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# CHARACTER TABLE OF $\mathrm{End}_{\mathrm{K}(\mathrm{G}\times\mathrm{H})}\mathrm{K}\Omega$ VIA COMPUTER ALGEBRA SYSTEM GAP

**SAAD. O. BEDAIWI[1], MAROA. S. AL-SHAMMERY*[2]**

[1,2]Department of Mathematics,
College of Science, University of Al-Mustansiryah. Baghdad- Iraq.

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***ABSTRACT***

*In our present work, we compute intersection matrices via regular representation for the $E \coloneqq End_{K(G\times \boldsymbol{H})}K\Omega$ of special k- algebras, and then we construct character tables of $E$. A computer algebra system GAP has been used intensively in our work.*

***Key Words:*** *Permutation representation, Character table, GAP.*

## 1. INTRODUCTION

In 1900, representation theory of finite groups have been developed essentially by Frobenius, Schur and Burnside. When dealing with fields of prime characteristic it turns to called modular representation theory. It started with the work of R.Brauer in 1930.

Representation theory is interested with the ways of writing a group as a group of matrices. Not only is the theory beautiful in its own just, but also provides one of the keys to a particular constructing character table of $End_{K(\boldsymbol{G}\times \boldsymbol{H})}K\Omega$.

In the last decades the existing of computers and developing of algorithms give rise to many computer algebra systems such as MAGMA, SINGULAR, GAP ...etc.

Soon people are capable of translating problems from representation theory into algorithms handled by computer algebra systems. W.R. Unger in 2006 [6] studied and computing the character table of finite group.

In our present work, we compute the intersection matrices via regular representation for the $E \coloneqq End_{K(G\times \boldsymbol{H})}K\Omega$ of special k- algebras, and then we construct character tables of $E$ in two Cases:

**Case-(I):** $D = G \times G$ , where $|\mathrm{G}| = 6$ , with attributes (non-abelian, solvable, non-perfect) acting on the set $\Omega \times \Omega$ where $\Omega = \{1,2,3,4,5,6\}$ and action = on pairs.

**Case-II:** $D = G \times H$ , where $|\mathrm{G}| = 42$, with attributes (non-abelian, solvable, non-perfect) and $|\mathrm{H}| = 120$ , with attributes (non-abelian, non-solvable, non-perfect) acting on the set $\binom{\Omega}{\mathrm{k}} \times \binom{\Omega}{\mathrm{k}}$ where $\Omega = \{1,2,3,4,5,6,7\}$, $\binom{\Omega}{\mathrm{k}}$ is the k-element subsets of the power set of $\Omega$ and action = On TuplesSets .

A computer algebra system GAP has been used intensively in our work.

## PRELIMINARIES

We give a brief discussion and basic notations which we may need throughout our work. Unless otherwise stated, K denotes a finite field, V a vector space over K.

*Corresponding Author: Maroa. S. AL-shammery*[2], [1,2]Department of Mathematics,
College of Science, University of Al-Mustansiryah. Baghdad- Iraq.*

**2.1 Definition: [3]** A group G is said to act (from left) on a non-empty finite set Ω if there exist a map $G \times \Omega \to \Omega$ ; $(g,\omega) \to g.\omega$ satisfies: $g_1.(g_2.\omega) = (g_1.g_2).\omega,\ \forall\, g_1, g_2 \in G, \omega \in \Omega\ and\ i_G.\omega = \omega$

- Ω is called ***a G-set.***
- For any $\omega \in \Omega$ the set $\omega^G := G.\omega = \{g.\omega | g \in G\}$ is called a ***G-orbit in Ω***.
- Ω is called ***transitive*** if $\Omega = \omega^G$.

**2.2 Definition: [1]** Let G be a finite group, A homomorphism $\delta: G \to GL(V)$ (or $\delta: G \to GL(n,K)$ of G into the group of isomorphism of V onto itself (or into group of all $n \times n$ non− singular matrices over K), is called ***a* representation** of G in V.

**2.3 Computer algebra system GAP**

The main tool in our study is the computer algebra system GAP [7]. A computer software GAP enables us to manipulating problems in Group theory as well as representation theory algorithmically. We refer to references [7], [8] and [9] for suitable introduction and more details.

**2.4 Remark:** In GAP a group act from right and product for $g_1, g_2$ is defined as $g_1.g_2 = g_2 * g_1$, The isomorphism $(G,.) \to (G,*)$, $g \to g^{-1}$, Provides a way to translate results.

**2.5 Definition: [5]** Let $(G,.)$ be a finite group, and K be a commutative ring, the **group algebra** of *G* over K which we will denote by $K^G$ (or simply$KG$), is the set of mappings from G to K (simply$KG = \{a | a: G \to K\}$).

And define for $a, b \in KG, \beta \in K$ and $g \in G$;

$$(a+b)(g) := a(g) + b(g),\ (ab)(g) := \sum_{h\in G} a(h)b(h^{-1}g), (\beta a)(g) := \beta a(g).$$

**2.6 Remark:**

1. The KG is a K-algebra with unit $1^{\circ}$ where 1 is the unit in G
2. If K=R, The group algebra is called G***roupRing***.

In GAP, fortunately we have the command GroupRing to construct such structure, for example:
gap> K:=Ring(0,1); G:=One Small Group(168);;
gap> # pc denote the polycyclic group
gap> KG:=Group ring (K,G); <free left module over Integers, and ring-with-one, with 5 generators>

**2.7 Definition [2]:** Let A and B be n×n matrices. We say that A and B are simultaneously diagonalizable if there exists an invertible matrix S such that S^(-1) AS and S^(-1) BS are diagonal matrices.

**2.8 Theorem [2]:** If A, B are both diagonalizable $n \times n$ matrices, then A and B are simultaneously diagonalizable if and only if AB and BA are commute.

**3. PERMUTATION REPRESENTATION**

**3.1 Definition: [4]** Let K be commutative ring andΩ is a finite G-set. We define KΩ to be the free K -module with basis Ω and consider KG-module by extending the action G×Ω→Ω to KG×KΩ→KΩ

Thus $\sum_{h\in G} a_h h \sum_{\omega\in\Omega} b_\omega \omega = \sum_{h\in G}\sum_{\omega\in\Omega} a_h b_\omega (g.\omega) \in K\Omega\ for\ a_h, b_\omega \in K$; KΩ is called the ***permutation module*** corresponding toΩ (and K), the corresponding representation $\delta_\Omega : KG \to End_{KG}\, k\Omega$ , is called ***permutation representation*** of KG.

**3.2 Remark:** The ***regular module*** $_{KG}$KG is itself a permutation module, with taking KΩ = KG.

**3.3 Definition: [4]** If W and V are a G-sets the Cartesian product $W \times V$is also a G- set in natural way define

$$g.(w,v) = (gw, gv)\ for\ g \in G, w \in W, v \in V.$$

−If O is G-orbit on $W \times V$ then

$$\acute{O} = \{(v,w) \in V \times W |\ (w,v) \in O\} \text{ is a G-orbit on} W \times V.$$

− If $a \in W, b \in V$, then we defines: $O(a) = \{v \in V | (a,v) \in O\}$, $\acute{O}(b) = \{w \in W |\ (w,b) \in O\}$.

- If $O = \acute{O}$ then O is called ***self-paired***.
- If $O_1, \dots, O_{r}$ are the orbits of G on Ω×Ω the cardinalities$|O_1(\omega)|, \dots, |O_r(\omega)|$ are called the ***subdegree*** of Ω.

**3.4 Definition: [4]** Let $W = \{w_1, \dots, w_n\}$and $V = \{v_1, \dots, v_m\}$be G-sets, then a KG-homomorphism $\varphi: KW \to KV$ is a K-linear map commuting with the action of the elements of G on KW and KV.

 

**3.5 Remark: [4]**

(1) The matrix associated to K-linear map in definition (3.4) takes the form

$$V[\varphi]W = [a_{vw}]_{w\in W, v\in V} \in K^{n\times m}$$

with respect to the K-bases W and V must satisfy:

$$[\delta_V(g)]_V . [a_{vw}] = [a_{vw}] . [\delta_W(g)]_W, \quad \forall\, g \in G.$$

That is $$a_{g^{-1}v,w} = a_{v,gw}, \ \forall\, g \in G, w \in W, v \in V.$$

(2) From part (1), the entries of $V[\varphi]W$ must be constant on the orbits of G on V×W.

**3.6 Definition: [4]** Let $W = \{w_1, \dots, w_n\}$ and $V = \{v_1, \dots, v_m\}$ be G-sets, the set of all KG-homomorphisms from KW into KV is denoted by $Hom_{KG}(KW, KV)$.

**3.7 Theorem: [4]** If K a commutative ring and W and V are G-sets then:
$Hom_{KG}$(KW, KV) = $\{\varphi \in\ Hom_K$(KW, KV)| $_W[\varphi]_V$=$[a_{vw}]$ with $a_{v,w} = a_{gv,gw}$ , $\forall\, g \in G, w \in W, v \in V\}$

For any G-orbit O on V×W let $\theta_O \in Hom_{KG}$(KW, KV) be defined by: $\theta_O(w) = \acute{O}(w)^+ := \sum_{v\in \acute{O}(w)} y$ , thus the matrix of $\theta_O$ with respect to the bases W and V is $_V[\theta_O]_W$=$[a_{vw}]$ with $a_{vw} = \begin{cases} 1 & for\ (v,w) \in O, \\ 0 & else\ . \end{cases}$ and $\{\theta_O$|O a G-orbit on V×W} is a K-basis for $Hom_{KG}$(KW, KV) often called ***standard basis*** of $Hom_{KG}$(KW,KV).

**3.8 Remark:** In the case that W=V the standard basis of $Hom_{KG}$(KW, KW) = $End_{KG}$(KW) is called ***schur basis.***

## 4. CHARACTER TABLE OF $End_{K(N\times H)}K\Omega$

**4.1 Theorem: [4]** Let Ω be a finite G-set and $O_i (i = 1, \dots, r)$ be the orbits of G on Ω ×Ω and $\theta_i = \theta_{O_i}$ be the schur basis elements of E: =$End_{KG}$K Ω then:

a) $\theta_i\theta_j = \sum_{K=1}^{r} a_{ijk}\,\theta_k$ $\ with\ a_{ijk} = |O_i(w) \cap \acute{O}_j(v)|\ for (w,v) \in O_k . a_{ijk}$ is independent of the choice of $(w,v) \in O_k$. The $a_{ijk}$ are called the ***intersection number*** of G-set Ω.

b) If Ω is transitive and $\theta_{\acute{i}} := \theta_{\acute{O}_i}$ then

$$a_{i\acute{j}1} := |\ O_i(\omega)|.\delta_{i,j} \quad for\ \omega \in \Omega$$

where $O_1 := \{(\omega,\omega)|\omega \in \Omega\}$ , so that $\theta_1 = 1_k$.Then the K-Linear map

$$\vartheta_1 : E \to K \quad with \quad \vartheta_1 : \theta_O \to |\theta(\omega)|.1_K \quad for\ any\ \omega \in \Omega$$

is a representation of E, as a rule called the ***principal character*** of E.

**4.2 Remark: [4]** The representation of $E := End_{KG}(K\Omega)$ given by:

$$End_{KG}K\Omega \to K^{r\times r},\ \ \theta_i \to A_i = [a_{ijk}]_{j,k=1,\dots,r} \qquad for\ i = 1, \dots, k$$

Is called the ***regular representation*** $A_i$ of $End_{KG}K\Omega$ with respect to the Schur basis, the matrices $A_i$ are called the ***intersection matrices*** of Ω.

**4.3 Cayley's Theorem [3]:** Every group of finite order is isomorphic to a subgroup of $S_n$.

**Proof:** see [3].

### 4.4 Finding character table of $End_{K(G\times H)}(K\Omega)$

**Case-I:** $\boldsymbol{D} = G \times G$, *where* $|G| = 6$, *with attributes (non-abelian, solvable, non-perfect) acting on the set* $\Omega \times \Omega$ *where* $\Omega = \{1,2,3,4,5,6\}$ *and left action.*

**Step -1:**

```
gap> alsma:=AllSmallGroups(6,IsAbelian,false,IsSolvable,true,IsPerfect,false);
[<pc group of size 6 with 2 generators> ]
gap> G:=alsma[1];;
gap> GxG:=DirectProduct(G,G); <pc group of size 36 with 4 generators>
```

By Cayley's Theorem look for $D$ is isomorphic to $G \times G$ with attributes (non-abelian, solvable, non-perfect).

```
gap> S:=SymmetricGroup(6);;AS:=AllSubgroups(S);;
gap> Fi:=Filtered(AS,i->Size(i)=36);;
gap> D:=Fi[1];
```

Group([ (1,3)(4,5), (1,4,3,5,2,6) ])
gap> IsSolvable(D);
true
gap> IsPerfect(D);
false

**Step -2:** Find orbits of $D = G \times G$ on $\Omega \times \Omega$
$O_1$=$D$.(1,1)={(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)}, $|O_1| = 6$
$O_2$=$D$.(1,2)={(1,2),(3,1),(2,3),(1,3),(2,1),(3,2),(4,5),(6,4),(5,6),(4,6),(5,4),(6,5)}, $|O_2| = 12$
$O_3$ =$D$ . (1, 4)={(1,4),(3,4),(2,4),(1,6),(3,6),(2,6),(1,5),(3,5) ,(2,5),(4,2),(6,2),(5,2), (4,1),(6,1),(5,1),(4,3) , (6,3),(5,3)}
$|O_3| = 18$

These can be seen as in the table (1) below, in which at position (i, j) the orbit of (i, j) can be found.

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **1** | $O_1$ | $O_2$ | $O_2$ | $O_3$ | $O_3$ | $\mathbf{O_3}$ |
| **2** | $O_2$ | $O_1$ | $O_2$ | $O_3$ | $O_3$ | $\mathbf{O_3}$ |
| **3** | $O_2$ | $O_2$ | $O_1$ | $O_3$ | $O_3$ | $\mathbf{O_3}$ |
| **4** | $O_3$ | $O_3$ | $O_3$ | $O_1$ | $O_2$ | $\mathbf{O_2}$ |
| **5** | $O_3$ | $O_3$ | $O_3$ | $O_2$ | $O_1$ | $\mathbf{O_2}$ |
| **6** | $O_3$ | $O_3$ | $O_3$ | $O_2$ | $O_2$ | $\mathbf{O_1}$ |

**Table-1**

Since the table is symmetric, then all orbits of on $\Omega\times\Omega$ are self-paired.

**Step-3:** Find the intersection numbers and intersection matrices. The intersection numbers can be read off from table (2):

| | $J$ | | | |
|---|---|---|---|---|
| | 1 | 2 | 4 | |
| $\mathbf{O_1(j)}$ | {1} | {2} | {4} | $(1,1) \in O_1 = \acute{O}_1$ |
| $\mathbf{O_2(j)}$ | {3,2} | {1,3} | {5,6} | $(1,2) \in O_2 = \acute{O}_2$ |
| $\mathbf{O_3(j)}$ | {4,6,5} | {4,5,6} | {1,3,2} | $(1,4) \in O_3 = \acute{O}_3$ |

**Table-2**

Let $a_i = [a_{i,j,,k}\ ]_{i,j,k=1,2,3}$ , so that regular representation of $E$ with respect to the schur basis is given by $O_i \to a_i$ $(for\ i = 1,2,3)$, Then we have three intersection matrices (matrix representation) of $E$.

$$a_1 = I_3 = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix},\ a_2 = \begin{bmatrix}0 & 1 & 0\\2 & 1 & 0\\0 & 0 & 2\end{bmatrix}, a_3 = \begin{bmatrix}0 & 0 & 1\\0 & 0 & 2\\3 & 3 & 0\end{bmatrix}.$$

**Step-4:** Check the above three intersection matrices of $E$ are "simultaneously diagonalizable".

The **eigenvalue** and **eigenvector** of $a_1$ are

$$\lambda_1 = \lambda_2 = \lambda_3 = 1\ with\ v_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}, v_2 = \begin{bmatrix}0\\1\\0\end{bmatrix}, v_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}.$$

The **eigenvalue** and **eigenvector** of $a_2$ are

$$\lambda_1 = -1\ with\ v_1 = \begin{bmatrix}1\\-1\\0\end{bmatrix}, \lambda_2 = \lambda_3 = 2\ with\ v_2 = \begin{bmatrix}1\\2\\0\end{bmatrix}, v_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}.$$

The **eigenvalue**s and **eigenvector**s of $a_3$ are

$$\lambda_1 = 0\ with\ v_1 = \begin{bmatrix}1\\-1\\0\end{bmatrix}, \lambda_2 = 3\ with\ v_2 = \begin{bmatrix}1\\2\\3\end{bmatrix}, \lambda_3 = -3\ with\ v_3 = \begin{bmatrix}1\\2\\-3\end{bmatrix}$$

Since the eigenvectors of $a_1$, $a_2$ and $a_3$ are linearly independent the intersection matrices are diagonalizable. The intersection matrices $a_1$, $a_2$ and $a_3$ are commute, which can therefore simultaneously diagonalizable. Since complete reducibility is the analog of diagonalizability in representation theory, thus in this case $End_{K(G\times G)}K\Omega$ has three irreducible representations $\vartheta_i$ , $i = 1,2,3$.

**Step-4:** Creation character table of $End_{K(G\times G)}K\Omega$.

Since the intersection matrices $a_i$ $(for\ i = 1,2,3)$ are commute, then the common eigenvectors of its

$$\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}1\\-1\\0\end{bmatrix}, \begin{bmatrix}1\\2\\0\end{bmatrix}$$

Represent rows of character table of $End_{K(G\times G)}K\Omega$.

| | $\boldsymbol{\theta_1}$ | $\boldsymbol{\theta_2}$ | $\boldsymbol{\theta_3}$ |
|---|---|---|---|
| $\boldsymbol{\vartheta_1}$ | 1 | 2 | 3 |
| $\boldsymbol{\vartheta_2}$ | 1 | -1 | 0 |
| $\boldsymbol{\vartheta_3}$ | 1 | 2 | 0 |

**Table-3**

- For the following case we use only GAP to construct the character table since it will be of large size and unpractical to handle with hand.

**Case-II:** $D = G \times H$ , *where* $|G| = 42$, *with attributes (non-abelian, solvable, non-perfect) and* $|H| = 120$ *with attributes (non-abelian, non-solvable, non-perfect) acting on the set* $\binom{\Omega}{k} \times \binom{\Omega}{k}$ *where* $\Omega = \{1,2,3,4,5,6,7\}$, $\binom{\Omega}{k}$ *is the k-element subsets of the power set of* $\Omega$ *and action = On TuplesSets.*

```
gap> alsma120:=AllSmallGroups(120,IsAbelian,false,IsSolvable,false,IsPerfect,false);
[ Group([ (1,2,3,4,5), (1,2) ]), Group([ (1,2,3,5,4), (1,3)(2,4)(6,7) ]) ]
gap > G:=alsma120[1];;
gap > alsma42:=AllSmallGroups(42,IsAbelian,false,IsSolvable,true,IsPerfect,false);;
gap > H:=alsma42[1]; <pc group of size 42 with 3 generators>
gap > GxH:=DirectProduct(G,H); <group of size 5040 with 5 generators>
gap > IsSolvable(DirectProduct(GxH));IsPerfect(DirectProduct(GxH)); false
false
```

By Cayley's Theorem look for $D$ is isomorphic to $G \times H$ with attributes (non-abelian, non-solvable, non-perfect).

```
gap> S:=SymmetricGroup(7);AS:=AllSubgroups(S);;
Sym( [ 1 .. 7 ] )
gap> F:=Filtered(AS,i->Size(i)=5040);
[ Group([ (1,3,2,7)(4,6), (1,4,5,2,3,7) ]) ]
gap> D:=F[1];
Group([ (1,3,2,7)(4,6), (1,4,5,2,3,7) ])
gap> IsSolvable(D);IsPerfect(D);
false
false
```

Find orbits of $D = G \times H$ on $\binom{\Omega}{k} \times \binom{\Omega}{k}$

```
gap> #k=4
gap> omg:=[1..7];;com:=Combinations( omg,4);
[[ 1, 2, 3, 4 ], [1, 2, 3, 5], [1, 2, 3, 6], [1, 2, 3, 7], [1, 2, 4, 5], [1, 2, 4, 6], [1, 2, 4, 7],[1, 2, 5, 6], [1, 2, 5, 7], [1, 2, 6, 7],
[1, 3, 4, 5], [1, 3, 4, 6], [1, 3, 4, 7], [1, 3, 5, 6],[1, 3, 5, 7], [1, 3, 6, 7], [1, 4, 5, 6], [1, 4, 5, 7], [1, 4, 6, 7], [1, 5, 6, 7],
[2, 3, 4, 5],[2, 3, 4, 6], [2, 3, 4, 7], [2, 3, 5, 6], [2, 3, 5, 7], [2, 3, 6, 7], [2, 4, 5, 6], [2, 4, 5, 7], [2, 4, 6, 7], [2, 5, 6, 7],
[3, 4, 5, 6],[3, 4, 5, 7],[3, 4, 6, 7], [3, 5, 6, 7], [4, 5, 6, 7]]
gap > orb := Orbits(D, Tuples(com,2) , OnTuplesSets);;
gap > List( orb , Length ); [35, 420, 630, 140]
gap > #principal character
gap > List([1,2,3,4],i -> Length(Filtered(orb[i],p -> p[1] = [1,2,3,4]))); [1, 12, 18, 4]
gap> #implementation"GAP_code" of The intersection matrices
gap > a := [];; x := 1;; y := 1;;
gap > for i in [1..Length(orb)] do
> a[i] := []; # a[i] will be the i-th intersection matrix
> for j in [1..Length(orb)] do
> a[i][j]:= []; # a[i][j] will be the j-th row of a[i]
```

```
> for k in [1..Length(orb)] do
> x:=orb[k][1][1]; y:=orb[k][1][2]; # [x,y] in orb[k]
> a[i][j][k] := Size( Intersection (
> Filtered(com,z->[x,z] in orb[i]),
> Filtered(com,z->[y,z] in orb[j]) ) );
> od;od;od; gap>Display(a[1]);Display(a[2]);Display(a[3]);Display(a[4]);
```

$$a_1 = I_4, a_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 12 & 5 & 4 & 0 \\ 0 & 6 & 6 & 9 \\ 0 & 0 & 6 & 3 \end{pmatrix}, a_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 6 & 6 & 9 \\ 18 & 9 & 9 & 9 \\ 0 & 3 & 2 & 0 \end{pmatrix}, a_4 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 3 \\ 0 & 3 & 2 & 0 \\ 4 & 1 & 0 & 0 \end{pmatrix}.$$

```
gap> e:= Exponent(D); 420
gap> p := 421;; K := GF(p);; Id:=Identity(K);;
gap> ev1 := Eigenvalues(K,TransposedMat(a[1])*Id); [Z(421)^0]
gap> ev2 := Eigenvalues(K,TransposedMat(a[2])*Id); [0*Z(421), Z(421)^278, Z(421)^406, Z(421)^194]
gap> ev3 := Eigenvalues(K,TransposedMat(a[3])*Id); [Z(421)^389, Z(421)^404, Z(421)^194]
gap> ev4 := Eigenvalues(K,TransposedMat(a[4])*Id); [Z(421)^210, Z(421), Z(421)^2, Z(421)^194]
gap> vs1:= List( Eigenspaces(K,TransposedMat(a[1])*Id),GeneratorsOfVectorSpace);
        [[[Z(421)^0, 0*Z(421),0*Z(421),0*Z(421)],[0*Z(421), Z(421)^0, 0*Z(421), 0*Z(421)],
        [0*Z(421), 0*Z(421), Z(421)^0, 0*Z(421)], [0*Z(421), 0*Z(421), 0*Z(421), Z(421)^0 ]]]
gap> vs2:= List( Eigenspaces(K,TransposedMat(a[2])*Id),GeneratorsOfVectorSpace );
        [[[Z(421)^0, 0*Z(421), Z(421)^194, Z(421)]], [[Z(421)^0, Z(421)^278, Z(421)^194, Z(421)^194]],
        [[Z(421)^0, Z(421)^406, Z(421)^389, Z(421)^2]], [[Z(421)^0, Z(421)^194, Z(421)^404, Z(421)^210]]]
gap> vs3:= List( Eigenspaces(K,TransposedMat(a[3])*Id),GeneratorsOfVectorSpace);
        [[[Z(421)^0, Z(421)^406, Z(421)^389, Z(421)^2]], [[Z(421)^0, Z(421)^194, Z(421)^404, Z(421)^210]],
        [[Z(421)^0, 0*Z(421), Z(421)^194, Z(421)], [0*Z(421), Z(421)^0, 0*Z(421), Z(421)^210]]]
gap> vs4:= List(Eigenspaces(K,TransposedMat(a[4])*Id),GeneratorsOfVectorSpace);
        [[[Z(421)^0, Z(421)^194, Z(421)^404, Z(421)^210]], [[Z(421)^0, 0*Z(421), Z(421)^194, Z(421)]],
        [[Z(421)^0, Z(421)^406, Z(421)^389, Z(421)^2]], [[Z(421)^0, Z(421)^278, Z(421)^194, Z(421)^194]]]
gap> #Int(elm);  #Int returns an integer
```

We find that the "character table of $End_{K(G\times H)}K\Omega$" over GF(421) is

| | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ |
|---|---|---|---|---|
| $\vartheta_1$ | 1 | 12 | 18 | 4 |
| $\vartheta_2$ | 1 | 0 | 418 | 2 |
| $\vartheta_3$ | 1 | 15 | 418 | 418 |
| $\vartheta_4$ | 1 | 418 | 3 | 420 |

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