

**I – CONTINUOUS - FUNCTIONS AND  $I^*$  – C CONTINUOUS –FUNCTIONS  
ON INFRA TOPOLOGICAL SPACES**

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**ABSTRACT**

*In this paper, we introduced a new class of function called I-continuous functions and  $I^*$  –continuous functions on infra Topological Spaces. We obtain several characterizations and some of their properties.*

**Keywords and phrases:** *Infra –Topological Space, I -continuous functions and  $I^*$  –continuous functions.*

**1. INTRODUCTION**

In 1983, A.S. Mashhour *et al.* [2] introduced the supra topological spaces and studied S-Continuous function and  $S^*$ -Continuous function on supra topological spaces. In this paper we introduced I-Continuous Functions and  $I^*$  –continuous functions on Infra -Topological Space (ITS) [1]. The analogue concepts associated with infra- topological space. Such as, infra- derived set (resp .infra-closure, infra-interior, infra-exterior and infra-boundary) of subset  $A$  of infra-space  $X$  will be denoted by  $ids(A)$  (resp.  $icp(A), iip(A), iep(A)$  and  $ibp(A)$ ) and studied by our work in [1]. As we shown, Various concepts like the interior, closure, derived set and boundary operators as well as set properties can be defined in infra topological space in analogy with topological spaces. At the same times many results of topological spaces Still valid in infra topological spaces, whereas some become invalid.

**2. PRELIMINARIES**

**Definition 2.1** [1]: Let  $X$  be any arbitrary set. An *Infra -topology* on  $X$  is a collection  $\tau_{iX}$  of subsets of  $X$  such that the following axioms are satisfying,

Ax-1  $\emptyset, X \in \tau_{iX}, .$

Ax-2 The intersection of the elements of any sub collection of  $\tau_{iX}$  in  $\tau_{iX}$ .i.e,

$$\text{If } O_i \in \tau_{iX}, \quad 1 \leq i \leq n \rightarrow \bigcap O_i \in \tau_{iX}.$$

The ordered pair  $(X, \tau_{iX})$  is called *Infra- Topological Space* (ITS), we simply say  $X$  is a *Infra -space*.

**Definition 2.2[1]:** Let  $(X, \tau_{iX})$  be an (ITS) and  $A \subset X$ .  $A$  is called *infra -open set* (IOS) if  $A \in \tau_{iX}$ .

**Theorem 2.1[1]:** Let  $(X, \tau_X)$  be a *topological –space* (TS), then  $(X, \tau_{iX})$  is an *infra-topological space* (IITS). But the conversely is not true.

**Theorem 2.2[1]:** Let  $(X, \tau_{iX})$  be *infra-topological space*. Then:

1.  $\emptyset, X$  are *infra -open set*.
2. Any arbitrary intersections of infra- open sets are infra- open sets.
3. Finite union of infra- *open sets* may not be infra- *open sets*.

**Theorem 2.3[1]:** let  $(X, \tau_{iX})$  and  $(X, \tau_{iX}^*)$  be two *infra topological Spaces* on set  $X$ . Then the intersection  $\tau_{iX}$  and  $\tau_{its}^*$  is an *infra topological space*.

**Theorem 2.4[1]:** let  $(X, \tau_{iX})$  and  $(X, \tau_{iX}^*)$  be two *infra topological Spaces* on set  $X$ . Then the union  $\tau_{iX}$  and  $\tau_{its}^*$  is an *infra topological space*.

**Remark:** In topological space the union of two topological spaces may not be topological space.

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### 3. INFRA-CONTINUOUS FUNCTIONS OR BRIEFLY I-CONTINUOUS FUNCTIONS

**Definition 3.1:** Let  $(X, \tau_X)$  be topological space and  $(X, \tau_{iX})$  be infra-topological Space. We say that  $\tau_{iX}$  is infra-topological space associated with  $\tau$ , if  $\tau_{iX} \subset \tau_X$ . That is,  $\tau_X$  is finer than  $\tau_{iX}$  or  $\tau_{iX}$  is coarser than  $\tau_X$ .

**Definition 3.2:** Let  $(X, \tau_X)$  and  $(Y, \tau_Y)$  be represent two topological Spaces and  $\tau_{iX}$  be associated infra-topologyspace with  $\tau_X$ . A function  $f: X \rightarrow Y$  is called I-continuous function at  $x \in X$ , if for all open set  $O$  containing  $f(x)$  in  $Y$ , then there exists infraopen set  $U$  containing  $x$  in  $\tau_{iX}$  such that  $f(U) \subset O$ .

**Remark:**

1. we say that  $f$  is an I-continuous function on  $A \subset X$ , if  $f$  is an I-continuous function at all points of  $A$ .
2. we say that  $f$  is an I-continuous function on  $X$ , if  $f$  is an I-continuous function at all points of  $X$ .

**Example 3.1:** Let  $(X, \tau_X)$  and  $(Y, \tau_Y)$  represents two topological Spaces such that,  $X = \{a, b, c\}$ ,  $Y = \{1, 2, 3\}$ ,  $\tau_X = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ ,  $\tau_Y = \{\emptyset, Y, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}\}$  and  $\tau_{iX} = \{\emptyset, X, \{a\}, \{b\}\}$ . Define  $f: X \rightarrow Y \ni f(a) = f(c) = 1, f(b) = 2$ . It is clear that  $f$  is I-continuous function.

**Example 3.2:** Let  $(X, \tau_X)$  and  $(Y, \tau_Y)$  represents two topological Space such that:  $X = \{a, b, c\}$ ,  $\tau_X = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ ,  $Y = \{1, 2, 3\}$ ,  $\tau_Y = \{\emptyset, Y, \{1, 2\}\}$ .  $\tau_{iX} = \{\emptyset, X, \{a\}, \{b\}\}$ . Define  $f: X \rightarrow Y \ni f(a) = 1, f(b) = 2, f(c) = 3$ . So that  $f$  is not I-continuous function.

**Theorem 3.1:** Let  $f: (X, \tau_X) \rightarrow (Y, \tau_Y)$  be a function between two topological spaces and  $\tau_{iX}$  be infra-topological space associated with  $\tau_X$ . Then the following statements are equivalent.

1.  $f$  is an I-continuous function.
2. The inverse image of each open set in  $Y$  is an  $\tau_{iX}$ -infraopen set in  $X$ .
3. The inverse image of each closed set in  $Y$  is  $\tau_{iX}$ -infraclosed set in  $X$ .

**Proof:**

1  $\Rightarrow$  2. Suppose that  $f$  is an I-continuous function. Let  $O$  be an open set in  $Y$ . To show that  $f^{-1}(O) \in \tau_{iX}$ . Assume that  $a \in f^{-1}(O) \rightarrow f(a) \in O$ . since  $f$  is an I-continuous function at  $a \in X$ , implies that, there exists infraopen set  $U$  containing  $a$  in  $\tau_{iX}$  such that  $f(U) \subset O$ . Therefore  $a \in U \subset f^{-1}(O)$ . Hence  $f^{-1}(O)$  is an infraopen set in  $X$ .

2  $\Rightarrow$  3. Let  $C \subset Y$  be a closed set in  $Y$ . Then  $Y - C$  is open set in  $Y$ , implies that:  $f^{-1}(Y - C) = f^{-1}(Y) - f^{-1}(C) = X - f^{-1}(C)$  is infraopen set in  $X$ . Therefore  $f^{-1}(C)$  is infraclosed set in  $X$ .

3  $\Rightarrow$  1. Suppose that  $f^{-1}(C)$  is infraclosed set in  $X$  for all closed set  $C \subset Y$ . Let  $O \subset Y$  be an open set, then  $Y - O$  is closed set in  $Y$ , then:  $f^{-1}(Y - O) = f^{-1}(Y) - f^{-1}(O) = X - f^{-1}(O)$  is infraclosed set in  $X$ , so  $f^{-1}(O)$  is infraopen set in  $X$ , consequently,  $f$  is I-Continuous function.

**Theorem 3.2:** Let  $f: (X, \tau_X) \rightarrow (Y, \tau_Y)$  and  $g: (Y, \tau_Y) \rightarrow (Z, \tau_Z)$  be two I-continuous function, then:  $gf: (X, \tau_X) \rightarrow (Z, \tau_Z)$  is an I-continuous function.

**Proof:** Suppose that  $f: (X, \tau_X) \rightarrow (Y, \tau_Y)$  and  $g: (Y, \tau_Y) \rightarrow (Z, \tau_Z)$  are I-continuous function. Let  $O$  be an open subset of  $Z$ . Since  $g$  I-continuous function, implies that:  $g^{-1}(O) \in \tau_{iY}$  - infraopen set in  $Y$ . But  $\tau_{iY} \subset \tau_Y$ . Therefore  $g^{-1}(O) \in \tau_Y$  is an open set in  $Y$ . Also  $f$  I-continuous function, implies that  $f^{-1}g^{-1}(O) \in \tau_{iX}$  - infraopen set in  $X$ , But  $(gf)^{-1}(O) = f^{-1}g^{-1}(O) \in \tau_{iX}$  - infraopen set in  $X$  whenever  $O$  is an open subset of  $Z$ . Hence  $gf$  is I-continuous function.

**Note that:** If  $f: (X, \tau_X) \rightarrow (Y, \tau_Y)$  and  $g: (Y, \tau_Y) \rightarrow (Z, \tau_Z)$  be two S-continuous function, then  $gf: (X, \tau_X) \rightarrow (Z, \tau_Z)$  may not be S-continuous function [2].

**Theorem 3.3:** Let  $f: (X, \tau_X) \rightarrow (Y, \tau_Y)$  be a constant function between two topological spaces and  $\tau_{iX}$  be infra-topological space associated with  $\tau_X$ . Then the constant function is an I-continuous function.

**Proof:** Suppose that  $f: (X, \tau_X) \rightarrow (Y, \tau_Y)$  be a constant function, then there exists  $y_0 \in Y$  such that  $f(x) = y_0, \forall x \in X$ .

Let  $O$  be an open subset of  $Y$ , then  $f^{-1}(O) = \begin{cases} X, & \text{if } y_0 \in O \\ \emptyset, & \text{if } y_0 \notin O \end{cases}$

Since  $X$  and  $\emptyset$  are infraopen sets, therefore  $g$  I-continuous function, implies that:  $g^{-1}(O) \in \tau_{iY}$  - infraopen set in  $Y$ . But  $\tau_{iY} \subset \tau_Y$ . Therefore  $f$  is an I-continuous function.

**Theorem 3.4:** Let  $f: (X, \tau_X) \rightarrow (Y, \tau_Y)$  be an I-continuous function and  $g: (Y, \tau_Y) \rightarrow (Z, \tau_Z)$  is a continuous function, then  $gf: (X, \tau_X) \rightarrow (Z, \tau_Z)$  is an I-continuous function.

**Proof:** Suppose that  $O$  is an open subset of  $Z$ . Since  $g$  is a continuous function, then  $g^{-1}(O) \in \tau_Y$  is an open set in  $Y$ , implies that  $f^{-1}g^{-1}(O) \in \tau_{iX}$  – infraopen set in  $X$ , But  $(gf)^{-1}(O) = f^{-1}g^{-1}(O) \in \tau_{iX}$ , whenever  $O$  is an open set  $Z$ . so that  $gf$  is an  $I$ -continuous function .

**Note that:**

1. If  $f: (X, \tau_X) \rightarrow (Y, \tau_Y)$  is an  $S$ -continuous function and  $g: (Y, \tau_Y) \rightarrow (Z, \tau_Z)$  is a continuous function, then  $gf: (X, \tau_X) \rightarrow (Z, \tau_Z)$  is a  $S$ -continuous function [2].
2. If  $f: (X, \tau_X) \rightarrow (Y, \tau_Y)$  is a continuous function and  $g: (Y, \tau_Y) \rightarrow (Z, \tau_Z)$  is an  $I$ - continuous function, then  $gf: (X, \tau_X) \rightarrow (Z, \tau_Z)$  is a continuous function.

**Theorem 3.5:** Let  $f: (X, \tau_1) \rightarrow (X, \tau_2)$  be the identity function, where  $f(x) = x, \forall x \in X$ . Let  $\tau_1^*$  associated infra-topology with  $\tau_1$ , then  $f$  is called  $I$ -continuous function if and only if  $\tau_2 \subset \tau_1^*$ . ( $\tau_1^*$  is finer than  $\tau_2$ ).

**Proof:**  $\Rightarrow$ : Suppose that  $f$  is  $I$ -continuous function. To show that  $\tau_2 \subset \tau_1^*$ . Let  $O$  be an open set in  $\tau_2$ , then  $f^{-1}(O) = O$  be infraopen set in  $\tau_1^*$ . Hence  $\tau_2 \subset \tau_1^*$ .  $\Leftarrow$ : Suppose that  $f: (X, \tau_1) \rightarrow (X, \tau_2)$  is the identity function, where,  $f(x) = x, \forall x \in X$  and  $\tau_2 \subset \tau_1^*$ . Now Let  $O$  be an open set in  $\tau_2$ , then  $f^{-1}(O) = O \in \tau_1^*$ , thus  $f$  is  $I$ - continuous function .

**Remark:** Consider the sequences of identity functions:

$$(X, \tau_1) \xrightarrow{id-fun} (X, \tau_2) \xrightarrow{id-fun} (X, \tau_3) \xrightarrow{id-fun} (X, \tau_4) \dots (X, \tau_{n-1}) \xrightarrow{id-fun} (X, \tau_n).$$

To reliable  $I$  – continuous function, we must have:  $\tau_1^*$  is finer than  $\tau_2, \tau_2^*$  is finer than  $\tau_3, \dots \tau_{n-1}^*$  is finer than  $\tau_n$ .

**Theorem 3.6:** Let  $f: (X, \tau_X) \rightarrow (Y, \tau_Y = \{\emptyset, y\})$  be a function between two topological Spaces and  $\tau_{iX}$  associated infra-topology with  $\tau_X$ , then  $f$  always  $I$  – continuous function.

**Proof:** Since  $\emptyset, y$  only two open set in  $Y$ , then  $f^{-1}(\emptyset) = \emptyset \in \tau_{iX}$  &  $f^{-1}(Y) = X \in \tau_{iX}$ , so that  $f$  always  $I$  – continuous function.

**Note that:**

1. If  $f: (X, \tau_X = P(X)) \rightarrow (Y, \tau_Y)$  is a function between two topological Spaces and  $\tau_X^*$  associated infra-topology with  $\tau_X$ , then  $f$  may not be  $I$  – continuous function. But in topological space is always continuous function.
2. Let  $f: (X, \tau_X) \rightarrow (Y, \tau_Y)$  be a continuous function: between two topological space and  $\tau_{iX}$  is an infra-topological space associated with  $\tau_X$ , then  $f$  may not be an  $I$  – continuous function.

**Example 3.3:** Let  $X = \{a, b, c, d\}, \tau_X = \{\emptyset, X, \{a\}, \{a, b\}, \{a, b, c\}\}$  and  $\tau_{iX} = \{\emptyset, X, \{a\}, \{a, b\}\}$ . Take  $Y = \{1, 2, 3, 4\}, \tau_Y = \{\emptyset, Y, \{2\}, \{2, 3, 4\}\}$ . Define  $f: X \rightarrow Y = f(a) = 1, f(b) = 2, f(c) = 4$  and  $f(d) = 3$ . It easy to check that  $f$  is a continuous function but not  $I$ -continuous function. If  $f$  is a continuous function, then  $f$  is a  $S$ -continuous function [2].

**Theorem 3.7:** Let  $f$  be a  $I$ -Continuous function from  $(X, \tau_X)$  to  $(Y, \tau_Y)$  and  $C \subset Y$ .

1. If  $C$  is a closed set in  $Y$ , then  $icp(f^{-1}(C)) \subset icp(f^{-1}(cl(C)))$ .
2. If  $C$  is an open set in  $Y$ , then  $iip(f^{-1}(int(C))) \subset iip(f^{-1}(C))$ .

**Proof:**

1. Let  $f$  be a  $I$ -Continuous function from  $(X, \tau_X)$  to  $(Y, \tau_Y)$  and  $C \subset Y$ , where  $C$  is a closed set in  $Y$ . Since  $C$  is a closed set in  $Y$ , then  $C \subset cl(C)$ , implies that  $f^{-1}(C) \subset f^{-1}(cl(C))$ , where  $f^{-1}(C)$  and  $f^{-1}(cl(C))$  are infraclosed sets in  $X$ . Therefore,  $icp(f^{-1}(C)) \subset icp(f^{-1}(cl(C)))$  by [1].
2. Let  $f$  be a  $I$ -Continuous function from  $(X, \tau_X)$  to  $(Y, \tau_Y)$  and  $C \subset Y$ , where  $C$  is an open set in  $Y$ . Since  $C$  is an open set in  $Y$ , then  $int(C) \subset C$ , implies that  $f^{-1}(int(C)) \subset f^{-1}(C)$ , where  $f^{-1}(int(C))$  and  $f^{-1}(C)$  are infraopen sets in  $X$ . Therefore,  $iip(f^{-1}(int(C))) \subset iip(f^{-1}(C))$  by [1].

**Theorem 3.8:** Let  $f$  be a  $I$ -Continuous function from  $(X, \tau_X)$  to  $(Y, \tau_Y)$  and  $A \subset X$ . Then:

1.  $f(iip(A)) \subset f(icp(A))$ .
2.  $f(ibp(A)) \subset f(icp(A))$ .
3.  $f(ids(A)) \subset f(ids(icp(A)))$ .

**Proof:** Let  $f$  be a  $I$ -Continuous function from  $(X, \tau_X)$  to  $(Y, \tau_Y)$  and  $A \subset X$ .

1. Since,  $iip(A) \subset A \subset icp(A)$  [1]  $\rightarrow iip(A) \subset icp(A) \rightarrow f(iip(A)) \subset f(icp(A))$ .
2. Since,  $ibp(A) \subset icp(A)$  [1], then  $f(ibp(A)) \subset f(icp(A))$ .
3. Since,  $A \subset icp(A) \rightarrow ids(A) \subset ids(icp(A))$  [1]  $\rightarrow f(ids(A)) \subset f(ids(icp(A)))$ .

#### 4. INFRA STAR-CONTINUOUS FUNCTIONS OR BRIEFLY $I^*$ -CONTINUOUS FUNCTIONS

**Definition 4.1:** Let  $(X, \tau_X)$  and  $(Y, \tau_Y)$  be represent two topological Spaces. Let  $\tau_{iX}$  and  $\tau_{iY}$  be two associated infra-topology space with  $\tau_X$  and  $\tau_Y$  respectively. A function  $f: X \rightarrow Y$  is called  $I^*$ -continuous function, if The inverse image of each infraopen set in  $\tau_{iY}$  is an  $\tau_{iX}$ -infraopen set in  $X$ .

**Example 4.1:** Let  $(X, \tau_X)$  and  $(Y, \tau_Y)$  represents two topological Spaces such that,  $X = \{a, b, c\}, Y = \{1, 2, 3\}$ ,  $\tau_X = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ ,  $\tau_Y = \{\emptyset, Y, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}\}$ ,  $\tau_{iX} = \{\emptyset, X, \{a\}, \{b\}\}$  and  $\tau_{iY} = \{\emptyset, Y, \{2\}, \{3\}\}$ . Define  $f: X \rightarrow Y \ni f(a) = f(c) = 1, f(b) = 2$ . It is clear that  $f$  is  $I^*$ -continuous function.

**Example 4.2:** Let  $(X, \tau_X)$  and  $(Y, \tau_Y)$  represents two topological Spaces such that,  $X = \{a, b, c\}, Y = \{1, 2, 3\}$ ,  $\tau_X = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ ,  $\tau_Y = \{\emptyset, Y, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}\}$ ,  $\tau_{iX} = \{\emptyset, X, \{a\}, \{b\}\}$  and  $\tau_{iY} = \{\emptyset, Y, \{2\}, \{3\}\}$ . Define  $f: X \rightarrow Y \ni f(a) = 1, f(b) = 2, f(c) = 3$ . So that  $f$  is not  $I^*$ -continuous function.

**Theorem 4.1:** Let  $f: (X, \tau_X) \rightarrow (Y, \tau_Y)$  be a function between two topological spaces and  $\tau_{iX}, \tau_{iY}$  be two associated infra-topology space with  $\tau_X$  and  $\tau_Y$  respectively. Then  $f$  is an  $I^*$ -continuous function, if and only if, the inverse image of each infraclosed set in  $\tau_{iY}$  is an  $\tau_{iX}$ -infraclosed set in  $X$ .

**Proof:**  $\Rightarrow$ : Suppose that  $f$  is an  $I^*$ -continuous function. Let  $C$  be an infraclosed set  $\in \tau_{iY}$ . Then  $Y - C$  is infraopen set in  $Y$ , implies that  $f^{-1}(Y - C) = f^{-1}(Y) - f^{-1}(C) = X - f^{-1}(C)$  is infraopen set in  $X$ . Therefore  $f^{-1}(C)$  is infraclosed set in  $X$ .

$\Leftarrow$ : Let  $O$  be an infraopen set  $\in \tau_{iY}$ , then  $Y - O$  is infraclosed set  $\in \tau_{iY}$ , this implies that,  $f^{-1}(Y - O) = f^{-1}(Y) - f^{-1}(O) = X - f^{-1}(O)$  is infraclosed set  $\in \tau_{iX}$ , so  $f^{-1}(O)$  is infraopen set  $\in \tau_{iX}$  and consequently,  $f$  is  $I^*$ -continuous function.

**Theorem 4.2:** Let  $f: (X, \tau_X) \rightarrow (Y, \tau_Y)$  and  $g: (Y, \tau_Y) \rightarrow (Z, \tau_Z)$  be two  $I^*$ -continuous function, then:  $gf: (X, \tau_X) \rightarrow (Z, \tau_Z)$  is an  $I^*$ -continuous function

**Proof:** Suppose that  $f: (X, \tau_X) \rightarrow (Y, \tau_Y)$  and  $g: (Y, \tau_Y) \rightarrow (Z, \tau_Z)$  are  $I^*$ -continuous function. Let  $O$  be an infraopen set  $\in \tau_{iZ}$ . Since  $g$  is  $I^*$ -continuous function, implies that:  $g^{-1}(O) \in \tau_{iY}$  - infraopen set in  $Y$ . since  $f$  is  $I^*$ -continuous. Therefore  $f^{-1}g^{-1}(O) \in \tau_{iX}$  - infraopen set in  $X$ . But  $(gf)^{-1}(O) = f^{-1}g^{-1}(O) \in \tau_{iX}$ . so that  $gf$  is  $I^*$ -continuous.

**Theorem 4.3:** Let  $f: (X, \tau_X) \rightarrow (Y, \tau_Y)$  be  $I$ -continuous function, and  $g: (Y, \tau_Y) \rightarrow (Z, \tau_Z)$  be  $I^*$ -continuous function, then:  $gf: (X, \tau_X) \rightarrow (Z, \tau_Z)$  is an  $I^*$ -continuous function

**Proof:** Suppose that  $f: (X, \tau_X) \rightarrow (Y, \tau_Y)$  an  $I$ -continuous function and  $g: (Y, \tau_Y) \rightarrow (Z, \tau_Z)$  is an  $I^*$ -continuous function. Let  $O$  be an infra open set  $\in \tau_{iZ}$ . Since  $g$  is an  $I^*$ -continuous function, implies that:  $g^{-1}(O) \in \tau_{iY}$  - infraopen set in  $Y$ . But  $\tau_{iY} \subset \tau_Y$ , so that  $g^{-1}(O) \in \tau_Y$  is open set in  $Y$ . As that  $f$  is  $I$ -continuous. Therefore  $f^{-1}g^{-1}(O) \in \tau_{iX}$  - infraopen set in  $X$ . Hence  $(gf)^{-1}(O) = f^{-1}g^{-1}(O) \in \tau_{iX}$ . so that  $gf$  is  $I^*$ -continuous.

**Theorem 4.4:** Let  $f: (X, \tau_X) \rightarrow (Y, \tau_Y)$  be  $I^*$ -continuous function, and  $g: (Y, \tau_Y) \rightarrow (Z, \tau_Z)$  be  $I$ -continuous function, then:  $gf: (X, \tau_X) \rightarrow (Z, \tau_Z)$  is an  $I$ -continuous function.

**Proof:** By the same techniques of theorem 4.3.

**Theorem 4.5:** Let  $f: (X, \tau_X) \rightarrow (Y, \tau_Y)$  be a constant function between two topological spaces and  $\tau_{iX}$  and  $\tau_{iY}$  are infra-topological space associated with  $\tau_X$  and  $\tau_Y$  respectively. Then the constant function is an  $I^*$ -continuous function. .

**Proof:** By the similar way for theorem 3.3.

**Theorem 4.6:** Let  $f$  be a  $I^*$ -Continuous function from  $(X, \tau_X)$  to  $(Y, \tau_Y)$  and  $C \subset Y$ .

1. If  $C$  is a infraclosed set in  $Y$ , then  $icp(f^{-1}(C)) \subset icp(f^{-1}(icp(C)))$ .
2. If  $C$  is an infraopen set in  $Y$ , then  $iip(f^{-1}(int(C))) \subset iip(f^{-1}(C))$ .

**Proof:**

1. Let  $f$  be a  $I^*$ -Continuous function from  $(X, \tau_X)$  to  $(Y, \tau_Y)$  and  $C \subset Y$ , where  $C$  is a infraclosed set in  $Y$ . Since  $C$  is a infraclosed set in  $Y$ , then  $C \subset icp(C)$ , implies that  $f^{-1}(C) \subset f^{-1}(icp(C))$ , where  $f^{-1}(C)$  and  $f^{-1}(icp(C))$  are infraclosed sets in  $X$ . Therefore,  $icp(f^{-1}(C)) \subset icp(f^{-1}(icp(C)))$  by [1].
2. Let  $f$  be a  $I^*$ -Continuous function from  $(X, \tau_X)$  to  $(Y, \tau_Y)$  and  $C \subset Y$ , where  $C$  is an infraopen set in  $Y$ . Since  $C$  is an infraopen set in  $Y$ , then :  
 $int(C) \subset C$ , implies that  $f^{-1}(iip(C)) \subset f^{-1}(C)$ , where  $f^{-1}(iip(C))$  and  $f^{-1}(C)$  are infraopen sets in  $X$ . Therefore,  $iip(f^{-1}(iip(C))) \subset iip(f^{-1}(C))$  by [1].

**Theorem 3.8:** Let  $f$  be a  $I^*$ -Continuous function from  $(X, \tau_X)$  to  $(Y, \tau_Y)$  and  $A \subset X$ . Then:

1.  $f(iip(A)) \subset f(icp(A))$ .
2.  $f(ibp(A)) \subset f(icp(A))$ .
3.  $f(ids(A)) \subset f(ids(icp(A)))$ .

**Proof:** Let  $f$  be a  $I^*$ -Continuous function from  $(X, \tau_X)$  to  $(Y, \tau_Y)$  and  $A \subset X$ .

1. Since,  $iip(A) \subset A \subset icp(A)[1] \rightarrow iip(A) \subset icp(A) \rightarrow f(iip(A)) \subset f(icp(A))$ .
2. Since,  $ibp(A) \subset icp(A)[1]$ , then  $f(ibp(A)) \subset f(icp(A))$ .
3. Since,  $A \subset icp(A) \rightarrow ids(A) \subset ids(icp(A)[1]) \rightarrow f(ids(A)) \subset f(ids(icp(A)))$ .

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