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FORECASTING MODEL BASED ON FUZZY TIME SERIES WITH FIRST ORDER DIFFERENCING

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ABSTRACT

This paper deals with the design of forecasting model for annual district outlay for district Nagpur using fuzzy time series with first order differencing. Paper suggested modification in Fuzzy Time Series algorithm while defuzzification to forecast values of annual outlay. In fuzzy time series numerous modifications were suggested by various authors to improve forecasting accuracy or computation methodology. It is shown that proposed model achieves a significant improvement in forecasting accuracy as compared to fuzzy time series forecasting with equal weight method while defuzzification. To illustrate the forecasting process, the historical annual district outlay for Nagpur district is used.

Keywords: *Fuzzy time series, Fuzzy logical Relationships, Forecast error.*

INTRODUCTION

Fuzzy set theory and fuzzy logic was first introduced by Zadeh (1965). Fuzzy time series originally proposed by Song and Chissom [1] to forecast student enrollment at the University of Albama. Significant drawbacks of the fuzzy time series model developed by the Song & Chissom is that they are associated with unnecessary high computational overheads due to complex matrix operations in the proposed algorithm, in order to reduce these computational overheads of Time-variant and time-invariant models chen[2] proposed a simplified model including only simple arithmetic operations. Selection of interval partition in universe of discourse and formulation of fuzzy relationships both of these highly influence forecast accuracy and thus are considered central to fuzzy time series. Wang, Chen, Lee [3] use high order time variant fuzzy time series model to deal with enrollment forecasting. Chen and Hwang [4] developed two algorithms for temperature prediction to deal with forecasting problem. Huarng [5] presented a heuristic model for fuzzy time series forecasting. Sullivan and Woodall proposed the 'Markov –based model' [6] by using conventional matrix multiplication. Saxena & Sharma[7] proposed percentage change as the universe of discourse and mean based partitioning using historical enrollment of University of Melike Sah and Konstantin Degtiarev[8] proposes novel improvement of forecasting approach based on time invariant fuzzy time series and shows that time-invariant method improves the performance of forecasting process. Tsaur[9] proposed a new residual analysis method using Fourier series transformation into fuzzy time series model for improving the forecasting performance. Zhang & Zhu [10] uses the K-means clustering algorithm to cluster the universe of discourse and then adjust the clusters into intervals. In this present paper predicted values are defuzzified using harmonic mean to arrive at crisp values and results are compared with fuzzy time series with first order differencing with defuzzification using equal weight Annual district outlay which is the annual allocation made by government for development work of district. Annual district outlay comprises various sectors which include Agriculture and allied Services, Rural Development, Irrigation, Energy, Mining and Industry, Transport, Social & Community Service and general service. Allocation of district outlay is made by government on the basis of State Plan outlay.

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1. FUZZY TIME SERIES

Basic Concepts of Fuzzy Time Series: Song & Chissom first proposed the definition of fuzzy time series in 1993(4).

Let U be the universe of discourse with

$U = \{u_1, u_2, u_3, ...u_n\}$ in which a fuzzy set A_i, i=1,2......n is defined as follows.

$A1= f_{Ai}(u_1)/u_1+f_{Ai}(u_2)/u_2+........+f_{Ai}(u_n)/u_n$ (1)

Where f_{Ai} is the membership function of the fuzzy set A_i, U_j is the element of fuzzy set A_i, and $fA_j(u_j)$ is the membership degree of U_j belonging to A_i, J=1.2.......,n.

Definition 1.1 Let the universe of discourse Y(t) (t =...,0, 1, 2...n,...) be a subset of R defined by the fuzzy set A_i, If F(t) consists of A_i (i=1,2.....,n), F(t) is defined as a fuzzy time series on Y(t) (t=...,0,1,2.....,n,...).

Definition 1.2: Suppose that F(t) is caused by F(t-1), then the relation of the first-order model F(t) can be expressed as F(t) F(t-1) 0 R(t,t-1), Where R(t,t-1) is the relation matrix to describe the fuzzy relationship between F(t-1) and F(t), and 'o' is the max-min operator.

Let the relationship between F(t) and F(t-1) be denoted by F(t-1) $\rightarrow$ F(t), (t =...,0,1,2,...,n,...).

Then, the fuzzy logical relationship between F(t) and F(t-1) is defined as follows.

Definition 1.3: Suppose F(t) = A_i is caused by F(t-1) =A_j, then the fuzzy logical relationship is defined as $A_i \longrightarrow A_j$.

If there are fuzzy logical relationships obtained from state A_2, Then a transition is made to another state A_j, j=1,2,...,n as $A_2 \longrightarrow A_3$, $A_2 \rightarrow A_2$,......, $A_2 \rightarrow A_1$; hence, the fuzzy logical relationships are grouped into a fuzzy logical relationship group as

$$A_2 \longrightarrow A_1, A_2, A_3,$$

Although, various models have been developed to establish fuzzy relationships, Chen's fuzzy logical relationship groups approach is easy to work with as compared to Song and Chissom. And this methodology is used in the proposed work.

Fuzzy Time Series Algorithm: Song and Chissom proposed the following method for solving fuzzy time series model.

Step-1: Define the universe of discourse U for the historical data. When defining the universe of discourse, the minimum value and maximum value of given historical data are obtained as D_{min} and D_{max} respectively. On the basis of D_{min} and D_{max}, we can define the universe of discourse U as $[D_{min}-D_1, D_{max}-D_2]$ where D_1, D_2 are proper positive numbers, then U is partitioned into n equal intervals with length l defined as $l=1/n\,[(D_{max}+D_2)-(D_{min}-D_1)]$.

Step-2: Define fuzzy sets on the universe of discourse U and fuzzifying the time series A fuzzy set Ai of U is defined by $Ai= f_{Ai}(u_1)/u_1+f_{Ai}(u_2)/u_2+........+f_{Ai}(u_n)/u_n$
Where fA_i is the membership function of fuzzy set A_i, fA_i: U $\longrightarrow$[0, 1], and $fA_i(u_j)$ indicates the grade of membership of u_j in A_i. Thus, the "district annual outlay" can be described by the fuzzy set of A_1 = (not many), A_2 = (not too many), A_3 = (many), A_4 = (many many), A_5 = (very many), A_6 = (too many), A_7 = (too many many). Each fuzzy set A_i (i=1,2,....,7) is defined on 7 intervals, which are $u_1 = [d_1,d_2]$, $u_2 = [d_2,d_3]$, $u_3 = [d_3,d_4]$, $u_4 = [d_4,d_5]$,......, $u_7 = [d_7,d_8]$. Hence, the fuzzy set A_1, A_2,........,A_7 are defined as follows.

A_1 = {1/u1, 0.5/u2, 0/u3, 0/u4, 0/u5, 0/u6, 0/u7},
A_2 = {0.5/u1,1/u2, 0.5/u3, 0/u4, 0/u5, 0/u6,0/u7},
A_3 = {0/u1, 0.5/u2, 1/u3, 0/u4, 0/u5, 0/u6, 0/u7},
A_4 = {0/u1, 0/u2, 0.5/u3, 1/u4, 0.5/u5, 0/u6, 0/u7},
A_5 = {0/u1, 0/u2, 0/u3, 0.5/u4, 1/u5, 0.5/u6, 0/u7},
A_6 = {0/u1, 0/u2, 0/u3, 0/u4, 0.5/u5, 1/u6, 0.5/u7},
A_7 = {0/u1, 0/u2, 0/u3, 0/u4, 0/u5, 0.5/u6, 1/u7},

Step-3: Fuzzify the historical data. This step aims to find an equivalent fuzzy set for each input data. The used method is to define a cut set for each A_i (i=1, 2....,7). If the collected time series data belongs to an interval ui, then it is fuzzified to the fuzzy set A_i.

Step-4: Determine the fuzzy logical relationship group, by the definition stated at 1.3 fuzzy logical relationship can be easily obtained from fuzzy set.

Step-5: Forcasting and defuzzifying outputs. The forcasting results of the first order forcasting model is based on the following rules.

Rule-1: If F (t-1) = A_i and the number of |Group (A_i)| = 0, then the predicted result at time t, m_i is the midpoint of interval u_i in which the maximum membership degree of A_i locates.

Rule-2: If F (t-1) = A_i and Group (A_i) = {A_{j1},A_{j2},......A_{jp}}, p>=1, then the predicted result at time t is $1/n\sum m_{ji}$, i=1,....,p

Predicted result at time t is modified using harmonic mean as; $n/\sum(1/m_{ji})$, i=1,2....,p

Where m_{j1}, m_{j2},.....m_{jp} is the mid point of the interval u_{j1},u_{j2},......,u_{jp} in which the maximum membership degree of A_{j1}, A_{j2},....., A_{jp} locates respectively.

MSE (Mean Squared Error) is defined as.

$$MSE = 1/n \sum|\text{forecasted } i - \text{Actual } i|^2, \; i= 1, 2,.....n$$

MAPE (Mean Absolute Percentage Error) is defined as.

$$MAPE = 1/n\sum|\text{forecasted}_t - \text{Actual}_t|/\text{actual}_t, \; i= 1, 2,.....n$$

2. NUMERICAL CALCULATION AND GRAPHICAL PRESENTATION

As shown in Table 1, there are 10 observation and two attribute District annual plan and year.

Year	Outlay(Rs in Crore)	Fuzzy set
2005-2006	25.53	A_1
2006-2007	36.89	A_1
2007-2008	35.73	A_1
2008-2009	75.00	A_2
2009-2010	96.80	A_2
2010-2011	122.35	A_3
2011-2012	160.00	A_4
2012-2013	160.00	A_4
2013-2014	175.00	A_4
2014-2015	214	A_5
2015-2016	300	A_7

Step-1: The universe of Discourse U is defined as
U = [D_{min}-D_1, D_{max}+D_2]
U = [25.53-5, 300+0] = [30.53, 300]

Step-2: The universe of discourse is partitioned in to seven equal length of interval.
U_1 = [20.53, 60], U_2 = [60,100], U_3 = [100,140], U_4 = [140, 180], U_5 = [180, 220],
U_6 = [220, 260], U_7 = [260, 300]

Step-3: The memberships of the linguistic variables are as follows.
A_1 = {1/u1, 0.5/u2, 0/u3, 0/u4, 0/u5, 0/u6, 0/u7},
A_2 = {0.5/u1,1/u2, 0.5/u3, 0/u4, 0/u5, 0/u6,0/u7},
A_3 = {0/u1, 0.5/u2, 1/u3, 0/u4, 0/u5, 0/u6, 0/u7},
A_4 = {0/u1, 0/u2, 0.5/u3, 1/u4, 0.5/u5, 0/u6, 0/u7},
A_5 = {0/u1, 0/u2, 0/u3, 0.5/u4, 1/u5, 0.5/u6, 0/u7},
A_6 = {0/u1, 0/u2, 0/u3, 0/u4, 0.5/u5, 1/u6, 0.5/u7},
A_7 = {0/u1, 0/u2, 0/u3, 0/u4, 0/u5, 0.5/u6, 1/u7},

Step-4: Fuzzy logical relationships.

$A_1 \rightarrow A_1$	$A_1 \rightarrow A_2$	$A_2 \rightarrow A_2$	$A_2 \rightarrow A_3$
$A_3 \rightarrow A_4$	$A_4 \rightarrow A_4$	$A_4 \rightarrow A_5$	$A_5 \rightarrow A_7$

Step-5: Fuzzy logical Relations Groups (FLRG's)

Groups	Fuzzy Logic Relationships
1	$A_1 \longrightarrow A_1, A_2$
2	$A_2 \longrightarrow A_2, A_3$
3	$A_3 \longrightarrow A_4$
4	$A_4 \longrightarrow A_4, A_5$
5	$A_5 \longrightarrow A_7$

Computation of Harmonic Mean;
HM= $n/\sum(1/m_{ji})$, i=1,2....,p
HM= 3/(1/15+1/20+1/25) for series of values, m_1=15, m_2=20 and m_3=25
HM= 19.10

Table-2: Forecasting Values of Annual District Plan

Year	Outlay (Rs in Crores)	Forecast (AM)	Forecast (HM)
2005-2006	25.53	60	53.33
2006-2007	36.89	60	53.33
2007-2008	35.73	60	53.33
2008-2009	75	100	96.15
2009-2010	96.80	100	96.15
2010-2011	122.35	160	160
2011-2012	160.00	180	177.78
2012-2013	160.00	180	177.78
2013-2014	175.00	180	177.8
2014-2015	214	280	280
2015-2016	300	280	280

Forecasting performance by MSE (Mean Squared Error) and MAPE (Mean Absolute Percentage Error).

Charecteristics	AM	HM
MSE (Mean Squared Error)	994.5	786.4
MAPE (Mean Absolute Percentage Error)	39.84	32.00

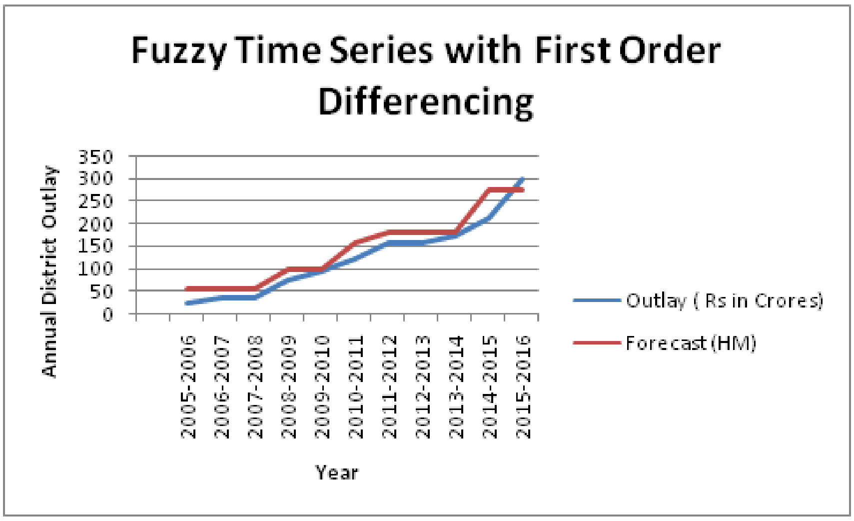

Figure-1: Actual and Forecasted value of Annual District Outlay.

CONCLUSION

Forecasting model for fuzzy time series for Annual district outlay for Nagpur district is computed by using first order differencing method, At step 5 of time series algorithm forecasting values are defuzzified using harmonic mean instead of equal weight. Study shows considerable decrease in mean square error and mean absolute percentage error when compared with equal weight method in the algorithm. Further studies will be carried out with fuzzy time series with higher order differencing.

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