1, - 1 >, < 3, 0, 0 >}. Let $\Im = \{0_X, 1_X, A\}$. Then (X, $\Im$) is bipolar valued fuzzy topological space. Define bipolar valued fuzzy set B in X by B= {<1, 0, 0 >, < 2, 1, -1 >, < 3, 0, 0 >}. Then B is bipolar valued fuzzy rw-open set but it is not bipolar valued fuzzy open set in bipolar valued fuzzy topological space X.

Remark: It has been proved that every bipolar valued fuzzy regular open set is bipolar valued fuzzy open set but not conversely. By Theorem 2.15, every bipolar valued fuzzy open set is bipolar valued fuzzy rw-open set but not conversely and hence every bipolar valued fuzzy regular open set is bipolar valued fuzzy rw-open set but not conversely.

2.17 Theorem: Bipolar valued fuzzy subset A of bipolar valued fuzzy topological space X is bipolar valued fuzzy rw-open if and only if D $\subseteq$ bint(A), whenever D $\subseteq$ A and D is bipolar valued fuzzy regular semi open set in bipolar valued fuzzy topological space X.

Proof: Suppose that D $\subseteq$ bint(A), whenever D $\subseteq$ A and D is bipolar valued fuzzy regular semi open set in bipolar valued fuzzy topological space X. To prove that A is bipolar valued fuzzy rw-open in bipolar valued fuzzy topological space X. Let $A^C \subseteq$ B and B is any bipolar valued fuzzy regular semi open set in bipolar valued fuzzy topological space X. Then $B^C \subseteq$ A. By Theorem 1.17**,** B^C is also bipolar valued fuzzy regular semi open set in bipolar valued fuzzy topological space X. By hypothesis, $B^C \subseteq$ bint(A) which implies $(\text{bint}(A))^C \subseteq$ B. That is bcl(A^C) $\subseteq$ B, since bcl(A^C) = $(\text{bint}(A))^C$. Thus A^C is bipolar valued fuzzy rw-closed and hence A is bipolar valued fuzzy rw-open in bipolar valued fuzzy topological space X.

Conversely, suppose that A is bipolar valued fuzzy rw-open. Let B $\subseteq$ A and B is any bipolar valued fuzzy regular semi open in bipolar valued fuzzy topological space X. Then $A^C \subseteq B^C$. By Theorem 1.9, B^C is also bipolar valued fuzzy regular semi open. Since A^C is bipolar valued fuzzy rw-closed, we have bcl(A^C) $\subseteq B^C$ and so B $\subseteq$ bint(A), since bcl(A^C) = $(\text{bint}(A))^C$.

2.18 Theorem: If A and B are bipolar valued fuzzy rw-open sets in bipolar valued fuzzy topological space X, then A$\cap$B is also bipolar valued fuzzy rw-open set in bipolar valued fuzzy topological space X.

Proof: Let A and B be two bipolar valued fuzzy rw-open sets in bipolar valued fuzzy topological space X. Then A^C and B^C are bipolar valued fuzzy rw-closed sets in bipolar valued fuzzy topological space X. By Theorem 1.20, $A^C \cup B^C$ is also bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space X. That is $(A^C \cup B^C) = (A\cap B)^C$ is bipolar valued fuzzy rw-closed set in X. Therefore A$\cap$B is also bipolar valued fuzzy rw-open set in bipolar valued fuzzy topological space X.

2.19 Remark: The union of any two bipolar valued fuzzy rw-open sets in bipolar valued fuzzy topological space X is generally not bipolar valued fuzzy rw-open set in bipolar valued fuzzy topological space X.

Proof: Consider the bipolar valued fuzzy topological space (X, $\Im$) defined as in previous example**.** In this bipolar valued fuzzy topological space X, the bipolar valued fuzzy sets D_1, D_2 are defined by D_1 = {< 1, 1, - 1 >, < 2, 0, 0 >, < 3, 0, 0 >} and D_2 = {< 1, 0, 0 >, < 2, 0, 0 >, < 3, 1, - 1 >}. Then D_1 and D_2 are the bipolar valued fuzzy rw-open sets in bipolar valued fuzzy topological space X. Let E = $D_1 \cup D_2$. Then E = {< 1, 1, - 1 >, < 2, 0, 0 >, < 3, 1, - 1 >}. Then E = $D_1 \cup D_2$ is not bipolar valued fuzzy rw-open set in bipolar valued fuzzy topological space X.

2.20 Theorem: If bint(A) $\subseteq$ B $\subseteq$ A and A is bipolar valued fuzzy rw-open set in bipolar valued fuzzy topological space X, then B is also bipolar valued fuzzy rw-open set in bipolar valued fuzzy topological space X.

Proof: Suppose bint(A) $\subseteq$ B $\subseteq$ A and A is bipolar valued fuzzy rw-open set in a bipolar valued fuzzy topological space X. To prove that B is bipolar valued fuzzy rw-open set in bipolar valued fuzzy topological space X. Let F be any bipolar valued fuzzy regular semi open set in bipolar valued fuzzy topological space X such that F $\subseteq$ B. Now F $\subseteq$ B $\subseteq$ A. That is F $\subseteq$ A. Since A is bipolar valued fuzzy rw-open set of bipolar valued fuzzy topological space X, F $\subseteq$ bint(A), by Theorem 2.17. By hypothesis bint(A) $\subseteq$ B. Then bint(bint(A)) $\subseteq$ bint(B). That is bint(A) $\subseteq$ bint(B). Then F $\subseteq$ bint(B). Again by Theorem 2.17, B is bipolar valued fuzzy rw-open set in bipolar valued fuzzy topological space X.

2.21 corollary: Let A and B be two bipolar valued fuzzy subsets of bipolar valued fuzzy topological space X. If A is bipolar valued fuzzy rw-open set and B $\supseteq$ bint(A), then A$\cap$B is a bipolar valued fuzzy rw-open set in bipolar valued fuzzy topological space X.

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Proof: Let A be bipolar valued fuzzy rw-open set of bipolar valued fuzzy topological space X and B ⊇ bint(A). That is bint(A) ⊆ (A∩B). Also bint(A) ⊆ (A∩B) ⊆ A and A is a bipolar valued fuzzy rw-open set. By Theorem 2.20, A∩B is also bipolar valued fuzzy rw-open set in bipolar valued fuzzy topological space X.

2.22 Theorem: If bipolar valued fuzzy subset A of bipolar valued fuzzy topological space X is bipolar valued fuzzy rw-closed and bcl(A) ∩ (1_X – bcl(A)) = 0_X, then bcl(A) –A is bipolar valued fuzzy rw-open set in bipolar valued fuzzy topological space X.

Proof: Let A be bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space X and bcl(A) ∩ (1_X – bcl(A)) = 0_X. Let B be any bipolar valued fuzzy regular semi open set of bipolar valued fuzzy topological space X such that B ⊆ (bcl(A) –A). Then by Theorem 2.11, bcl(A) –A does not contain any non-zero bipolar valued fuzzy regular semi open set and so B = 0_X. Therefore B ⊆ bint(bcl(A) –A). By Theorem 1.19, bcl(A) –A is bipolar valued fuzzy rw-open.

Remark: Every bipolar valued fuzzy w-open set is bipolar valued fuzzy rw-open but its converse may not be true.

2.23 Example: Let X = {1, 2} and ℑ = {1_X, 0_X, A} be bipolar valued fuzzy topology on X, where A = {< 1, 0.7, - 0.7 >, < 2, 0.6, - 0.6 >}. Then the bipolar valued fuzzy set B = {< 1, 0.2, - 0.2 >, < 2, 0.1, - 0.1 >} is bipolar valued fuzzy rw-open in (X, ℑ) but it is not bipolar valued fuzzy w-open in (X, ℑ).

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