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SOME CHARACTERISTIC PROPERTIES OF FUZZY b-OPEN SETS

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ABSTRACT

The purpose of the paper is to obtain some of interesting characteristic properties of fuzzy b-open set in fuzzy topological space by using fuzzy b-open set introduce by Benchalli and Karnel [3].

Keyword: *fuzzy b-open set, fuzzy maximal open set, fuzzy minimal closed set, fuzzy topological space.*

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1. INTRODUCTION

The study of fuzzy set was initiated with the famous paper of Zadeh [1] in 1965. Since then the notion of fuzziness has been applied in all the branches of science and technology. Chang [2] in1968 introduced the notion of fuzzy topological space and opened the way for the subsequent growth of the numerous fuzzy topological concepts. In this paper we introduce some interesting properties of fuzzy b-open set in fuzzy topological space.

2. PRELIMINARIES

Let X be a non-empty set and I, the unit interval [0, 1]. A fuzzy set A in X has been characterized by a function $\mu_A : X \to I$, where μ_A is called the membership function of A. $\mu_A(x)$ represents the membership grade of x in A. The empty fuzzy set is defined by $\mu_\phi(x) = 0$ for all $x \in X$, denoted by 0_X. Also X can be regarded as a fuzzy set in itself defined $\mu_X(t) = 1$ for all $t \in X$, denoted by 1_X.

Throughout this paper X denotes a nonempty fuzzy set. (X, τ) denotes a fuzzy topological space (fts in short).

A family $\tau \leq I^X$ of fuzzy sets is called a fuzzy topology for X if it satisfies the following three axioms;

1. $0_X, 1_X \in \tau$
2. $\forall A, B \in \tau \Rightarrow A \wedge B \in \tau$
3. $\forall (A_j)_{j\in J} \in \tau \Rightarrow \vee_{j\in J} A_j \in \tau$

The pair (X, τ) is called a *fuzzy topological space or fts*.

Fuzzy operations: The closer and interior of a fuzzy set A of X are define by
cl(A) = inf $\{K: A \leq K, K^c \in \tau\}$
int(A)= sup$\{O: O \leq A, O \in \tau\}$

Fuzzy minimal open set and fuzzy minimal closed sets:

Definition 2.1 [4]: A nonzero fuzzy open set A ($\neq$1) of a fuzzy topological space (X, τ) is said to be a fuzzy minimal open set if any open set which is contained in A is either 0 or A.

Similarly a nonzero fuzzy closed set B ($\neq$1) of a fuzzy topological space (X, τ) is said to be fuzzy minimal closed set if any fuzzy closed set which is contained in B is either 0 or B.

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Fuzzy b-open set and fuzzy b-closed sets:

Definition 2.2[3]: A fuzzy set A in a fts X is called fuzzy b-open (fb-open) set if and only if A≤int (cl A) ∨ cl (int A)

Definition 2.3[3]: A fuzzy set A in fts X is called fuzzy b-closed (fb-closed) set if and only if int (cl A) ∨ cl (int A) ≤A

Definition 2.3[5]: A fuzzy set A of a fts X is called fuzzy pre-open (fuzzy pre-closed) if A≤int cl A (cl int A≤A)

Definition 2.4 [5]: A fuzzy set A of a fts X is called fuzzy semi-open (fuzzy semi- closed) if A≤ cl int A (int cl A≤A)

Definition 2.5[5]: A fuzzy set A of a fts X is called fuzzy semi-pre-open (fuzzy semi-pre closed) if A≤cl int cl A (int cl int A≤ A)

Lemma 2.1: Let (X, δ) be a Fuzzy topological space, then
(i) An arbitrary union of fb-open set is fb-open set.
(ii) An arbitrary intersection of fb-closed set is fb-closed.

Proof:
(i) Let $\{A_\alpha\}$ be the collection of fb-open sets. Then for each α , $A_\alpha \leq$ (cl int A_α) ∨ (int cl A_α).
Now ∨ A_α ≤∨((cl int, A_α)∨(int cl , A_α)) ≤(((cl int(∨A_α))∨(cl int (∨, A_α))). Thus ∨A_α is fuzzy b-open set.
(ii) Similar by taking complement.

Proposition 2.1: Let A be a fuzzy subset of a fuzzy topological space (X, τ), then
(1) fbcl(int A)=int(fbcl A)=int (cl(int A))
(2) fbint(cl A)=cl(fbint A)=cl(int(cl A))
(3) fbcl(fsint A)=fscl(fsint A)=fsint(fbcl A)
(4) fbint(fscl A)=fsint(fscl A)
(5) fsint(fbcl A)=fscl A ∩ cl(int A)
(6) fscl(fbint A)=fsint A ∪ int(cl A)

3. SOME CHARACTERISTIC PROPERTIES OF FUZZY b-OPEN SETS

The following theorems procure beforehand a unique property which characterizes fuzzy b-open sets:

Theorem 3.1: A subset A of a topological space (X, τ) is fuzzy b-open set if and only if every fuzzy closed set F containing A, there exist the union of fuzzy maximal open set M contained in a cl(A) and the fuzzy minimal closed set N containing int(A) such that A≤M∪N≤F

Proof: Let A be a fuzzy b-open set in a fuzzy topological space (X, τ).

Then A≤ cl(int(A))∪int(cl(A)) (1)

Let A≤F and F is fuzzy closed so that cl(A) ≤F.

Let M=int(cl(A)), then M is the fuzzy maximal open set contained in cl(A)

Let N=cl(int(A)), then N is the fuzzy minimal closed set containing int(A)

Again, A≤cl(A) ≤F & int(cl(A)) ≤cl(A).

i.e. int(cl(A)) ≤ F (2)

Next, int(A)<A
i.e. cl(int (A)) ≤cl(A) & cl(A) ≤F

i.e. cl(int(A)) ≤A (3)

int(cl(A))∪cl(int(A)) ≤ F (4)

Combining (1) & (4), we have

A≤ int(cl(A))∪cl(int(A)) ≤ F
Or , A≤M ∪ N≤ F

Conversely, assume that the condition hold good i.e. $A \leq M \cup N \leq F$ where A is a fuzzy subset in a fuzzy topological space, F is fuzzy closed & M is the fuzzy maximal open set contained in a cl(A), N is the fuzzy minimal closed set containing int(A).

Therefore, M=int(cl(A)) & N=cl(int(A))

Thus the above condition reduce to $A \leq int(cl(A)) \cup cl(int(A)) \leq F$

This means that A is a fuzzy b-open set.

Hence, the theorem.

Theorem 3.2: A fuzzy subset A in a topological space (X, τ) is fuzzy b-open set if and only if there exists a fuzzy pre-open set U in (X, τ) such that $U \leq A \leq fpcl(U)$.

Proof: Let A be a fuzzy sub set of X. Since

$A \leq fpcl\,(fpint\,(A))$ (1)

Now, as usual fpint $A \leq A$ and U=fpint (A)= a fuzzy pre-open set. Hence, from (1) it follows that $U \leq A \leq fpcl(U)$.

Conversely, for a fuzzy set A there exist fuzzy pre-open set U such that

$U \leq A \leq fpcl(U)$ (2)

Science, fpint(A) is the fuzzy maximal pre-open set contained in A, hence,

$U \leq f\,pint(A) \leq A$ (3)

Now, $fpcl(U) \leq fpcl(fpint(A))$ [from (3)] (4)

Combining (2) & (3) we get,
$A \leq fpcl(fpint\ A)$, which means that A is a fuzzy b-open set.

Hence, the theorem.

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