

**A COMMON FIXED POINT THEOREM
USING IMPLICIT RELATION AND PROPERTY (Clrg) IN FUZZY METRIC SPACES**

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ABSTRACT

In this note we generalize the results of S.Kumar and B.Fisher [S.Kumar and B.Fisher, A common fixed point theorem in fuzzy metric space using property (E.A.) and implicit relation, Thai J. of Mathematics, Vol 8(2010) Number: 439-446] and we generalize the results by using weakly compatible mappings along with the property (CLRg). We also demonstrate an example in support our main result.

Key words: fuzzy metric space, common fixed point, weakly compatible maps, implicit relation and property CLRg.

Mathematics Subject Classifications: 54H25, 47H10.

1. INTRODUCTION AND BRIEF HISTORICAL BACKGROUND

In the year 1965 Lotfi A Zadeh [1] introduced the concept of fuzzy sets. Kramosil and Michalek [2] introduced the notion of a fuzzy metric space (in short we say FM-spaces) by generalizing the concept of the probabilistic metric space to the fuzzy situation. Consequently, many authors viz George and Veeramani [3], Grabiec [4], Subrahmanyam [5] and others generalized some metric fixed point results to FM-spaces. George and Veeramani [3] modified the concept of fuzzy metric spaces introduced by Kramosil and Michalek [2]. Mishra *et al.* [6] introduced the concept of compatible mappings in fuzzy metric spaces further Singh and Jain [7] generalized the results of Mishra *et al.* [6]. Jungck and Rhoades [8] introduced the notion of weak compatibility in metric spaces which was further generalized by Singh and Jain [7] in FM-spaces.

In 2002, Aamri and El- Moutawakil [9] defined the notion of property (E-A) in metric spaces for self mappings which contained the class of non compatible mappings in metric spaces. Pant and Pant [10] proved some common fixed points for a pair of maps under the notion of property (E.A.) and non-compatible maps.

Popa [11, 12] introduced the idea of implicit function to prove a common fixed point theorem in metric spaces. Jain [13] further extended the result of Popa [11, 12] in fuzzy metric spaces. Recently, many authors have used implicit relations as a tool for finding common fixed point of contraction maps (see, [14-27]).

Sharma and Bamboria [28] defined a property (S-B) in fuzzy metric spaces for self maps and obtained some common fixed point theorems in IFMS for such mappings under strict contractive conditions, further Sharma and Sharma [29,30] proved some common fixed point theorems in IFMS by using this concept. The class of (S-B) maps contains the class of non compatible maps.

Most recently, Sintunavarat and Kumam [31] defined the notion of “common limit in the range” property or CLR property in fuzzy metric spaces. It is observed that the notion of CLR property never requires the condition of the closedness of the subspace while (E-A), common (E-A) and (S-B) property require this condition for the existence of the fixed point and hence, now a days, authors are giving much attention to this property for generalizing the results present in the literature. Works noted in the references [32–38] are some examples.

In [39], Altun and Turkoglu proved two common fixed point theorems on complete FM-space with an implicit relation, in which they proved common fixed point theorems for continuous compatible maps of type (α) or (β) .

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In [40], S.Kumar and B.Fisher proved a common fixed point theorem by removing the assumption of continuity, relaxing compatibility to weak compatibility and replacing the completeness of the space with a set of four alternative conditions for functions satisfying an implicit relation in FM-space. They generalized the result of Altun and Turkoglu [39].

In our main result we generalize the results of S.Kumar and B.Fisher [40] by using the property (CLRg) and relaxing (EA) and many conditions taken by Kumar and Fisher to prove the result.

2. PRELIMINARIES

For proving main result we need the following definitions:

Definition 2.1 [1]: Let X be any set. A fuzzy set A in X is a function with domain X and values in $[0, 1]$.

Definition 2.2 [41]: A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a continuous t-norm if $([0, 1], *)$ is an abelian topological monoid with the unit 1 such that $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Note that among a number of possible choices for $*$, $a*b = \min \{a, b\}$ or simply “ $*$ = min” is the strongest possible universal t-norm (see [41]).

Definition 2.3 [2]: The triplet $(X, M, *)$ is a FM-space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times [0, \infty)$ satisfying the following conditions for all $x, y, z \in X$ and $t, s > 0$,

- (FM1) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$;
- (FM2) $M(x, y, 0) = 0$;
- (FM3) $M(x, y, t) = M(y, x, t)$;
- (FM4) $M(x, y, t) * M(y, z, t) \leq M(x, z, t + s)$;
- (FM5) $M(x, y, \cdot): [0, \infty) \rightarrow [0, 1]$ is left continuous.

Throughout this paper, we consider M to be a fuzzy metric space with condition:

(FM-6) $\lim_{t \rightarrow \infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$.

In the following example (see [3]), we know that every metric induces a fuzzy metric:

Example 2.4: Let (X, d) be a metric space. Define $a*b = ab$ (or $a*b = \min \{a, b\}$) for all $x, y \in X$ and

$$t > 0, M(x, y, z) = \frac{t}{t + |x - y|}$$

Then $(X, M, *)$ is a FM-space and the fuzzy metric M induced by the metric d is often referred to as the standard fuzzy metric.

Definition 2.5 [6]: Let A and B maps from a FM-space $(X, M, *)$ into itself. The maps A and B are said to be compatible (or asymptotically commuting), if for all t , $\lim_{n \rightarrow \infty} M(AB x_n, BA x_n, t) = 1$, whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n \rightarrow \infty} A x_n = \lim_{n \rightarrow \infty} B x_n = z \text{ for some } z \in X.$$

From the above definition it is inferred that A and B are noncompatible maps from a FM-space $(X, M, *)$ into itself if $\lim_{n \rightarrow \infty} A x_n = \lim_{n \rightarrow \infty} B x_n = z$ for some $z \in X$, but either $\lim_{n \rightarrow \infty} M(AB x_n, BA x_n, t) \neq 1$ or the limit does not exist.

Definition 2.6 [7]: Let A and B be maps from a FM-space $(X, M, *)$ into itself. The maps are said to be weakly compatible if they commute at their coincidence points, that is, $Az = Bz$ implies that $ABz = BAZ$. Note that compatible mappings are weakly compatible but converse is not true in general.

Definition 2.7 [10]: Let A and B be two self-maps of a FM-space $(X, M, *)$. We say that A and B satisfy the property (E.A) if there exists a sequence $\{x_n\}$ such that

$$\lim_{n \rightarrow \infty} A x_n = \lim_{n \rightarrow \infty} B x_n = z \text{ for some } z \in X.$$

Note that weakly compatible and property (E.A) are independent to each other (see [16], Ex. 2.2).

Definition 2.8: Mappings A, B, S and T on a fuzzy metric space $(X, M, *)$ are said to satisfy common (E.A.) property if there exists sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \rightarrow \infty} A x_n = \lim_{n \rightarrow \infty} S x_n = \lim_{n \rightarrow \infty} B y_n = \lim_{n \rightarrow \infty} T y_n = z \text{ for some } z \in X.$$

For more on (E.A) and common (E.A) properties, we refer to [42] and [43].

Definition 2.9 [31]: Let (X, d) be a metric space. Two mappings $f: X \rightarrow X$ and $g: X \rightarrow X$ are said to satisfy property (CLR_g) if there exists sequences $\{x_n\}$ in X such that

$$\lim_{n \rightarrow \infty} f(x_n) = \lim_{n \rightarrow \infty} g(x_n) = g(p), \text{ for some } p \text{ in } X.$$

Similarly, we can have the property (CLR_T) and the property (CLR_S) if in the definition 2.9, the mapping $g: X \rightarrow X$ has been replaced by the mapping $T: X \rightarrow X$ and $S: X \rightarrow X$ respectively.

Definition 2.10 [44]: Let $(X, M, *)$ be fuzzy metric space. A sequence $\{x_n\}$ in X is said to be

- (i) Convergent to a point $x \in X$, if $\lim_{n \rightarrow \infty} M(x_n, x, t) = 1$ for all $t > 0$;
- (ii) Cauchy sequence if $\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 1$, for all $t > 0$ and $p > 0$.

Definition 2.11 [44]: A fuzzy metric space $(X, M, *)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Lemma 2.12 [45]: $M(x, y, \cdot)$ is non-decreasing for all $x, y \in X$.

Lemma 2.13 [45]: Let $x_n \rightarrow x$ and $y_n \rightarrow y$, then

- (i) $\lim_{n \rightarrow \infty} M(x_n, y_n, t) \geq M(x, y, t)$, for all $t > 0$,
- (ii) $\lim_{n \rightarrow \infty} M(x_n, y_n, t) = M(x, y, t)$, for all $t > 0$, if $M(x, y, t)$ is continuous.

Lemma 2.14 [6]: If for all $x, y \in X, t > 0$ and for a number $k \in (0, 1)$;

$$M(x, y, kt) \geq M(x, y, t), \text{ then } x = y.$$

3. IMPLICIT RELATIONS

In [39], Altun and Turkoglu used the following implicit relation: Let $I = [0, 1]$, $*$ be a continuous t-norm and F be the set of all real continuous functions $F: I^6 \rightarrow \mathbb{R}$ satisfying the following conditions:

(F-1) F is non-increasing in the fifth and sixth variables,

(F-2) if, for some constant $k \in (0, 1)$ we have

(F-a) $F(u(kt), v(t), v(t), u(t), 1, u(t/2) * v(t/2)) \geq 1$, or (F-b) $F(u(kt), v(t), u(t), v(t), u(t/2) * v(t/2), 1) \geq 1$ for any fixed $t > 0$ and any non-decreasing functions $u, v: (0, \infty) \rightarrow I$ with $0 \leq u(t), v(t) \leq 1$ then there exists $h \in (0, 1)$ with $u(ht) \geq v(t) * u(t)$,

(F-3) if, for some constant $k \in (0, 1)$ we have $F(u(kt), u(t), 1, 1, u(t), u(t)) \geq 1$ for any fixed $t > 0$ and any non-decreasing function $u: (0, \infty) \rightarrow I$ then $u(kt) \geq u(t)$.

4. MAIN RESULTS

In [39], Altun and Turkoglu proved the following:

Theorem A: Let $(X, M, *)$ be a complete fuzzy metric space with $a * b = \min\{a, b\}$ for all $a, b \in I$ and A, B, S and T be maps from X into itself satisfying the conditions:

(A.1) $A(X) \subseteq T(X), B(X) \subseteq S(X)$;

(A.2) one of the maps A, B, S and T is continuous;

(A.3) (A, S) and (B, T) are compatible of type (α) ;

(A.4) there exist $k \in (0, 1)$ and $F \in \mathcal{F}$ such that

$$F\{M(Ax, By, kt), M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Ax, Ty, t), M(By, Sx, t)\} \geq 1 \text{ for all } x, y \in X, t > 0.$$

Then A, B, S and T have a unique common fixed point in X .

By generalizing the results of [39], Kumar and Fisher [40] in [40] proved the following:

Theorem B: Let $(X, M, *)$ be a fuzzy metric space with $a * b = \min\{a, b\}$ for all $a, b \in I$ Further, let (A, S) and (B, T) be weakly compatible pairs of self-maps of X satisfying (A.1), (A.4) and (A.5), (A, S) or (B, T) satisfies the property (E.A). If the range of one of the maps A, B, S or T is a complete subspace of X then A, B, S and T have a unique common fixed point in X .

We now generalize the theorems A & B as follows;

Theorem 4: Let $(X, M, *)$ be a fuzzy metric space with $a * b = \min\{a, b\}$. Let A, B, S, T be maps from X into itself satisfying the following conditions:

(4.1) $B(X) \subseteq S(X)$ and the pair (B, T) satisfies property (CLR_T)

Or $A(X) \subseteq T(X)$ and the pair (A, S) satisfies property (CLR_S)

(4.2) the pairs (A, S) and (B, T) are weakly compatible.

(4.3) there exist $k \in (0, 1)$ and $F \in \mathcal{F}$ such that

$$F(M(Ax, By, kt), M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Ax, Ty, t), M(By, Sx, t)) \geq 1 \text{ for all } x, y \in X, t > 0.$$

Then $A, B, S,$ and T have a unique common fixed point in X .

Proof: Assume that $B(X) \subseteq S(X)$ and the pair (B, T) satisfies property (CLR_T) , then there exists a sequence $\{x_n\}$ in X such that Bx_n and Tx_n converges to Tx , for some x in X as $n \rightarrow \infty$. Since $B(X) \subseteq S(X)$, so there exists a sequence $\{y_n\}$ in X such that

$$Bx_n = Sy_n, \text{ hence } Sy_n \rightarrow Tx \text{ as } n \rightarrow \infty.$$

We shall show that $\lim_{n \rightarrow \infty} Ay_n = Tx$. Let $\lim_{n \rightarrow \infty} Ay_n = z$. Taking $x = y_n$, and $y = x_n$ in (4.3), we have

$$F\{M(Ay_n, Bx_n, kt), M(Sy_n, Tx_n, t), M(Ay_n, Sy_n, t), M(Bx_n, Tx_n, t), M(Ay_n, Tx_n, t), M(Bx_n, Sy_n, t)\} \geq 1.$$

Taking $n \rightarrow \infty$, we have

$$F\{M(z, Tx, kt), M(Tx, Tx, t), M(z, Tx, t), M(Tx, Tx, t), M(z, Tx, t), M(Bx_n, Bx_n, t)\} \geq 1$$

$$F\{M(z, Tx, kt), 1, M(z, Tx, t), 1, M(z, Tx, t), 1\} \geq 1$$

Since, $M(z, Tx, t) \geq M(z, Tx, t/2) = M(z, Tx, t/2) * 1$, and F is non-increasing in the fifth variable, so we have for any $t > 0$

$$F\{M(z, Tx, kt), 1, M(z, Tx, t), 1, M(z, Tx, t/2), 1\} \geq F\{M(z, Tx, kt), 1, M(z, Tx, t), 1, M(z, Tx, t), 1\} \geq 1$$

which implies by the definition of implicit relation (F-2) that $z = Tx$

or $\lim_{n \rightarrow \infty} Ay_n = Tx$.

Subsequently, we have Bx_n, Tx_n, Sy_n, Ay_n converges to z .

Now, we shall show that $Bx = z$.

Taking $x = y_n$, and $y = x$ in (4.3), we have

$$F\{M(Ay_n, Bx, kt), M(Sy_n, Tx, t), M(Ay_n, Sy_n, t), M(Bx, Tx, t), M(Ay_n, Tx, t), M(Bx, Sy_n, t)\} \geq 1$$

Taking limit $n \rightarrow \infty$, we obtain

$$F\{M(z, Bx, kt), M(z, z, t), M(z, z, t), M(Bx, z, t), M(z, z, t), M(Bx, z, t)\} \geq 1$$

$$F\{M(z, Bx, kt), 1, 1, M(z, Bx, t), 1, M(z, Bx, t)\} \geq 1$$

Since, $M(z, Bx, t) \geq M(z, Bx, t/2) = M(z, Bx, t/2) * 1$, and F is non-increasing in the sixth variable, so we have for any $t > 0$

$$F\{M(z, Bx, kt), 1, 1, M(z, Bx, t), 1, M(z, Bx, t/2) * 1\} \geq F\{M(z, Bx, kt), 1, 1, M(z, Bx, t), 1, M(z, Bx, t)\} \geq 1$$

which implies by the definition of implicit relation (F-2) that, $z = Bx$.

Hence, $z = Bx = Tx$.

Since, the pair (B, T) is weak compatible, it follows that $Bz = Tz$.

Also, since $B(X) \subseteq S(X)$, there exists some y in X such that $Bx = Sy (= z)$.

We next show that $Sy = Ay (= z)$.

Taking $y = x_n$, and $x = y$ in (4.3), we have

$$F\{M(Ay, Bx_n, kt), M(Sy, Tx_n, t), M(Ay, Sy, t), M(Bx_n, Tx_n, t), M(Ay, Tx_n, t), M(Bx_n, Sy, t)\} \geq 1$$

Taking limit $n \rightarrow \infty$, we have

$$F\{M(Ay, z, kt), M(z, z, t), M(Ay, z, t), M(z, z, t), M(Ay, z, t), M(z, z, t)\} \geq 1$$

$$F\{M(Ay, z, kt), 1, M(Ay, z, t), 1, M(Ay, z, t), 1\} \geq 1$$

Since, $M(Ay, z, t) \geq M(Ay, z, t/2) = M(Ay, z, t/2) * 1$, and F is non-increasing in the fifth variable, so we have for any $t > 0$

$$F\{M(Ay, z, kt), 1, M(Ay, z, t), 1, M(Ay, z, t/2) * 1, 1\} \geq F\{M(Ay, z, kt), 1, M(Ay, z, t), 1, M(Ay, z, t), 1\} \geq 1$$

which implies by the definition of implicit relation (F-2) that, $Ay = z$

which implies that $Ay = z = Sy$.

But the pair (A, S) is weakly compatible, it follows that $Az = Sz$.

Next, we claim that $Az = Bz$.

Taking $x = z$, and $y = z$ in (4.3), we have

$$\begin{aligned} &F\{M(Az, Bz, kt), M(Sz, Tz, t), M(Az, Sz, t), M(Bz, Tz, t), M(Az, Tz, t), M(Bz, Sz, t)\} \geq 1 \\ &F\{M(Az, Bz, kt), M(Az, Bz, t), M(Az, Az, t), M(Bz, Bz, t), M(Az, Bz, t), M(Bz, Az, t)\} \geq 1 \\ &F\{M(Az, Bz, kt), M(Az, Bz, t), 1, 1, M(Az, Bz, t), M(Az, Bz, t)\} \geq 1 \end{aligned}$$

which implies by the definition of implicit relation (F-3) that $Az = Bz$.

Hence, $Az = Bz = Sz = Tz$.

We now show that $z = Az$.

Taking $x = z$, and $y = x$ in (4.3), we have

$$\begin{aligned} &F\{M(Az, Bx, kt), M(Sz, Tx, t), M(Az, Sz, t), M(Bx, Tx, t), M(Az, Tx, t), M(Bx, Sz, t)\} \geq 1 \text{ that is,} \\ &F\{M(Az, z, kt), M(Az, z, t), M(Az, Az, t), M(z, z, t), M(Az, z, t), M(z, Az, t)\} \geq 1 \\ &F\{M(Az, z, kt), M(Az, z, t), 1, 1, M(Az, z, t), M(Az, z, t)\} \geq 1 \end{aligned}$$

which implies by the definition of implicit relation (F-3) that $z = Az$.

Therefore, $z = Az = Bz = Sz = Tz$, that is z is the common fixed point of the maps A, B, S , and T .

Uniqueness: the uniqueness of common fixed point of the mappings A, B, S and T be easily verified by using (4.3); In fact, if w be another fixed point for mappings A, B, S and T then for $x = z$, and $y = w$ in (4.3) we have,

$$\begin{aligned} &F\{M(z, w, kt), M(z, w, t), M(z, z, t), M(w, w, t), M(z, w, t), M(w, z, t)\} \geq 1 \\ &F\{M(z, w, kt), M(z, w, t), 1, 1, M(z, w, t), M(z, w, t)\} \geq 1 \end{aligned}$$

which implies by the definition of implicit relation (F-3) that $z = w$.

Hence common fixed point is unique. This completely establishes the theorem.

Example 4.1: Let $X = [0, 1]$ and M be the usual fuzzy metric space on $(X, M, *)$ with minimum t-norm, defined by

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0 \\ 0, & \text{if } t = 0 \end{cases} \quad \text{for all } x, y \in X.$$

Then $(X, M, *)$ is a fuzzy metric space, where $*$ is continuous t-norm defined by $a * b = \min\{a, b\}$ for all a, b in $[0, 1]$.

Define the mappings $A, B, S, T: X \rightarrow X$ by $Ax = x/75, Bx = x/15, Sx = x/5, Tx = x$ respectively. Then, for some $k \in [1/15, 1)$, we have

$$\begin{aligned} M(Ax, By, kt) &= \left[\frac{kt}{kt + \left| \frac{x}{75} - \frac{y}{15} \right|} \right] \geq \left[\frac{t}{t + \left| \frac{x}{5} - y \right|} \right] \\ &= M(Sx, Ty, t) \geq \min\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Ax, Ty, t), M(By, Sx, t)\}. \end{aligned}$$

Thus the condition (4.3) of theorem [4] is satisfied.

Further the pairs (A, S) and (B, T) are weakly compatible.

Also $B(X) = [0, 1/15] \subseteq [0, 1/5] = S(X)$.

Consider the sequence $\{x_n\} = \left\{ \frac{1}{n} \right\}$ so that $\lim_{n \rightarrow \infty} Bx_n = \lim_{n \rightarrow \infty} Tx_n = 0 = T(0)$.

Hence the pair (B, T) satisfies property (CLR_T) .

Therefore all the conditions of theorem [4] are satisfied.

CONCLUSION

Theorem [4] is proved for weakly compatible mappings using the (CLR_T) / (CLR_s) property in FM-space without any requirement of completeness of any subspace. An example [4.1] validates the theorem [4]. Theorem [4] improves and generalizes the results of S.Kumar and B.Fisher [40] and some earlier results.

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