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A NEW ALGORITHM FOR SOLVING FUZZY ASSIGNMENT PROBLEM USING BRANCH AND BOUND METHOD

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ABSTRACT

In the literature, there are various methods to solve assignment problems in which parameters are represented by triangular or trapezoidal fuzzy numbers. This paper presents an assignment problem with fuzzy costs, where the objective is to minimize the cost. Here each fuzzy cost is assumed as trapezoidal fuzzy number. A new ranking method has been used for ranking the fuzzy trapezoidal numbers. The fuzzy assignment problem has been transformed into a crisp one using ranking function and solved by branch and bound method. A numerical example is provided to demonstrate the proposed approach.

Keywords: *Trapezoidal fuzzy numbers, ranking of fuzzy numbers, fuzzy assignment problem, optimal solution.*

1. INTRODUCTION

The assignment problem is a special type of transportation problem and a linear zero-one programming problem. It is a one of the well studied optimization problems in management science and has been widely applied in both manufacturing and service systems. In an assignment problem, n jobs are to be performed by n persons depending on their efficiency to do the job. All the algorithms developed to find the optimal solution of transportation problems are applicable to assignment problems. However, due to its high degeneracy nature, Kuhn H.W. [5] introduced a specially designed algorithm so called Hungarian method for solving Assignment problems in crisp environment.

In recent years, Fuzzy Assignment Problems have received much attention. Lin & Wen [7] solved the assignment problem with fuzzy interval number costs by a labeling algorithm. Kumar and Gupta [6] proposed two new methods for solving fuzzy assignment problems and fuzzy travelling salesman problems. Kalaiarasi *et.al* [4] Optimization of fuzzy assignment model with triangular fuzzy numbers. Bai *et.al* [1] proposed a method for solving fuzzy generalized assignment problem. Chen [2] proved some theorems and proved a fuzzy assignment model that considers all individuals to have same skills. Wang [10] solved a similar model by graph theory. The assignment problems introduced by Votaw and Orden [9] can be solved using the linear programming technique. Sriniwas B. and Ganesan G. [8] introduced a new method for solving branch and bound technique for assignment problem using triangular and trapezoidal fuzzy numbers. Hussein Iden Hasan and Dheyab Anfal Hasan [3] proposed a new algorithm using ranking function to find solution for fuzzy transportation problem.

The objective of this paper is propose a new algorithm depending on ranking function to solve fully fuzzy assignment problem using trapezoidal fuzzy numbers for the jobs and machines(persons).This paper contain five section:-in section 2 review some concept of fuzzy theory, in section 3 define fuzzy assignment problem and its formula. In section 4 The Computational Procedure for Fuzzy Assignment Problem in section 5 takes numerical example and applied a new algorithm depending on ranking function.

2. CONCEPT OF FUZZY THEORY

In this section we will introduce some definitions of fuzzy theory.

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Fuzzy set
Let Ω be a nonempty set. A fuzzy set A in Ω is characterized by its membership function, $\mu_A : \Omega \to [0,1]$ and denoted by $\tilde{A}$ and $\mu_A(x)$ is interpreted as the degree of membership of element a in fuzzy set A for each $x \in \Omega, \tilde{A} = \{(x, \mu_A(x)) : x \in \Omega\}$.

Fuzzy number
The fuzzy set A defined on the set of real numbers is said to be a fuzzy number if its membership function $\mu_A : \Omega \to [0,1]$ has the following characteristics

1. A is normal. It means that there exists an x∈R such that $\mu_A(x) = 1$.
2. A is convex it means that for every $x_1, x_2 \in R$
3. $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2), \lambda \in [0,1]\}$.
4. 3- μ_A is upper semi-continuous.
5. Supp (A) is bounded in R.

Trapezoidal fuzzy numbers:
A fuzzy number A is defined to be a trapezoidal fuzzy number if its membership functions $\mu_{\tilde{A}} : R \to [0,1]$ is equal to

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & if \;\; x \in [a_1, a_2] \\ \dfrac{a_4-x}{a_4-a_3} & if \;\; x \in [a_2, a_3] \\ 0 & otherwise \end{cases}$$

A new algorithm ranking function:
We use the following trapezoidal membership function

$$\mu_A(X) = \begin{cases} \dfrac{\lambda(x-a)}{b-a} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \dfrac{\lambda(x-d)}{c-d} & c \leq x \leq d \end{cases}$$

By using $\alpha - cut$, where $\alpha \in [0,1]$ and $0 \leq \alpha \leq \lambda$, then

$$\alpha = \frac{\lambda(x-a)}{b-a} \qquad \alpha = \frac{\lambda(x-d)}{c-d}$$

$$x = \frac{\alpha(b-a)}{\lambda} + a \qquad x = \frac{\alpha(c-d)}{\lambda} + d$$

$$\tilde{A}^l_{(\alpha)} = a + \frac{\alpha}{\lambda}(b-a) \qquad \tilde{A}^u_{(\alpha)} = d + \frac{\alpha}{\lambda}(c-d)$$

$$R(\tilde{A}_{(\alpha)}) = \left[\frac{\frac{1}{2}\int_0^\lambda \alpha^2 \left[\tilde{A}^l_{(\alpha)} + \tilde{A}^u_{(\alpha)}\right] d\alpha}{\int_0^\lambda \alpha^2 d\alpha} \right]$$

$$\Rightarrow \quad R\left(\tilde{A}_{(\alpha)}\right)=\frac{\frac{1}{2}\int_0^{\lambda}\alpha^2\left[a+\frac{\alpha}{\lambda}(b-a)+d+\frac{\alpha}{\lambda}(c-d)\right]d\alpha}{\int_0^{\lambda}\alpha^2 d\alpha}$$

$$R\left(\tilde{A}_{(\alpha)}\right)=\frac{\frac{1}{2}\int_0^{\lambda}\left[\ \alpha^2(a+d)+\frac{\alpha^3}{\lambda}(b-a+c-d)\ \right]d\alpha}{\int_0^{\lambda}\alpha^2 d\alpha}$$

$$R\left(\tilde{A}_{(\alpha)}\right)=\frac{\frac{1}{2}\left[\frac{\alpha^3}{3}(a+d)+\frac{\alpha^4}{4\lambda}(b-a+c-d)\right]_0^{\lambda}}{\left[\frac{\alpha^3}{3}\right]_0^{\lambda}}$$

$$\Rightarrow R\left(\tilde{A}_{(\alpha)}\right)=\frac{\frac{\lambda^3}{2}\left[\frac{(a+d)}{3}+\frac{(b-a+c-d)}{4}\right]}{\frac{\lambda^3}{3}}$$

$$R\left(\tilde{A}_{(\alpha)}\right)=\frac{1}{8}(a+3b+3c+d)$$

Ranking properties of trapezoidal fuzzy numbers:

The ranking function defined as $R : F(\mu) \to R$ which maps each fuzzy number into the real line. $F(\mu)$ represent the set of trapezoidal fuzzy number.

There are many properties for ranking function, any two trapezoidal fuzzy numbers A and B we have the following comparison.

1. $A \prec B \quad iff \quad R(A) \prec R(B)$
2. $A \succ B \quad iff \quad R(A) \succ R(B)$
3. $A \approx B \quad iff \quad R(A) \approx R(B)$
4. $A - B = 0 \ iff \ R(A) - R(B) = 0$

A trapezoidal fuzzy number $A = (a,b,c,d)$ in $F(\mu)$ is said to be positive if $R(A) \geq 0$.

3. FUZZY ASSIGNMENT PROBLEM

Suppose there are n jobs for a factory and has n machines to process those jobs. A job $i\ (i = 1,2,\ldots n)$ when processed by machine $j\ (j = 1,2,\ldots n)$ is assumed to incur a cost C_{ij}. The assignment is to be made in such a way that each job can be associated with one and only one machine. Determine an assignment of jobs to machine so as to minimize the overall cost. We can define

$$x_{ij} = \begin{cases} 1 & if\ ith\ person\ is\ assigned\ to\ jth\ job \\ 0 & otherwise \end{cases}$$

Mathematically assignment problem can be stated as

Minimize $Z = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_{ij}$

Subject to $\sum_{i=1}^{n} x_{ij} = 1$

$$\sum_{j=1}^{n} x_{ij} = 1$$

i.e. we can assign one job to each machine.

4. THE COMPUTATIONAL PROCEDURE FOR FUZZY ASSIGNMENT PROBLEM

BRANCH BOUND TECHNIQE AND BOUND TECHNIQUE FOR ASSIGNMENT PROBLEM:

The assignment problem can also be solved using a branch and bound algorithm:

It is a curtailed enumeration technique. The terminologies of branch and bound technique applied to the assignment problem are presented below.

1. Let k be the level number in the branch tree (for root node it is 0) σ be an assignment in the current node of a branching tree.
2. P_σ^k be an assignment at level k of the branching tree. A is the set of assigned cells (partial assignment) up to the node P_σ^k from the root node (set of i and j values with respect to the assigned cells up to the node P_σ^k from the root node) and V_σ be the lower bound of the partial assignment up to P_σ^k such that

$$V_\sigma = \sum_{i,j \in A} C_{i,j} + \sum_{i \in X} \left(\sum_{j \in Y} \min C_{ij} \right)$$

Where C_{ij} is the cell entry of the cost matrix with respect to the i^{th} row and j^{th} column. X is the set of rows which are not deleted up to the node P_σ^k from the root node in the branching node.

Branching guidelines:

1. At Level k, the row marked as k of the assignment assigned problem, will be assigned with the best column of the assignment problem.
2. If there is tie on the lower bound. Then the terminal node at the lower most is to be considered for the further branching.
3. Stooping rule: If the minimum lower bound happiness to be at any of the terminal node at the $(n-1)^{th}$ level, the optimality is reached. Then the assignments on the path of the node to that node along with the missing pair of row-column combination from the optimum solution.

5. NUMERICAL EXAMPLE

A company has four sources S1, S2, S3, S4 and destinations D1, D2, D3, D4. All the data in this example are trapezoidal fuzzy numbers. We desire to solve this fuzzy assignment problem with proposed algorithm.

	D1	D2	D3	D4
S1	(1,2,4,5)	(4,3,1,0)	(8,5,1,6)	(9,8,7,2)
S2	(3,5,5,7)	(5,3,2,4)	(15,10,5,4)	(11,11,9,1)
S3	(15,13,4,6)	(7,6,6,5)	(2,6,5,5)	(4,3,1,0)
S4	(12,11,8,3)	(9,8,7,2)	(7,6,6,5)	(15,13,4,6)

The above fuzzy assignment problem can be formulated in the following mathematical programming form

$$\min (1,2,4,5)x_{11} + (4,3,1,0)x_{12} + (8,5,1,6)x_{13} + (9,8,7,2)x_{14} + (3,5,5,7)x_{21} + (5,3,2,4)x_{22} + (15,10,5,4)x_{23}$$
$$+ (11,11,9,1)x_{24} + (15,13,4,6)x_{31} + (7,6,6,5)x_{32} + (2,6,5,5)x_{33} + (4,3,1,0)x_{34} + (12,11,8,3)x_{41} + (9,8,7,2)x_{42}$$
$$+ (7,6,6,5)x_{43} + (15,13,4,6)x_{44}$$

Subject to

$$x_{11}+x_{12}+x_{13}+x_{14}=1 \qquad x_{11}+x_{21}+x_{31}+x_{41}=1$$
$$x_{21}+x_{22}+x_{23}+x_{24}=1 \qquad x_{12}+x_{22}+x_{32}+x_{42}=1$$
$$x_{31}+x_{32}+x_{33}+x_{34}=1 \qquad x_{13}+x_{23}+x_{33}+x_{43}=1$$
$$x_{41}+x_{42}+x_{43}+x_{44}=1 \qquad x_{14}+x_{24}+x_{34}+x_{44}=1$$

Where $x_{ij}\in[0,1]$

Now we calculate R (1, 2, 4, 5) by proposed method.

$$R(C_{11})=\frac{1+3(2)+3(4)+5}{8}=3$$

Similarly,

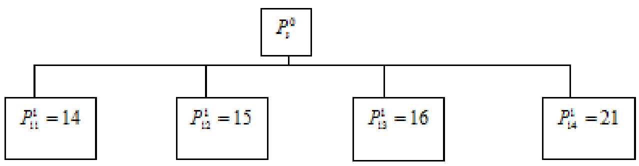
$$R(C_{12})=2 \quad R(C_{13})=4 \quad R(C_{14})=7 \quad R(C_{21})=5 \quad R(C_{22})=3 \quad R(C_{23})=8 \quad R(C_{24})=9 \quad R(C_{31})=10$$
$$R(C_{32})=6 \quad R(C_{33})=5 \quad R(C_{34})=2 \quad R(C_{41})=9 \quad R(C_{42})=7 \quad R(C_{43})=6 \quad R(C_{44})=10$$

We replace these values for their corresponding to given problem which results in a conventional assignment problem in the LPP form. Solving it we get the solution a crisp form of the problem:

	D1	D2	D3	D4
S1	3	2	4	7
S2	5	3	8	9
S3	10	6	5	2
S4	9	7	6	10

Initially, no job is assigned to any operator, so the assignment (σ) at the root (level 0) of the branching tree is a null set and the corresponding lower bound is also 0 for each.

Further branching: the four sub problem under the root nodes are shown as in figure lower bound the solution problems shown on its right hand side.



Compute the lower bound for P_{11}^1

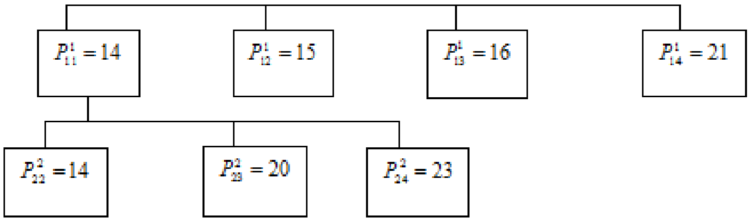
$$V_\sigma=\sum_{i,j\in A}C_{i,j}+\sum_{i\in X}\left(\sum_{j\in Y}\min C_{ij}\right)$$

Where $\sigma=\{(11)\}$ $\quad A=\{(11)\}$ $\quad X=\{2,3,4\}$ $\quad Y=\{2,3,4\}$ then

$$V_{11}=C_{11}+\sum_{i\in 2,3,4}\left(\sum_{j\in 2,3,4}\min C_{ij}\right)$$

$$P_{11}^1=3+(3+2+6)=14 \qquad P_{12}^1=2+(5+2+6)=15 \qquad P_{13}^1=4+(3+2+7)=16 \qquad P_{14}^1=7+(3+5+6)=21$$

Further branching: further branching is done from the terminal node which has the least lower bound at this stage; the nodes $P_{11}^1, P_{12}^1, P_{13}^1, P_{14}^1$ are the terminal nodes. The node P_{11}^1 has the least lower bound. Hence, further branching from this node is shown as in figure

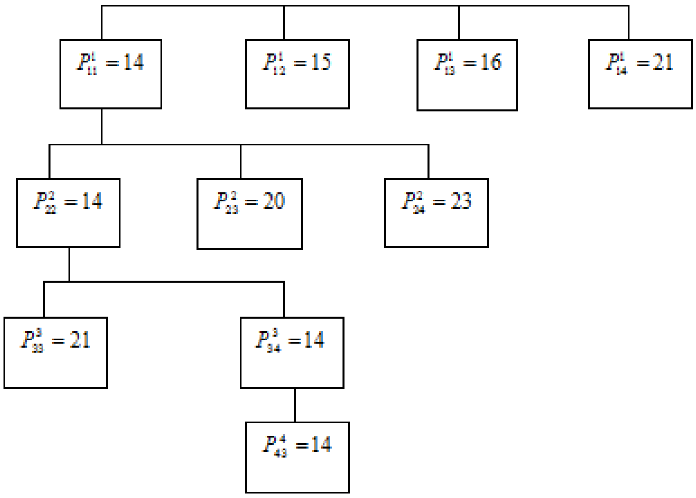


$$V_{11} = C_{11} + C_{22} + \sum_{i \in 3,4} \left(\sum_{j \in 3,4} \min C_{ij} \right)$$

$$P_{22}^2 = 3 + 3 + (2 + 6) = 14 \qquad P_{23}^2 = 3 + 8 + (2 + 7) = 20 \qquad P_{24}^2 = 3 + 9 + (5 + 6) = 23$$

Further branching: At this stage the nodes $P_{22}^2, P_{23}^2, P_{24}^2$ are the terminal node. The node P_{22}^2 has the least lower bound. Hence, further branching from this node is shown as in figure.

$$V_{33} = C_{11} + C_{22} + C_{33} + \sum_{i \in 4} \left(\sum_{j \in 4} \min C_{ij} \right) \qquad P_{33}^3 = 3 + 3 + 5 + 10 \qquad P_{34}^3 = 3 + 3 + 2 + 6 = 14$$



The optimal assignment is (1,1),(2,2),(3,4),(4,3). The fuzzy optimal cost is calculated
(1, 2, 4, 5) + (5, 3, 2, 4) + (4, 3, 1, 0) + (7, 6, 6, 5) = (17, 14, 13, 14)
3 + 3 + 2 + 6 = 14

CONCLUSION

In this paper, the assignment cost has been considered as trapezoidal fuzzy numbers which are more realistic and general in nature. Here the fuzzy assignment problem has been converted into crisp assignment problem using the proposed algorithm with ranking function and branch and bound method has been applied to find an optimal solution. Numerical example has been shown that the total cost obtained is optimal. This method is systematic procedure, easy to apply and can be utilized for all type of assignment problem whether maximize or minimize objective function.

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