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# BOOLEAN LIKE SEMIRINGS

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*ABSTRACT*

*Semiring theory is one of the most developing branch of Mathematics with wide application in many disciplines such as Computer science, Coding theory, Topological space and many researchers studies different structure of semirings like complemented semirings, ternary semirings, complemented ternary semirings, gamma semirings, Boolean like semirings etc. In this paper, we discuss some properties of Boolean like semirings. We determine the additive and multiplicative structures of a Boolean like semirings by assuming different properties on the additive (multiplicative) structures.*

***Key words:** Band, Regular, Multiplicatively sub idempotent, E-inverse semi group.*

## I. INTRODUCTION

Historically semirings first appear implicitly in Dedekind and later in Macaulay, Noether and Lorenzen in connection with the study of a ring. However semirings first appear explicitly in Vandiver, also in connection with the axiomatization of arithmetic of natural numbers. Semirings have been studied by various researchers in an attempt to broaden techniques coming from semigroup theory or ring theory or in connection with applications.

However in semi rings it is possible to derive the additive structures from their special multiplicative structures and vice versa. The semiring identities are taken from the book of Jonathan S. Golan [2], entitled "emirings and their Applications''. In this paper we investigate the additive and multiplicative properties of Boolean like semi rings.

## II. PRELIMINARIES

**Definition 2.1:** A Triple $(S,+,.)$ is said to be a semi ring if $S$ is a non-empty set and '+', '.' are binary operations on $S$ satisfying that

(i) $(S,+)$ is a semigroup
(ii) $(S,.)$ is a semigroup
(iii) *a (b+c) = ab+ac* and *(b+c) a= ba+ca* for all *a, b, c in* $S$..

**Definition 2.2:** A non- empty set $S$ together with two binary operations '+ 'and '.' Satisfying the following conditions is called a Boolean like semiring.

(i) $(S,+)$ is a semigroup
(ii) $(S,.)$ is a semigroup
(iii) *a ( b+c)= ab+ac and (b+c) a= ba+ca*
(iv) *ab( a+ab)= ab for all* $a,b$ in $S$. *and a.0=0.a=0*
(v) Weak commutative $abc = bac$ for all *a, b, c* in $S$.

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**Definition 2.3:** An element '*a*' of a semiring $S$ is multiplicatively sub idempotent if and only if $a + a^2 = a$ and $S$ is multiplicatively sub idempotent if and only if each of its elements is multiplicatively sub idempotent.

**Definition 2.4:** An element '*a*' of a multiplicative semigroup $S$ is called an *E*-inverse if there is an element '*x*' in $S$ such that $ax+ ax = ax$, that is $ax$ in $E (+)$, where $E (+)$ is the set of all multiplicative idempotent of $S$. A semigroup $S$ is called an *E*-inverse semigroup if every element of $S$ is an *E*-inverse.

**Definition 2.5:** An element '*a*' of a multiplicative semigroup $S$ is called an *E*-inverse if there is an element '*x*' in $S$ such that $ax.\ ax=ax$. That is $ax$ in $E (.)$, where $E (.)$ is the set of all multiplicative idempotent elements of $S$. A semi group $S$ is called an *E*-inverse semi group if every element of $S$ is an *E*-inverse.

**Definition 2.6:** A semigroup $(S, .)$ is said to be

(i) Left regular, if it satisfies the identity $aba= ab$ for all $a,b$ in $S$.

(ii) Right regular, if it satisfies the identity $aba = ba$ for all $a,b$ in $S$.

(iii) Regular if it is both left as well as right regular.

**Definition 2.7:** A semiring $(S,+,.)$ with multiplicative zero is said to be zero square semi ring if $x^2=0$ for all $x$ in $S$.

**Definition 2.8:** A semiring $(S,+,.)$ with additive identity zero is said to be zero sum free semiring if $x+x=0$ for all $x$ in $S$.

**Definition 2.9:** A semigroup $(S, .)$ is said to be weak commutative if $abc = bac$ for all $a, b, c$ in $S$.

**Definition 2.10:** A Semigroup $(S, .)$ is said to be a band if it satisfies the identity $a^2=a$ for all $a$ in $S$.

**Definition 2.11:** A semigroup $(S, .)$ is called commutative, if it $ab=ba$ for all $a,b$ in $S$.

**Definition 2.12:** A semigroup $(S, .)$ is said to be a rectangular band if it satisfies the identity $aba = a$ for all $a,b$ in $S$.

## III. MAIN RESULTS

**Theorem 3.1:** Let $(S,+,.)$ be a Boolean like semiring containing the multiplicative identity 1. If $(S,+)$ is a band, then $(S, .)$ is a band if and only if $(S,+)$ is a band

**Proof:** Given that $(S,+,.)$ is Boolean like semiring and $(S,+)$ is a band that is $a + a = a$ for all $a$ in $S$.
Consider $ab(a + ab) = ab$ for all $a,b$ in $S$.

Taking b=1

$$\Rightarrow a1(a + a1) = a1$$
$$\Rightarrow a(a + a) = a$$
$$\Rightarrow a(a) = a \quad \text{since } (S,+) \text{ is a band}$$
$$\Rightarrow a^2 = a$$

Therefore $(S, \cdot)$ is a band

Conversely $ab(a + ab) = ab$ for all $a,b$ in $S$.

$$\Rightarrow aba + abab = ab$$

Taking b=1 on both side we get

$$\Rightarrow a1a + a1a1 = a1$$
$$\Rightarrow aa + aa = a$$
$$\Rightarrow a^2 + a^2 = a$$
$$\Rightarrow a + a = a \quad \text{Since } (S, .) \text{ is a band}$$

Therefore $(S,+)$ is a band

 

**Theorem3.2:** Let $(S,+,.)$ be a Boolean like semiring. If $(S,\ .)$ is a band, then $(S,+)$ $E$-inverse semigroup

**Proof:** By hypothesis $(S,+,.)$ is a Boolean like semiring and $(S,\ .)$ is a band that is $a^2 = a$ for all $a$ in $S$.
Consider

$ab(a+ab) = ab$ for all $a,b$ in $S$.

$\Rightarrow\ aba + abab = ab$

$\Rightarrow\ aba + (aba)b = ab$

$\Rightarrow\ baa + baab = ab$ Since weak commutative $abc = bac$

$\Rightarrow\ ba^2 + ba^2 b = ab$

$\Rightarrow\ ba + bab = ab$ Since $(S,\ .)$ is a band $a^2 = a$

$\Rightarrow\ ba + abb = ab$ Since weak commutative

$\Rightarrow\ ba + ab^2 = ab$

$\Rightarrow\ ba + ab = ab$ Since $(S,\ .)$ is a band

Multiplying '$a$' on both sides we get

$\Rightarrow\ (ba+ab)a = aba$

$\Rightarrow\ baa + aba = aba$

$\Rightarrow\ ba^2 + aba = aba$

$\Rightarrow\ ba^2 + baa = baa$ Since weak commutative

$\Rightarrow\ ba^2 + ba^2 = ba^2$

$\Rightarrow\ ba + ba = ba$ Since $(S,\ .)$ is a band

Therefore $(S,+)$ is $E$-inverse semigroup

**Theorem3.3:** Let $(S,+,.)$ be a Boolean like semiring containing the multiplicative identity which is also an additive identity. Then $(S,\ .)$ is an $E$ - inverse semigroup.

**Proof**: By hypothesis $(S,+,.)$ is a Boolean like semiring. Let 'e' be the multiplicative identity is also an additive identity

Consider $ab(a+ab) = ab$ for all $a,b$ in $S$.

$\Rightarrow\ ab(a(e+b)) = ab$

$\Rightarrow\ ab(ab) = ab$

$\Rightarrow\ ab.ab = ab$

Therefore $(S,\ .)$ is an E-inverse semigroup

**Theorem3.4:** Let $(S,+,.)$ be is a Boolean like semiring containing the multiplicative identity 1. If $(S,+)$ is a band, then $(S,\ .)$ is multiplicatively sub idempotent.

**Proof**: Given that $(S,+,.)$ is a Boolean like semiring and $(S,+)$ is a band that is $a+a=a$ for all $a$ in $S$.

We have $ab(a+ab) = ab$ for all $a,b$ in $S$.
Taking b=1

$\Rightarrow\ a1(a+a1)a1$

$\Rightarrow\ a(a+a) = a$

$\Rightarrow\ a(a) = a$ since $(S,+)$ is a band

$\Rightarrow\ a^2 = a$

On both sides adding '$a$' we get

$\Rightarrow\ a + a^2 = a + a$

$\Rightarrow\ a + a^2 = a$ since $(S,+)$ is a band

Therefore $(S,\ .)$ is multiplicatively sub idempotent.

**Theorem3.5**: Let $(S,+,.)$ be a Boolean like semiring containing the multiplicative identity which is also an additive identity. If $(S,\ .)$ is a band, then i. $(S,\ .)$ is right regular. and ii. $(S,\ .)$ is left regular.

**Proof**: i). By hypothesis $(S,+,.)$ is a Boolean like semiring.

Let 'e' be the multiplicative identity is also an additive identity.

Consider $ab(a+ab)=ab$ for all $a,b$ in $S.$

$\Rightarrow\ aba+abab=ab$
$\Rightarrow\ aba(e+b)=ab$
$\Rightarrow\ aba(b)=ab$
$\Rightarrow\ baab=ab$ since weak commutative $abc=bac$
$\Rightarrow\ ba^2\,b=ab$
$\Rightarrow\ bab=ab$ since $(S,\ .)$ is a band

Therefore $(S,\ .)$ is right regular.

ii) Now consider $ab(a+ab)=ab$ for all $a,b$ in $S.$

$\Rightarrow\ aba+abab=ab$
$\Rightarrow\ aba+a(bab)=ab$
$\Rightarrow\ aba+aabb=ab$ since weak commutative $abc=bac$
$\Rightarrow\ aba+a^2\,b^2=ab$
$\Rightarrow\ aba+ab=ab$ since $(S,\ .)$ is a band.
$\Rightarrow\ ab(a+e)=ab$
$\Rightarrow\ aba=ab$

Therefore $(S,\ .)$ is left regular

**Theorem3.6:** Let $(S,+,.)$ be a Boolean like semiring containing the multiplicative identity which is also an additive identity. If $(S,\ .)$ is a band, then $(S,\ .)$ is a commutative.

**Proof:** Since $(S,+,.)$ is a Boolean like semiring, and $(S,\ .)$ is a band that is $a^2=a$ for all $a$ in $S.$

Let 'e' be the multiplicative identity is also an additive identity.

We have $ab(a+ab)=ab$ for all $a,b$ in $S.$

$\Rightarrow\ aba+abab=ab$
$\Rightarrow\ aba+a(bab)=ab$
$\Rightarrow\ aba+aabb=ab$ since weak commutative $abc=bac$
$\Rightarrow\ aba+a^2\,b^2=ab$
$\Rightarrow\ aba+ab=ab$ since $(S,\ .)$ is a band
$\Rightarrow\ ab(a+e)=ab$
$\Rightarrow\ aba=ab$
$\Rightarrow\ baa=ab$ since weak commutative $abc=bac$
$\Rightarrow\ ba^2=ab$
$\Rightarrow\ ba=ab$ since $(S,\ .)$ is a band

Therefore $(S,\ .)$ is a commutative.

**Theorem3.7:** Let $(S,+,.)$ be a Boolean like semiring. If $(S,\ .)$ is a rectangular band, then $a+ab=ab$ for all $a,b$ in $S.$ conversely is also true if $(S,\ .)$ is right cancellative.

**Proof:** By hypothesis $(S,+,.)$ is a Boolean like semiring and $(S,\ .)$ is a rectangular band that is $aba=a$ for all $a,b$ in $S.$

Consider $ab(a+ab)=ab$ for all $a,b$ in $S$.

$$\Rightarrow aba+abab=ab$$
$$\Rightarrow aba+a(bab)=ab$$
$$\Rightarrow a+ab=ab \qquad \text{since } (S,\,.) \text{ is a rectangular band } aba=a$$

Therefore $a+ab=ab$

Conversely $ab(a+ab)=ab$

$$\Rightarrow abab=ab \qquad \text{since } a+ab=ab$$
$$\Rightarrow aba=a \qquad \text{since } (S,\,.) \text{ is right cancellative}$$

Therefore $(S,\,.)$ is rectangular band.

**Theorem3.8:** Let $(S,+,.)$ be a Boolean like semiring containing multiplicative identity which is also an additive identity. Then $a^2=a^{2n}$ and $a^3=a^{2n+1}$ and soon for $n>1$.

**Proof**: Given that $(S,+,.)$ is a Boolean like semiring.

Let 'e' be the multiplicative identity is also an additive identity.

We have $ab(a+ab)=ab$ for all $a,b$ in $S$.

$$\Rightarrow ab(a(e+b))=ab$$
$$\Rightarrow ab(ab)=ab$$

Taking $b=a$ on both sides we get

$$\Rightarrow aa(aa)=aa$$
$$\Rightarrow a^2\,a^2=a^2$$
$$\Rightarrow a^4=a^2 \qquad 1$$

On both sides multiplying '*a*' we get

$$\Rightarrow a^4\,a=a^2\,a$$
$$\Rightarrow a^5=a^3 \qquad 2$$

On both sides multiplying '*a*' we get

$$\Rightarrow a^5\,a=a^3\,a$$
$$\Rightarrow a^6=a^4 \qquad 3$$

On both sides multiplying '*a*' we get

$$\Rightarrow a^6\,a=a^4\,a$$
$$\Rightarrow a^7=a^5 \qquad 4$$

Continuing this process from 1, 2, 3 and 4 we get

$$a^2=a^4=a^6=\ldots\ldots\ldots. \quad \text{and} \quad a^3=a^5=a^7=\ldots\ldots\ldots\ldots$$

Therefore $a^2=a^{2n}$ and $a^3=a^{2n+1}$ for $n>1$.

## IV. CONCLUSION

In Boolean like semirings, the algebraic structure of a multiplicative semi group $(S,\,.)$ determine the additive structure of $(S,+)$ and vice versa.

In a Boolean like semiring $(S,+,.)$, if 1 is the multiplicative identity, then the multiplicative structure satisfies the band property. If the Boolean like semiring $(S,+,.)$ contains the multiplicative identity which is also an additive identity then the multiplicative structure satisfies the properties likes band, E-inverse, and left (right) regular.

 

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