

## MULTIPLE ENCRYPTIONS OF INDEPENDENT CIPHERS

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### ABSTRACT

The main aim of Cryptography is to protect user privacy. The modern ciphers which were employed in wireless networks, digital content and financial systems are rarely get cracked. In order to enhance the security levels, recent research work suggests multiple encryptions. For these issues designing of mathematical models from Algebraic Structures, Number Theoretic concepts play a vital role. In this paper we proposed multiple encryptions using Lucas, Pell numbers, Affine and Vignere transformations as layers of encryption.

**Keywords:** Pell number, Lucas numbers, Affine, Vignere transformations, encryption algorithms, decryption algorithms.

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### INTRODUCTION

Super-encryption is a process of encrypting the information that is already encrypted [5][13]. Multiple encryptions is the process of employing multiple ciphers. The generation of key for each cipher plays the pivotal role. Designing an algorithm for each layer of encryption requires independency. If one depends on the other then security levels are very low. Designing of mathematical models for multiple encryptions mostly based on time and space complexity.

#### Lucas Numbers

The Lucas number is defined to be the sum of its two immediate previous terms, thereby forming a Fibonacci integer sequence. The first two Lucas numbers are  $L_0 = 2$  and  $L_1 = 1$  as opposed to the first two Fibonacci numbers  $F_0 = 0$  and  $F_1 = 1$ . Though closely related in definition, Lucas and Fibonacci numbers exhibit distinct properties. The Lucas numbers may thus be defined as follows:

$$L_n = \begin{cases} 2 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ L_{n-1} + L_{n-2} & \text{if } n > 1 \end{cases}$$

The sequence of Lucas numbers is: 2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 189.....

#### Pell Numbers

The Pell numbers are defined by the recurrence relation

$$P_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ 2P_{n-1} + P_{n-2} & \text{other wise} \end{cases}$$

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In words, the sequence of Pell numbers starts with 0 and 1, and then each Pell number is the sum of twice the previous Pell number and the Pell number before that. The first few terms of the sequence are 0,1,2,5,12,29,70,169, 408,985, 2378, 5741, 13890,...

### Affine Transformation

An affine enciphering transformation is  $C \equiv aP + b \pmod{N}$  where the pair (a, b) is the encrypting key and  $\gcd(a,N)=1$ . If  $y = E(x) = (ax+b) \pmod{26}$ , then we can “solve for x in terms of y” and so  $E^{-1}(y)$  that is, if  $y \equiv (ax + b) \pmod{26}$  then  $y - b \equiv ax \pmod{26}$  or equivalently  $ax \equiv (y - b) \pmod{26}$ .

### Vigenere Transformation

The Vigenere cipher was generated by Giovan Batista Belaso in 1553[9]. This cipher uses a secret keyword to encrypt the plaintext. First, each letter in the plaintext is converted into a number. Then this numerical value for each letter of the plaintext is added to the numerical value of each letter of a secret keyword to get the ciphertext. The Vigenere ciphers are more powerful than substitution ciphers.

### Super encryption

Super-encryption is a process of encryption information that is already encrypted. Super encryption is simply the use of multiple ciphers, usually in multiple steps, as a singular encryption scheme and is a very important technique and many modern strong encryption algorithms can be regarded as resulting from super-encryption using a number of comparatively weak algorithm.

### Present Work

An Algorithm for multi encryption using offset rule with Fibonacci numbers as the first layer of encryption and the affine transformation for super encryption

### Multiple Encryption with Lucas numbers

#### Encryption algorithm:

**Step-1:** Alice creates plaintexts  $P = p_1 p_2 p_3 \dots p_m$

**Step-2:** Alice use off set rule with Lucas numbers  $F=f_1, f_2, f_3, \dots, f_n$  to each value in sequential order and get 1<sup>st</sup> ciphertext

**Step-3:** Alice computes  $C_i = P_i + F_i$  for  $i=1,2,3, \dots, m$  where  $C_{1i}$  is the first construct cipher text.

**Step-4:** Now Alice perform super encryption with the Affine transformation  $E(x) = (ax+b) \pmod{26}$ ,  $\gcd(a, N)=1$  and take a and b are secret, from the first level encryption message.

**Step-5:** Alice sends super encrypted message to Bob.

#### Decryption algorithm:

**Step-1:** Bob receives the super encrypted message.

**Step-2:** Bob decrypts the super encrypted message by using  $E^{-1}(y) = a^{-1}(y - b) \pmod{26}$

**Step-3:** Using reverse offset rule with Lucas number Bob decrypts first decrypted message to get original plaintext.

### SUPER ENCRYPTION OF VIGENERE CIPHER

#### Encryption algorithm:

**Step-1:** Alice creates plaintexts  $P = p_1 p_2 p_3 \dots p_m$

**Step-2:** Alice use off set rule with Lucas numbers  $F=f_1 f_2 f_3, \dots, f_n$  to each value in sequential order and get 1<sup>st</sup> ciphertext

**Step-3:** Alice computes  $C_i=P_i+F_i$  for  $i=1, 2, 3, \dots, m$  where  $C_{1i}$  is the first constructed cipher text.

**Step-4:** Alice super encrypts the first encrypted message with vigenere transformation using a secret key.

**Step-5:** Alice sends super encrypted message to Bob.

**Decryption algorithm:**

**Step-1:** Bob receives the super encrypted message.

**Step-2:** Bob decrypts with reverse offset rule vigenere transformation to super encryption message. It is the first decryption message

**Step-3:** Using reverse offset rule with Lucas number Bob decrypts first decrypted message to get original plaintext.

**Multiple Encryption with Pell numbers**

**Encryption algorithm:**

**Step-1:** Alice creates plaintexts  $P = p_1, p_2, p_3 \dots p_m$

**Step-2:** Alice use off set rule with Pell numbers  $F=f_1, f_2, f_3 \dots f_n$  to each value in sequential order and get 1<sup>st</sup> cipher text

**Step-3:** Alice computes  $C_i=P_i+F_i$  for  $i=1, 2, 3, \dots, m$  where  $C_{1i}$  is the first construct cipher text.

**Step-4:** Now Alice apply super encryption of Affine transformation  $E(x) = (ax+b) \bmod 26$ ,  $\gcd(a, N) = 1$  and take a and b are secret, from the first level encryption message.

**Step-5:** Alice sends super encrypted message to Bob.

**Decryption algorithm:**

**Step-1:** Bob receives the super encrypted message.

**Step-2:** Bob decrypts with the inverse affine transformation by using  $E^{-1}(y) = a^{-1}(y-b) \bmod 26$  as super encrypted message. It is the first decryption message

**Step-3:** Using reverse offset rule with Pell number Bob decrypts first decrypted message to get original plaintext.

**SUPER ENCRYPTION OF VIGENERE CIPHER**

**Encryption algorithm:**

**Step-1:** Alice creates plaintexts  $P = p_1 p_2, p_3 \dots p_m$

**Step-2:** Alice use off set rule with Pell numbers  $F=f_1, f_2, f_3 \dots f_n$  to each value in sequential order and get 1<sup>st</sup> ciphertext

**Step-3:** Alice computes  $C_i=P_i+F_i$  for  $i=1,2,3, \dots, m$  where  $C_{1i}$  is the first construct cipher text.

**Step-4:** Alice super encrypts the first encrypted message with vigenere transformation using a secret key

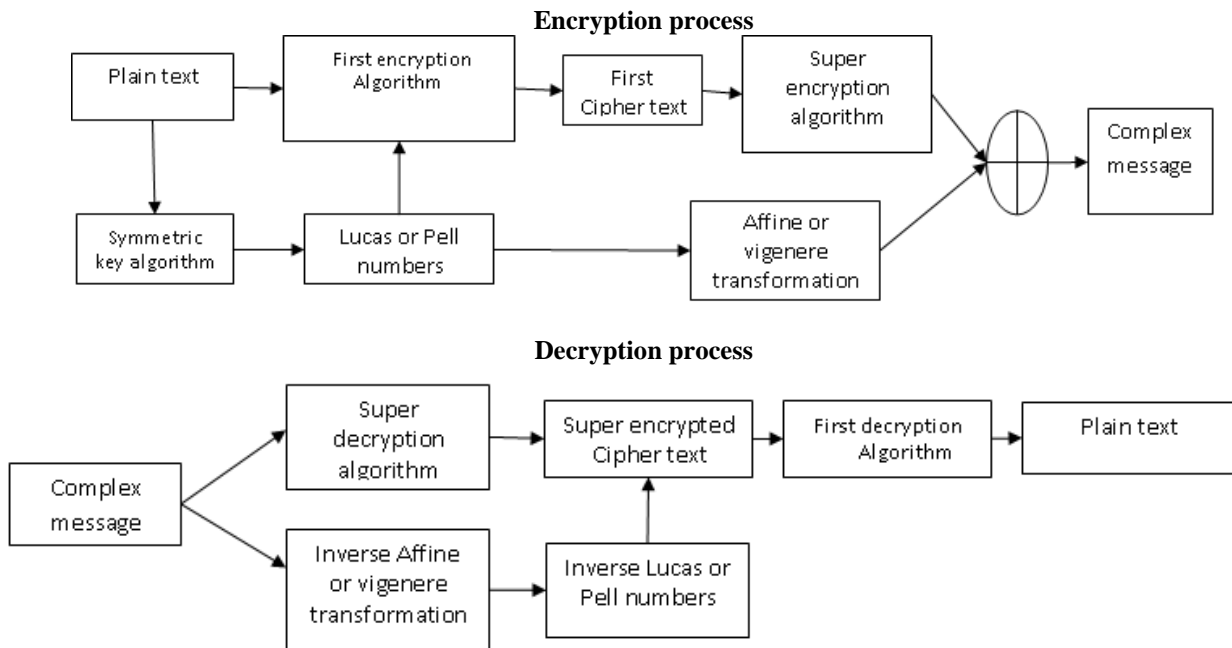
**Step-5:** Alice sends super encrypted message to Bob.

**Decryption algorithm:**

**Step-1:** Bob receives the super encrypted message.

**Step-2:** Bob decrypts with the reverse off set rule of vigenere transformation to super encryption message. It is the first decryption message

**Step-3:** Using reverse offset rule with Pell number Bob decrypts first decrypted message to get original plaintext.



A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

**EXAMPLE**

**Encryption algorithm:**

**Step-1:** Let the Plain text be NUMBERTHEORY

**Step-2:** Using Offset rule with Lucas numbers

N	U	M	B	E	R	T	H	E	O	R	Y
13	20	12	1	4	17	19	7	4	14	17	24
+	+	+	+	+	+	+	+	+	+	+	+
2	1	3	4	7	11	18	29	47	76	123	199
15	21	15	5	11	28	37	36	51	90	140	223

**Step-3:** Now applying affine transformation  $E(x) = (ax+b) \pmod{26}$  for  $a = 5$  &  $b = 20$

<b>x</b>	15	21	15	5	11	28	37	36	51	90	140	223
<b>5x+20</b>	95	125	95	45	75	160	205	200	275	470	720	1135
<b>(5x+20) mod 26</b>	17	21	17	19	23	4	23	18	15	2	18	17
<b>Second Encrypted message is</b>	R	V	R	T		E	X	S	P	C	S	R

**Step-4:** Encrypted message is RVRTXEXSPCSR

**Decryption algorithm:**

**Step-1:** First Decrypted Message is RVRTXEXSPCSR

**Step-2:** Find Inverse of Affine transformation  $E^{-1}(y) = a^{-1}(y - b) \pmod{26}$

<b>Message</b>	R	V	R	T	X	E	X	S	P	C	S	R
<b>y</b>	17	21	17	19	23	4	23	18	15	2	18	17
<b>y-20</b>	-3	1	-3	-1	3	-16	3	-2	-5	-18	-2	-3
<b>21(y-20)</b>	-63	21	-63	-21	63	-336	63	-42	-105	-378	-42	-63
<b>21(y-20)mod26</b>	15	21	15	5	11	2	11	10	25	12	10	15
<b>First Decrypted text</b>	P	V	P	F	L	C	L	K	Z	M	K	P

**Step-3:** Reverse Offset rule with the first decrypted message

<b>Message</b>	P	V	P	F	L	C	L	K	Z	M	K	P
	15	21	15	5	11	2	11	10	25	12	10	15
<b>Reverse offset rule with Lucas number</b>	15	21	15	5	11	2	11	10	25	12	10	15
	-	-	-	-	-	-	-	-	-	-	-	-
	2	1	3	4	7	11	18	29	47	76	123	199
	13	20	12	1	4	-9	-7	-19	-22	-64	-113	-184
<b>Mod 26</b>	13	20	12	1	4	17	19	7	4	14	17	24
<b>Second Decrypted message is</b>	N	U	M	B	E	R	T	H	E	O	R	Y

## VIGENERE CIPHER

**Encryption algorithm:**

**Step-1:** Let the Plain text be LUCASSERIES

**Step-2:** Offset rule with Lucas number

L	U	C	A	S	S	E	R	I	E	S
11	20	2	0	18	18	4	17	8	4	18
+	+	+	+	+	+	+	+	+	+	+
2	1	3	4	7	11	18	29	47	76	123
13	21	5	4	25	29	22	46	55	80	141

Using vigenere cipher as second key

C	A	R	D
2	0	17	3

**Step-3:** Offset rule with the first encrypted message

	13	21	5	4	25	29	22	46	55	80	141
<b>Offset rule with key</b>	13	21	5	4	25	29	22	46	55	80	141
	+	+	+	+	+	+	+	+	+	+	+
	2	0	17	3	2	0	17	3	2	0	17
	15	21	22	7	27	29	19	49	57	80	158
<b>Mod 26</b>	15	21	22	7	1	3	13	23	5	2	2
<b>Second Encrypted message is</b>	P	V	W	H	B	D	N	X	F	C	C

**Step-4:** Encrypted message is PVWHBDNXFCC

**Decryption algorithm:**

**Step-1:** First Decrypted Message is PVWHBDNXFCC

**Step-2:** Decrypts with the inverse of vigenere transformation

<b>Message</b>	P	V	W	H	B	D	N	X	F	C	C
	15	21	22	7	1	3	13	23	5	2	2
<b>Reverse Offset rule with key</b>	15	21	22	7	1	3	13	23	5	2	2
	-	-	-	-	-	-	-	-	-	-	-
	2	0	17	3	2	0	17	3	2	0	17
	13	21	5	4	-1	3	-4	20	3	2	11
<b>Mod 26</b>	13	21	5	4	25	3	22	20	3	2	11
<b>First Decrypted message is</b>	N	V	F	E	Z	D	W	U	D	C	L

**Step-3:** Reverse Offset rule with the first decrypted message

Message	N	V	F	E	Z	D	W	U	D	C	L
	13	21	5	4	25	3	22	20	3	2	11
Reverse Offset rule with Lucas number	13	21	5	4	25	3	22	20	3	2	11
	-	-	-	-	-	-	-	-	-	-	-
	2	1	3	4	7	11	18	14	47	76	123
	11	20	2	0	18	-8	4	-9	-44	-74	-112
Mod 26	11	20	2	0	18	18	4	17	8	4	18
Second Decrypted message is	L	U	C	A	S	S	E	R	I	E	S

## PELL NUMBERS

**Encryption algorithm:**

**Step-1:** Plain text is SECRETMATHS

**Step-2:** Offset rule with Pell number

S	E	C	R	E	T	M	A	T	H	S
18	4	2	17	4	19	12	0	19	7	18
+	+	+	+	+	+	+	+	+	+	+
0	1	2	5	12	29	70	169	408	985	2378
18	5	4	22	16	48	82	169	427	992	2396

**Step-3:** Now applying affine transformation  $E(x) = (ax+b) \bmod 26$  for  $a = 5$  &  $b = 12$

X	18	5	4	22	16	48	82	169	427	992	2396
$5x+12$	102	37	32	122	92	252	422	857	2147	4972	11992
$(5x+12) \bmod 26$	24	11	6	18	14	18	6	25	15	6	6
Second Encrypted message is	Y	L	G	S	O	S	G	Z	P	G	G

**Step-4:** Encrypted message is YLGSOSGZPGG

**Decryption algorithm:**

**Step-1:** First Decrypted Message is YLGSOSGZPGG

**Step-2:** Find Inverse of Affine transformation  $E^{-1}(y) = a^{-1}(y-b) \bmod 26$

Message	Y	L	G	S	O	S	G	Z	P	G	G
y	24	11	6	18	14	18	6	25	15	6	6
$y-12$	12	-1	-6	6	2	6	-6	13	3	-6	-6
$21(y-12)$	252	-21	-126	126	42	126	126	273	63	-126	-126
$21(y-12) \bmod 26$	18	5	4	22	16	22	4	13	11	4	4
First Decrypted text	S	F	E	W	Q	W	E	N	L	E	E

**Step-3:** Reverse Offset rule with the first decrypted message

Message	S	F	E	W	Q	W	E	N	L	E	E
	18	5	4	22	16	22	4	13	11	4	4
Reverse offset rule with Pell number	18	5	4	22	16	22	4	13	11	4	4
	-	-	-	-	-	-	-	-	-	-	-
	0	1	2	5	12	29	70	169	408	985	2378
	18	4	2	17	4	-7	-66	-156	-397	-981	-2357
Mod 26	18	4	2	17	4	19	12	0	19	7	18
Second Decrypted message is	S	E	C	R	E	T	M	A	T	H	S

## VIGENERE CIPHERE

### Encryption algorithm:

**Step-1:** Let the Plain text be PELLNUMBERS

**Step-2:** Offset rule with Pell number

P	E	L	L	N	U	M	B	E	R	S
15	4	11	11	13	20	12	1	4	17	18
+	+	+	+	+	+	+	+	+	+	+
0	1	2	5	12	29	70	169	408	985	2375
15	5	13	16	25	49	82	170	412	1002	2396

Using vigenere cipher as second key

M	A	T	H
12	0	19	7

**Step-3:** Offset rule with the first decrypted message

	15	5	13	16	25	49	82	170	412	1002	2396
<b>Offset rule with Key</b>	15	5	13	16	25	49	82	170	412	1002	2396
	+	+	+	+	+	+	+	+	+	+	+
	12	0	19	7	12	0	19	7	12	0	19
	27	5	32	23	37	49	101	177	424	1002	2415
<b>Mod 26</b>	1	5	6	23	11	23	23	21	8	14	23
<b>Second Encrypted message is</b>	B	F	G	X	L	X	X	V	I	O	X

**Step-4:** Encrypted message is BFGXLXXVIOX

### Decryption algorithm:

**Step-1:** First Decrypted Message is BFGXLXXVIOX

**Step-2:** Decrypts with the inverse of vigenere transformation

<b>Message</b>	B	F	G	X	L	X	X	V	I	O	X
	1	5	6	23	11	23	23	21	8	14	23
<b>Reverse offset rule with Key</b>	1	5	6	23	11	23	23	21	8	14	23
	-	-	-	-	-	-	-	-	-	-	-
	12	0	19	7	12	0	19	7	12	0	19
	-11	5	-13	16	-1	23	4	14	-4	14	4
<b>Mod 26</b>	15	5	13	16	25	23	4	14	22	14	4
<b>First Decrypted message is</b>	P	F	N	Q	Z	X	E	O	W	O	E

**Step-3:** Reverse Offset rule with the first decrypted message

<b>Message</b>	P	F	N	Q	Z	X	E	O	W	O	E
	15	5	13	16	25	23	4	14	22	14	4
<b>Reverse offset rule with Pell number</b>	15	5	13	16	25	23	4	14	22	14	4
	-	-	-	-	-	-	-	-	-	-	-
	0	1	2	5	12	29	70	169	408	985	2378
	15	4	11	11	13	-6	-66	-155	-386	-971	-2371
<b>Mod 26</b>	15	4	11	11	13	20	12	1	4	17	18
<b>Second Decrypted message is</b>	P	E	L	L	N	U	M	B	E	R	S

## 1. Result analysis

S.No	First encryption	Time for first encryption	Super encryption	Time for super encryption
1	PELL NUMBER	1.01 MILLI SEC	Affine or vigenere	3.01 MILLI SEC
2	LUCAS NUMBER	1.3 MILLI SEC	Affine or vigenere	3.35 MILLI SEC

## CONCLUSIONS

For multiple encryptions, two different algorithms were performed. Initial encryption is performed with Lucas or Pell numbers using offset rule whereas for super encryption two different ciphers either Affine or Vigenere are performed. In view of the time both are more or less the same.

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