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APPROXIMATE APPROACH TO SUM OF n!

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ABSTRACT

The factorial was known since 12th century. The factorial applied to many mathematics branches. This paper gives proof to n! Formula, factorial relation with stirling first kind numbers and approximate approach to sum of n! was explained.

Keywords: *n! Formula, sum of (n! ×n) with Stirling first kind numbers, sum of n! Approximate approach.*

1. INTRODUCTION

The factorial concept was known since 12th century. The factorial operation is involved in many areas of mathematics like number theory, algebra, probability theory, and exponential problems etc. Basically n! is explain ways to arrange n distinct object into a sequence. i.e., permutation of the set of objects .the notation n! was introduced by Christian kramp in 1808.the factorial denoted with gamma function (Γ) also.

The factorial (n!) is defined for a positive integer's n as

n! =n× (n-1) ×......×2×1.

Or

n! $=\prod_{k=1}^{n} k$

To calculate n! Many formulas are derived. In this paper n! Formula given with proof as well as approximate approach to sum of n! was explained.

2.1. Statement: Prove that $m!^k \times (m^k-1)! = m^k! \times (m-1)!^k$

Proof: $m!^k \times (m^k-1)! = m^k \times (m-1)!^k \times (m^k-1)!$
$= (m-1)!^k \times m^k!$

Hence $m!^k \times (m^k-1)! = m^k! \times (m-1)!^k$ is proved.

2.2. Statement: Prove that k! = $\sum_{n=0}^{k}(-1)^n \times c_n^k \times (k+1-n)^k$

Proof: Given statement is k! = $\sum_{n=0}^{k}(-1)^n \times c_n^k \times (k+1-n)^k$.

This can be proving with the mathematics induction method.

For that let us take k=1

=>1! = $\sum_{n=0}^{1}(-1)^n \times c_n^1 \times (1+1-n)^1$
$= (-1)^0 \times c_0^1 \times (2-0)^1 + (-1)^1 \times c_1^1 \times (2-1)^1$
$= c_0^1$×2-c_1^1 $= 1$

Such that at k=1 given statement is true.

Assume at k=k given statement is true.

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Such that k! = $\sum_{n=0}^{k}(-1)^n \times c_n^k \times (k+1-n)^k$ is true.

$$
\begin{aligned}
(k+1)! &= (k+1) \times k! \\
&= (k+1)\times \sum_{n=0}^{k}(-1)^n \times c_n^k \times (k+1-n)^k \\
&= k\times\sum_{n=0}^{k}(-1)^n \times c_n^k \times (k+1-n)^k + \sum_{n=0}^{k}(-1)^n \times c_n^k \times (k+1-n)^k \\
&= \sum_{n=0}^{k+1}(-1)^n \times c_n^{k+1} \times ((k+1)+1-n)^{k+1} \quad [\text{W.K .T } k\times c_n^k + c_n^k = c_n^{k+1}]
\end{aligned}
$$

Such that at k=k+1 given statement is true.

Hence k! =$\sum_{n=0}^{k}(-1)^n \times c_n^k \times (k+1-n)^k$ is proved.

2.1 Statement: Prove that $\sum_{n=0}^{k}(-1)^n \times c_n^k \times (k+1-n) = 0 \qquad where\ k \geq 2$

Proof: Given is

$$
\begin{aligned}
\sum_{n=0}^{k}(-1)^n \times c_n^k \times (k+1-n) &= \sum_{n=0}^{k} (-1)^n \times \frac{k!}{(k-r)!r!} \times (k+1-n) \\
&= \sum_{n=0}^{k}(-1)^n \times \frac{k\times(k-1)\times(k-2)\times\ldots\ldots\times(k-(r-1))(k-r)!}{(k-r)!\times r!} \times (k+1-r) \\
&= \sum_{n=0}^{k}(-1)^n \times \frac{k\times(k-1)\times(k-2)\times\ldots\ldots\times(k-(r-1))}{r!} \times (k+1-r)
\end{aligned}
$$

From above equation when k>1 then $k \times (k-1) \times (k-2) \times \ldots\ldots\ldots \times \left(k-(r-1)\right) = 0.$

$$=> \sum_{n=0}^{k}(-1)^n \times \frac{0}{(k-r)!\times r!} \times (k+1-r) = 0$$

Hence $\sum_{n=0}^{k}(-1)^n \times c_n^k \times (k+1-n) = 0 \quad where\ k \geq 2$ are proved.

2.2 Statement: Prove that $\sum_{n=1}^{k} n! \times n = \sum_{n=2}^{k+1} s(k+1,r)$

Proof: We know that $\sum_{n=1}^{k} n! \times n = (k+1)! - 1$ (1)

and $\sum_{n=1}^{k+1} s(k+1,r) = (k+1)!$ (2)

From equation (2)

$$s(k+1,1)+ \sum_{n=2}^{k+1} s(k+1,r) = (k+1)!$$

[From Stirling first kind numbers s(k+1,1)=1]

$$=> \sum_{n=2}^{k+1} s(k+1,r) = (k+1)! - 1 \qquad (3)$$

From equation (1), (3)

$$\sum_{n=1}^{k} n! \times n = \sum_{n=2}^{k+1} s(k+1,r)$$

Hence $\sum_{n=1}^{k} n! \times n = \sum_{n=2}^{k+1} s(k+1,r)$ is proved.

3.1. Approximate approach for sum of n!:

There is no any direct formula for sum of n! but approximate approach is possible. Here sum of n! is derived by follow step by step result of $\frac{(N+1)!}{\sum_{n=0}^{N} n!}$.

We know that (N+1)! Is always grater then to sum of n! where N≥2.

$$\sum_{n=0}^{N} n! < (N+1)!$$

Let us take $Z_N = \frac{(N+1)!}{\sum_{n=0}^{N} n!}$

For N=1, 2, 3, 4 ... note down N, Z_N value in a table. Z_N Consist integer and fraction values.

Example: for N=3

$$Z_3 = \frac{(3+1)!}{\sum_{n=0}^{3} n!} = 2.4$$ Here 2 is integer and 0.4 is fraction value.

N	Z_N
0	1
1	1
2	1.5
3	2.4
4	3.529411765
5	4.675324675
6	5.766590389
7	6.817720663
8	7.848769304
9	8.869899343
10	9.885500286
11	10.89761612
12	11.90734414
13	12.91534619
14	13.92205341
15	14.92776151
16	15.93268116
17	16.93696693
18	17.94073507
19	18.94407477
20	19.94705569
21	20.94973306
22	21.95215127
23	22.95434638
24	23.95634806
25	24.9581809
26	25.95986548
27	26.96141915
28	27.96285665
29	28.96419057
30	29.96543176

Table for $N \; and \; Z_N$ values

Form the table conclude that $Z_N aN, \lfloor Z_N \rfloor = N - 1$and fraction values are considerably varying.

So that $Z_N = (N-1) + fractional\ values(\ g(n))$ where n=N-2

Considered function is $Z_N = \frac{(N+1)!}{\sum_{n=0}^{N} n!}$ from this we can write as

$$Z_N = \frac{(N+1)!}{\sum_{n=0}^{N} n!}$$

$$=> \sum_{n=0}^{N} n! = \frac{(N+1)!}{Z_N}$$

So if we now Z_N value then simple to calculate sum of n! But fraction values are not following any functions so we can go through approximate function generating function for fraction values.

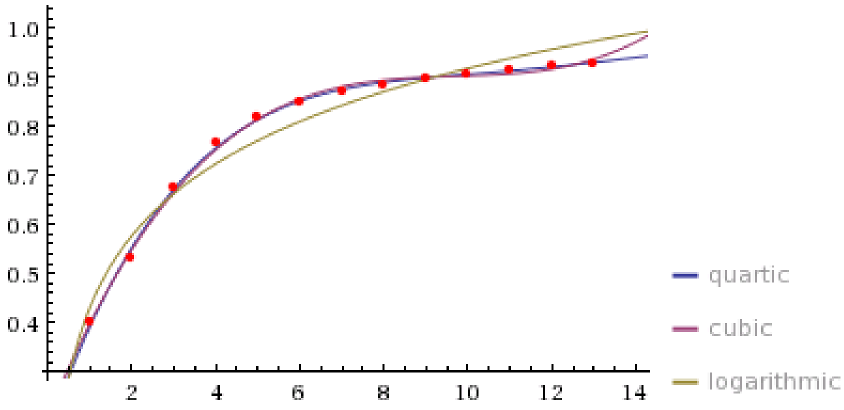
In sum of n! Fraction values (g (n)) approximate approaches generating functions:

Generating function for 0.4, 0.529411765, 0.675324675, 0.766590389, 0.817720663, 0.848769304, 0.869899343, ... where $N \geq 3$.

Approximate generating functions are

1. $g(n) = -0.0000386644n^4 + 0.00182067n^3 - 0.0314408n^2 + 0.243991n^1 + 0.175762\ (in\ quartic)$
2. $g(n) = 0.000738061n^3 - 0.0214378n^2 + 0.210043n + 0.207312\ (in\ cubic)$
3. $g(n) = 0.212168\log(n) + 0.429115\ (in\ logarthmic)$ Where n=N-2, N>2.

Generating functions graphical representation:



Approximate generating functions and Generating functions graphical representations are done with the help of Wolfram Math World software.

Hence $\sum_{n=0}^{N} n! = \frac{(N+1)!}{Z_N} = \frac{(N+1)!}{(N-1)+g(n)}$

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