International Journal of Mathematical Archive-7(1), 2016, 200-208
Available online through www.ijma.info ISSN 2229 – 5046

ON ANTI FUZZY IDEALS OF LATTICES

DHANANI S. H.*
Department of Mathematics, K. I. T.'s College of Engineering, Kolhapur. India.

PAWAR Y. S.
Department of Mathematics, Shivaji University, Kolhapur. India.

(Received On: 15-12-15; Revised & Accepted On: 25-01-16)

ABSTRACT

In this paper, we introduce the notions of anti fuzzy ideals and investigated some of its basic properties. We also study the homomorphic anti image, pre-image of anti fuzzy ideal. We introduce the notion of anti fuzzy prime ideals of a lattice and some related properties of it are discussed.

AMS Classification: *06D72.*

Keywords: *Fuzzy ideal, Anti fuzzy ideal, Anti fuzzy prime ideal.*

1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [19]. Rosenfeld [15] used this concept to formulate the notion of fuzzy groups. Since then many other fuzzy algebraic concepts based on the Rosenfeld's fuzzy groups were developed. Yuan and Wu [18] introduced the concepts of fuzzy sublattices and fuzzy ideals of a lattice. Ajmal and Thomas [1] defined a fuzzy lattice as a fuzzy algebra and characterized fuzzy sublattices.

In [4], Biswas introduced the concept of anti fuzzy subgroups of groups. M. Shabir and Y. Nawaz[16] introduced the concept of anti fuzzy ideals in semigroups and M. Khan and T. Asif [9] characterized different classes of semigroups by the properties of their anti fuzzy ideals. Lekkoksung and Lekkoksung [13] introduced the concept of an anti fuzzy bi-ideal of ordered Γ-Semigroups. Kim and Jun studied the notion of anti fuzzy ideals of a near-ring in [10]. In [5], Datta introduced the concept of anti fuzzy bi-ideals in rings. Anti fuzzy ideals of Γ - rings were studied by Zhou et.al. in [20]. In [17], Srinivas et. al. introduced the concept of anti fuzzy ideals of Γ- near-ring. Dheena and Mohanraaj [6] introduced the notion of anti fuzzy right ideal, anti fuzzy right k-ideal and intuitionistic fuzzy right k-ideal in semiring. In [8], Hong and Jun introduced the notion of anti fuzzy ideals of *BCK* – algebras. Al-Shehri [3] introduced the notion of anti fuzzy implicative ideal of *BCK* - algebras. Mostafa *et.al* [14] introduced the concept of anti-fuzzy sub implicative ideal of BCI-algebra. Akram [2] introduce the notion of anti fuzzy ideals in Lie algebras. In this paper, we introduce the notion of anti fuzzy ideals of lattices and investigate some related properties.

2. PRELIMINARIES

Now onwards X denotes a bounded lattice with the least element 0 and the greatest element 1 unless otherwise stated. For the undefined terms in this paper, the reader is referred to [7] and [11] respectively.

A non - empty subset I of X is called an ideal of X if, for any $a, b \in I$ and $x \in X$, $a \vee b \in I$, $a \wedge b \in I$ and for $a \in I$, $x \wedge a = x$ implies $x \in I$. A non-empty subset D of X is called a dual ideal of X if, for any $a, b \in D$ and $x \in X$, $a \wedge b \in D$, $a \vee b \in D$ and for $a \in D$, $x \vee a = x$ implies $x \in D$. A proper ideal P of X is called a prime ideal of X if, for any $a, b \in X$, $a \wedge b \in P$ implies $a \in P$ or $b \in P$.

A fuzzy set μ of X is a function $\mu: X \to [0, 1]$.

*Corresponding Author: Dhanani S. H.**
Department of Mathematics, K. I. T.'s College of Engineering, Kolhapur. India.

Let μ be a fuzzy set of X. Then the complement of μ, denoted by μ^c, is the fuzzy set of X given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in X$. For $t \in [0, 1]$, the set, $\mu_t^{\leq} = \{ x \in X / \mu(x) \leq t \}$ is called a lower t-level cut of μ and $\mu_t^{\geq} = \{ x \in X / \mu(x) \geq t \}$ is called an upper t-level cut of μ. If $t_1 < t_2$, then $\mu_{t_1}^{\leq} \subseteq \mu_{t_2}^{\leq}$ and $\mu_{t_2}^{\geq} \subseteq \mu_{t_1}^{\geq}$ for $t_1, t_2 \in [0, 1]$.
Also $\mu_t^{\leq} = \left(\mu^c\right)_{1-t}^{\geq}$ for all $t \in [0, 1]$.

A fuzzy set μ of X is proper if it is a non constant function.

A fuzzy set μ of X is called a fuzzy sublattice of X if for all $x, y \in X$,
$\mu(x \wedge y) \geq \min\{\mu(x), \mu(y)\}$ and $\mu(x \vee y) \geq \min\{\mu(x), \mu(y)\}$ (see [12]).

A fuzzy sublattice μ of X is a fuzzy ideal of X if for all $x, y \in X$,
$\mu(x \vee y) = \min\{\mu(x), \mu(y)\}$ (see [12]).

A proper fuzzy ideal μ of X is called fuzzy prime ideal of X if for all $x, y \in X$,
$\mu(x \vee y) \leq \max\{\mu(x), \mu(y)\}$ (see [12]).

3. ANTI FUZZY IDEALS

We introduce the notion of anti fuzzy sublattice of X.

Definition 3.1: A fuzzy set μ of X is an anti fuzzy sublattice of X if for all $x, y \in X$,
$\mu(x \wedge y) \leq \max\{\mu(x), \mu(y)\}$ and $\mu(x \vee y) \leq \max\{\mu(x), \mu(y)\}$.

Example 3.2: Consider the lattice X = {0, a, b, 1} as shown by the Hasse diagram of Figure 1.

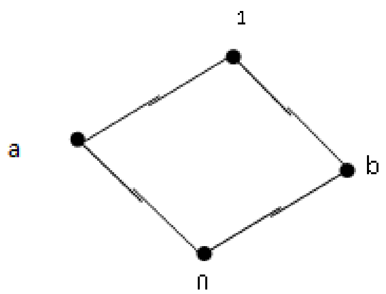

Figure 1

Define a fuzzy set μ of X by $\mu(0) = 0.1$, $\mu(a) = 0.2$, $\mu(b) = 0.3$ and $\mu(1) = 0.3$. Then μ is an anti fuzzy sublattice of X. Every fuzzy set of X need not be an anti fuzzy sublattice of X. For this consider the following example.

Example 3.3: Consider the lattice X = {0, a, b, 1} as shown by the Hasse diagram of Figure 1. Define a fuzzy set μ of X by $\mu(0) = 0.5$, $\mu(a) = 0.2$, $\mu(b) = 0.3$ and $\mu(1) = 0.2$. μ is not an anti fuzzy sublattice of X as $\mu(a \wedge b) = \mu(0) = 0.5 \nleq \max\{\mu(a), \mu(b)\} = 0.3$.

In Theorem 3.4 we prove that the complement of an anti fuzzy sublattice of X is a fuzzy sublattice of X.

Theorem 3.4: A fuzzy set μ of X is an anti fuzzy sublattice of X if and only if μ^c is a fuzzy sublattice of X.

Proof: Let μ be an anti fuzzy sublattice of X. Then for $x, y \in X$,
$\mu^c(x \wedge y) = 1 - \mu(x \wedge y)$
$\geq 1 - \max\{\mu(x), \mu(y)\}$ (since μ is an anti fuzzy sublattice of X)
$= \min\{1 - \mu(x), 1 - \mu(y)\}$
$= \min\{\mu^c(x), \mu^c(y)\}$.

and $\mu^c(x \vee y) = 1 - \mu(x \vee y)$
$\geq 1 - \max\{\mu(x), \mu(y)\}$ (since μ is an anti fuzzy sublattice of X)
$= \min\{1 - \mu(x), 1 - \mu(y)\}$
$= \min\{\mu^c(x), \mu^c(y)\}$.

Hence μ^c is a fuzzy sublattice of X. The converse is proved similarly.

Definition 3.5: Let μ be an anti fuzzy sublattice of X. Then μ is an anti fuzzy ideal of X if $\mu(x \vee y) = \max\{\mu(x), \mu(y)\}$ for all x, y $\in$ X.

Example 3.6: Consider the lattice X = {0, a, b, 1} as shown by the Hasse diagram of Figure 1. Define a fuzzy set μ of X by $\mu(0) = 0.1$, $\mu(a) = 0.2$, $\mu(b) = 0.3$ and $\mu(1) = 0.3$. Then μ is an anti fuzzy ideal of X.

Every anti fuzzy sublattice of X need not be an anti fuzzy ideal of X. For this consider Example 3.7.

Example 3.7: Consider the lattice X = {0, a, b, 1} as shown by the Hasse diagram of Figure 1. Define a fuzzy set μ of X by $\mu(0) = 0$, $\mu(a) = 0.2$, $\mu(b) = 0.3$ and $\mu(1) = 0.2$. Then μ is an anti fuzzy sublattice of X. But μ is not an anti fuzzy ideal of X as $\mu(a \vee b) = \mu(1) \neq \max\{\mu(a), \mu(b)\}$.

Remark 3.8: Let μ be an anti fuzzy ideal of X. As $\mu(x) = \mu(x \vee 0) = \max\{\mu(x), \mu(0)\}$. We get $\mu(0) \leq \mu(x)$ for any x $\in$ X.

In the following theorem we give a necessary and sufficient condition of a fuzzy set of X to be an anti fuzzy ideal of X.

Theorem 3.9: A fuzzy set μ of X is an anti fuzzy ideal of X if and only if μ^c is a fuzzy ideal of X.

Proof: Let μ be an anti fuzzy ideal of X. By Theorem 3.4 μ^c is a fuzzy sublattice of X.

For x, y $\in$ X, $\mu^c(x \vee y) = 1 - \mu(x \vee y)$
$= 1 - \max\{\mu(x), \mu(y)\}$(since μ is an anti fuzzy ideal of X)
$= \min\{1 - \mu(x), 1 - \mu(y)\}$
$= \min\{\mu^c(x), \mu^c(y)\}$

Thus μ^c is a fuzzy ideal of X. The converse is proved similarly.

In the following theorem we give a characterization of an anti fuzzy ideal of X.

Theorem 3.10: A fuzzy set μ of X is an anti fuzzy ideal of X if and only if lower t-level cut of μ, $\mu_t^{\leq}$ is an ideal of X for each t $\in$ [$\mu(0)$, 1].

Proof: Let μ be an anti fuzzy ideal of X and t $\in$ [$\mu(0)$, 1]. Then by Theorem 3.9, μ^c is a fuzzy ideal of X. Hence $\mu_t^{\leq} = \left(\mu^c\right)_{1-t}^{\geq}$ is an ideal of X [see 6, Lemma 3.1].

Conversely, let lower t-level cut of μ, $\mu_t^{\leq}$ is an ideal of X for each t$\in$[$\mu(0)$, 1] and s $\in$ [0, $1-\mu(0)$] = [0, $\mu^c(0)$]. Then $1-s \in [\mu(0), 1]$ and $\left(\mu^c\right)_s^{\geq} = \mu_{1-s}^{\leq}$ is an ideal of X. Hence $\left(\mu^c\right)_s^{\geq}$ is an ideal of X for all s $\in$[0, $\mu^c(0)$], and μ^c is a fuzzy ideal of X [see 1, Lemma 3.1]. This shows that μ is an anti fuzzy ideal of X.

Theorem 3.11: If I is an ideal of X, then for each t $\in$ [0, 1], there exists an anti fuzzy ideal μ of X such that $\mu_t^{\leq}$ = I.

Proof: Let I be an ideal of X. Let t $\in$ [0, 1]. Define a fuzzy set μ of X by

$$\mu(x) = \begin{cases} t & if \ x \in I \\ 1 & if \ x \notin I \end{cases}$$ for each x $\in$ X.

Then $\mu_s^{\leq}$ = I for any s $\in$[t, 1) = [μ (0), 1), and $\mu_1^{\leq}$ = X. Thus $\mu_s^{\leq}$ is an ideal of X for all s $\in$ [μ(0), 1]. Hence μ is an anti fuzzy ideal of X from Theorem 3.10, and $\mu_t^{\leq}$ = I.

Let μ be a fuzzy set of X. Define X μ = {x $\in$ X / μ (x) = μ (0)}. Then we have

Theorem 3.12: If μ is an anti fuzzy ideal of X, then X μ is an ideal of X.

Proof: Let μ be an anti fuzzy ideal of X. Let x, y $\in$ X μ then μ (x) = μ (0) and μ(y)= μ(0). Then μ (x $\vee$ y) = max { μ (x), μ (y)} = μ (0). Hence x $\vee$ y $\in$ X μ.

Let x $\leq$ a, x $\in$ X and a $\in$ X μ. Then x $\vee$ a = a and μ(a) = μ (0).

As μ is an anti fuzzy ideal of X, μ (x $\vee$ a) = max { μ (x), μ (a)}.

Thus μ (a) = max { μ (x), μ (a)}. Therefore μ (x) $\leq$ μ(a) = μ(0).

Also by Remark 3.8, μ (0) $\leq$ μ(x). So we get μ(x) = μ(0). Hence x $\in$ X μ .

This shows that X μ is an ideal of X.

For a family of fuzzy sets { μ_i | i $\in$ Λ} in X, the union $\bigvee_{i\in\Lambda} \mu_i$ is defined by

$\bigvee_{i\in\Lambda} \mu_i$(x) = sup { μ_i(x) / i $\in$ Λ} for each x $\in$ X.

Theorem 3.13: If { μ_i / i $\in$ Λ} is a family of anti fuzzy ideals of X, then so is $\bigvee_{i\in\Lambda} \mu_i$.

Proof: Let { μ_i / i $\in$ Λ } be a family of anti fuzzy ideals of X. Let x, y$\in$X.

$(\bigvee_{i\in\Lambda} \mu_i)$ (x $\wedge$ y) = sup { μ_i(x $\wedge$ y) / i $\in$ Λ}
$\leq$ sup { max{ μ_i(x), μ_i(y)} (since μ_i is an anti fuzzy sublattice of X
= max {sup { μ_i(x) }, sup { μ_i(y)}
= max { $\bigvee_{i\in\Lambda} \mu_i$(x), $\bigvee_{i\in\Lambda} \mu_i$ (y)}

Also $(\bigvee_{i\in\Lambda} \mu_i)$ (x $\vee$ y) = sup { μ_i(x $\vee$ y) / i $\in$ Λ}
$\leq$ sup{max { μ_i(x), μ_i(y)} (since μ_i is an anti fuzzy sublattice of X)
= max{sup { μ_i(x)}, sup { μ_i(y)}
= max{ $\bigvee_{i\in\Lambda} \mu_i$(x) $\bigvee_{i\in\Lambda} \mu_i$ (y)}

Hence $\bigvee_{i\in\Lambda} \mu_i$ is an anti fuzzy sublattice of X.

For x, y $\in$ X, $(\bigvee_{i\in\Lambda} \mu_i)$ (x $\vee$ y) = sup { μ_i(x $\vee$ y) / i $\in$ Λ}
= sup {max { μ_i(x), μ_i(y)} (since μ_i is an anti fuzzy ideal of X)
= max {sup { μ_i(x) }, sup { μ_i(y)}
= max { $\bigvee_{i\in\Lambda} \mu_i$(x), $\bigvee_{i\in\Lambda} \mu_i$ (y)}

Hence $\bigvee_{i\in\Lambda} \mu_i$ is an anti fuzzy ideal of X.

Let f: X $\to$ Y be a mapping where Y is a non – empty set. Let μ be a fuzzy set of Y. Then $f^{-1}(\mu)$ is a fuzzy set of X and is defined by $f^{-1}(\mu)(x) = \mu(f(x))$ for all $x \in X$.

Theorem 3.14: Let f: X $\to$Y be a lattice homomorphism where Y is a bounded lattice. Let μ be an anti fuzzy ideal of Y. Then $f^{-1}(\mu)$ is an anti - fuzzy ideal of X.

Proof: Let $x, y \in X$.

Then $f^{-1}(\mu)(x \wedge y) = \mu(f(x \wedge y))$
$= \mu(f(x) \wedge f(y))$ (since f is a lattice homomorphism)
$\leq \max\{\mu(f(x)), \mu(f(y))\}$ (since μ is an anti fuzzy sublattice of Y)
$= \max\{f^{-1}(\mu)(x), f^{-1}(\mu)(y)\}$.

Thus $f^{-1}(\mu)(x \wedge y) \leq \max\{f^{-1}(\mu)(x), f^{-1}(\mu)(y)\}$.

Similarly we can prove $f^{-1}(\mu)(x \vee y) \leq \max\{f^{-1}(\mu)(x), f^{-1}(\mu)(y)\}$.

Hence $f^{-1}(\mu)$ is an anti fuzzy sublattice of X.

Also for $x, y \in X$, $f^{-1}(\mu)(x \vee y) = \mu(f(x \vee y))$
$= \mu(f(x) \vee f(y))$ (since f is a lattice homomorphism)
$= \max\{\mu(f(x)), \mu(f(y))\}$ (since μ is an anti fuzzy ideal of Y)
$= \max\{f^{-1}(\mu)(x), f^{-1}(\mu)(y)\}$.

Thus $f^{-1}(\mu)(x \vee y) = \max\{f^{-1}(\mu)(x), f^{-1}(\mu)(y)\}$.

This shows that $f^{-1}(\mu)$ is an anti fuzzy ideal of X.

Let f: X $\to$ Y be a mapping where Y is a non – empty set. Let μ be a fuzzy set of X. Then anti image of μ under f is a fuzzy set $f(\mu)$ of Y defined by $f(\mu)(y) = \inf\{\mu(x) / x \in X \text{ and } f(x) = y\}$ for all $y \in Y$. For $f(\mu)$ we have

Theorem 3.15: Let f: X $\to$ Y be an onto lattice homomorphism where Y is a bounded lattice. Let μ be an anti fuzzy ideal of X. Then $f(\mu)$ is an anti - fuzzy ideal of Y.

Proof: Let μ be an anti fuzzy ideal of X. Let $a, b \in Y$. As f is onto, there exist $p, q \in X$ such that $f(p) = a$ and $f(q) = b$. $a \wedge b = f(p) \wedge f(q) = f(p \wedge q)$ (As f is a lattice homomorphism).

Now $f(\mu)(a \wedge b) = \inf\{\mu(z) / z \in X \text{ and } f(z) = a \wedge b\}$
$\leq \inf\{\mu(p \wedge q) / f(p) = a \text{ and } f(q) = b\}$
$\leq \inf\{\max\{\mu(p), \mu(q)\} / f(p) = a \text{ and } f(q) = b\}$ (As μ is an anti fuzzy sublattice of X)
$= \max \inf\{\mu(p) / f(p) = a\}, \inf\{\mu(q) / f(q) = b\}\}$
$= \max\{f(\mu)(a), f(\mu)(b)\}$.

Thus $f(\mu)(a \wedge b) \leq \max\{f(\mu)(a), f(\mu)(b)\}$.

Similarly we can prove that $f(\mu)(a \vee b) \leq \max\{f(\mu)(a), f(\mu)(b)\}$.

Hence $f(\mu)$ is an anti fuzzy sublattice of Y.

Let $x, y \in Y$. As f is onto, there exist $r, s \in X$ such that $f(r) = x$ and $f(s) = y$.

Also $x \vee y = f(r) \vee f(s) = f(r \vee s)$ (As f is a lattice homomorphism).

Now $f(\mu)(x \vee y) = \inf\{\mu(z) / z \in X \text{ and } f(z) = x \vee y\}$
$= \inf\{\mu(r \vee s) / f(r) = x \text{ and } f(s) = y\}$
$= \inf\{\max\{\mu(r), \mu(s)\} / f(r) = x \text{ and } f(s) = y\}$ (As μ is an anti fuzzy ideal of X)
$= \max\{\inf\{\mu(r) / f(r) = x\}, \inf\{\mu(s) / f(s) = y\}\}$
$= \max\{f(\mu)(x), f(\mu)(y)\}$.
Thus $f(\mu)(x \vee y) = \max\{f(\mu)(x), f(\mu)(y)\}$.

This shows that $f(\mu)$ is an anti fuzzy ideal of Y.

Theorem 3.16: If μ is any fuzzy set of X, then $\mu(x) = \inf\{t \in [0, 1] / x \in \mu_t^{\leq}\}$ for each $x \in X$.

Proof: For $x \in X$, let $T_x = \{t \in [0, 1] / x \in \mu_t^{\leq}\}$ and $\alpha = \inf T_x$. Then for any $t \in T_x$, $\mu(x) \leq t$. Hence $\mu(x)$ is a lower bound of T_x. Therefore $\mu(x) \leq \inf T_x = \alpha$. Let $\beta = \mu(x)$. Then $x \in \mu_\beta^{\leq}$ and $\beta \in T_x$. Hence $\alpha = \inf T_x \leq \beta = \mu(x)$.

Theorem 3.17: Every anti fuzzy ideal of X is order preserving.

Proof: Let μ be an anti fuzzy ideal of X. Let $x, y \in X$ such that $x \leq y$.
Then $\mu(y) = \mu(x \vee y) = \max\{\mu(x), \mu(y)\}$. Thus $\mu(x) \leq \mu(y)$.

This shows that μ is order preserving.

Definition 3.18: Let λ and μ be fuzzy sets of X. The anti Cartesian product
$\lambda \times \mu : X \times X \to [0, 1]$ is defined by $\lambda \times \mu(x, y) = \max\{\lambda(x), \mu(y)\}$ for all $x, y \in X$.

Theorem 3.19: If λ and μ are anti fuzzy ideals of X, then $\lambda \times \mu$ is an anti fuzzy ideal of $X \times X$.

Proof: Let (x_1, y_1) and $(x_2, y_2) \in X \times X$. Then
$\lambda \times \mu((x_1, y_1) \wedge (x_2, y_2)) = \lambda \times \mu(x_1 \wedge x_2, y_1 \wedge y_2)$
$= \max\{\lambda(x_1 \wedge x_2), \mu(y_1 \wedge y_2)\}$
$\leq \max\{\max\{\lambda(x_1), \lambda(x_2)\}, \max\{\mu(y_1), \mu(y_2)\}$(since λ and μ are anti fuzzy sublattices of X)
$= \max\{\max\{\lambda(x_1), \mu(y_1)\}, \max\{\lambda(x_2), \mu(y_2)\}\}$
$= \max\{\lambda \times \mu(x_1, y_1), \lambda \times \mu(x_2, y_2)\}$.

Also $\lambda \times \mu((x_1, y_1) \vee (x_2, y_2)) = \lambda \times \mu(x_1 \vee x_2, y_1 \vee y_2)$
$= \max\{\lambda(x_1 \vee x_2), \mu(y_1 \vee y_2)\}$
$\leq \max\{\max\{\lambda(x_1), \lambda(x_2)\}, \max\{\mu(y_1), \mu(y_2)\}\}$
(Since λ and μ are anti fuzzy sublattices of X)
$= \max\{\max\{\lambda(x_1), \mu(y_1)\}, \max\{\lambda(x_2), \mu(y_2)\}\}$
$= \max\{\lambda \times \mu(x_1, y_1), \lambda \times \mu(x_2, y_2)\}$.

Thus $\lambda \times \mu$ is an anti fuzzy sublattice of X x X.
For $x, y \in X$, $\lambda \times \mu((x_1, y_1) \vee (x_2, y_2))$
$= \lambda \times \mu(x_1 \vee x_2, y_1 \vee y_2)$
$= \max\{\lambda(x_1 \vee x_2), \mu(y_1 \vee y_2)\}$ (since λ and μ are anti fuzzy ideals of X)
$= \max\{\max\{\lambda(x_1), \lambda(x_2)\}, \max\{\mu(y_1), \mu(y_2)\}\}$
$= \max\{\max\{\lambda(x_1), \mu(y_1)\}, \max\{\lambda(x_2), \mu(y_2)\}\}$
$= \max\{\lambda \times \mu(x_1, y_1), \lambda \times \mu(x_2, y_2)\}$.
Hence $\lambda \times \mu$ is an anti fuzzy ideal of X x X.

Definition 3.20: An ideal I of X is said to be a characteristic if $f(I) = I$ for all $f \in \text{Aut}(X)$ where Aut (X) is the set of all automorphisms of X. An anti fuzzy ideal μ of X is a fuzzy characteristic, if $\mu(f(x)) = \mu(x)$ for all $x \in X$ and $f \in \text{Aut}(X)$.

Lemma 3.21: Let μ be a fuzzy set of X. Let x∈X. μ (x) = s, if and only if x∈ $\mu_s^{\le}$ and x∉ $\mu_t^{\le}$ for s > t and s, t∈ [0, 1].

Proof: Proof is straightforward.

Theorem 3.22: Let μ be an anti fuzzy ideal of X. μ is a fuzzy characteristic if and only if for t∈[0, 1], each non - empty lower t-level cut of μ, $\mu_t^{\le}$ is characteristic.

Proof: Let μ be a fuzzy characteristic. Select t∈ [0, 1] such that $\mu_t^{\le} \ne \emptyset$. As μ is an anti fuzzy ideal of X, $\mu_t^{\le}$ is an ideal of X (by Theorem 3.10). Let f∈Aut(X) and y∈f ($\mu_t^{\le}$). Thus y=f(x) for some x∈ $\mu_t^{\le}$. Therefore μ (x) ≤ t. As μ is a fuzzy characteristic, μ (y) = μ (f (x)) = μ (x) ≤ t. Hence y∈ $\mu_t^{\le}$. So f ($\mu_t^{\le}$) ⊆ $\mu_t^{\le}$. y∈ $\mu_t^{\le}$ implies μ (y) ≤ t. As f is an automorphism of X, there exists x ∈ X such that f (x) = y. Therefore μ (f (x)) ≤ t. As μ is a fuzzy characteristic, μ (f (x)) = μ (x) ≤ t. Hence x ∈ $\mu_t^{\le}$.

So we get y = f (x) ∈ f ($\mu_t^{\le}$). Therefore $\mu_t^{\le}$ ⊆ f ($\mu_t^{\le}$). Combining both inclusions f ($\mu_t^{\le}$) = $\mu_t^{\le}$.
This shows that $\mu_t^{\le}$ is characteristic.
Conversely, assume that each non - empty lower t-level cut of μ, $\mu_t^{\le}$ is characteristic for t ∈ [0, 1]. Select x∈ X and f ∈ Aut (X). If μ (x) = s, then x∈ $\mu_s^{\le}$. Hence by assumption $\mu_s^{\le}$ is characteristic i.e. f ($\mu_s^{\le}$) = $\mu_s^{\le}$. x ∈ $\mu_s^{\le}$ ⇒ f (x) ∈ f ($\mu_s^{\le}$) = $\mu_s^{\le}$ ⇒ μ (f (x)) ≤ s.
If μ (f (x)) < s, then f (x) ∈ $\mu_t^{\le}$ where μ (f(x)) = t. Again by assumption $\mu_t^{\le}$ is characteristic i.e. f($\mu_t^{\le}$) = $\mu_t^{\le}$. Thus f(x) ∈ f($\mu_t^{\le}$). Hence x∈ $\mu_t^{\le}$; a contradiction by lemma 3.22. Thus μ (f (x)) = s i.e., μ (f(x)) = μ (x). This shows that μ is fuzzy characteristic.

We introduce an anti fuzzy prime ideal of a lattice.

Definition 3.23: Let μ be an anti fuzzy ideal of X. μ is an anti fuzzy prime ideal of X if μ (x ∧ y) ≥ min{ μ (x), μ (y)} for all x, y ∈ X.

Example 3.24: Consider the lattice X = {0, a, b, 1} as shown by the Hasse diagram of Figure 1. Define a fuzzy set μ of X by μ (0) = 0.2, μ (a) = 0.2, μ (b) = 0.3 and μ (1) = 0.3. Then μ is an anti fuzzy prime ideal of X. Every anti fuzzy ideal of X need not be an anti fuzzy prime ideal of X. For this consider the following example.

Example 3.25: Consider the lattice X = {0, a, b, 1} as shown by the Hasse diagram of Figure 1. Define a fuzzy set μ of X by μ (0) = 0.1, μ (a) = 0.2, μ (b) = 0.3 and μ (1) = 0.3. Then μ is an anti fuzzy ideal of X. But μ is not an anti fuzzy prime ideal of X as μ (a ∧ b) = μ (0) ≱ min { μ (a), μ (b)}.

Theorem 3.26: Let P be a non-empty subset of X. Let r, t ∈ [0, 1] such that r < t. Let μ_p be a fuzzy subset of X such that

$$\mu_P(x) = \begin{cases} r & \text{if } x \in P \\ t & \text{if } x \notin P \end{cases} \quad \text{for all } x \in X.$$

P is a prime ideal of X if and only if μ_p is an anti fuzzy prime ideal of X.

Proof: Let P be a prime ideal of X. Let x, y∈X. If x ∧ y∈P, then μ_p (x ∧ y) = r ≤ max{ μ_p (x), μ_p (y)}. If x ∧ y∉ P, then x∉P and y∉P (since P is a sublattice of X). Then μ_p (x ∧ y) = t, μ_p (x) = t and μ_p (y) = t.
Hence μ_p (x ∧ y) ≤ max{ μ_p (x), μ_p (y)}. Therefore we have μ_p (x ∧ y) ≤ max{ μ_p (x), μ_p (y)}.

Similarly we can prove that μ_p (x ∨ y) ≤ max { μ_p (x), μ_p (y)}.

This shows that μ_p is an anti fuzzy sublattice of X.

Let x, y ∈ X. If x ∨ y∈P, then x ∈ P and y ∈ P (since P is an ideal of X).

Therefore μ_p (x ∧ y) = r, μ_p (x) = r and μ_p (y) = r.

Hence μ_p (x ∨ y) = max { μ_p (x), μ_p (y)}. If x ∨ y ∉ P, then x∉P or y∉P (since P is an ideal of X). Therefore μ_p (x ∧ y) = t, μ_p (x) = t or μ_p (y) = t.

Hence μ_p (x ∨ y) = max { μ_p (x), μ_p (y)}. This shows that μ_p is an anti fuzzy ideal of X.

Let x, y ∈ X. If x ∧ y ∈ P, then x ∈ P or y ∈ P (since P is a prime ideal of X). Therefore μ_p (x ∧ y) = r, μ_p (x) = r or μ_p (y) = r.

Hence μ_p (x ∧ y) ≥ min { μ_p (x), μ_p (y)}. If x ∧ y∉P, then x∉P and y∉P (since P is an ideal of X).
Therefore μ_p (x ∧ y) = t, μ_p (x) = t and μ_p (y) = t.

Hence μ_p (x ∧ y) ≥ min { μ_p (x), μ_p (y)}. This shows that μ_p is an anti fuzzy prime ideal of X.

Conversly, let μ_p be an anti fuzzy prime ideal of X. Let x, y ∈ P. As μ_p is an anti fuzzy sublattice of X,

μ_p (x ∧ y) ≤ max { μ_p (x), μ_p (y)} = r. Hence x ∧ y ∈ P. Similarly we can prove that x ∨ y∈ P. This shows that P is a sublattice of X.

Let a ∈ P, x ∈ X such that x ∧ a = x. Thus x ∨ a = (x ∧ a) ∨ a = a.

Therefore r = μ_p (a) = μ_p (x ∨ a) = max { μ_p (x), μ_p (a)} (As μ_p is an anti fuzzy ideal of X).

Hence μ_p (x) = r and so x ∈ P. This shows that P is an ideal of X.

Let x ∧ y ∈ P. Then μ_p (x ∧ y) = r. As μ_p is an anti fuzzy prime ideal of X,

μ_p (x ∧ y) ≥ min { μ_p (x), μ_p (y)}. Therefore μ_p (x) = r or μ_p (y) = r. Hence x ∈ P or y ∈ P.

This shows that P is a prime ideal of X.

Using Theorem 3.9 and Theorem 3.26 we get the following corollary.

Corollary 3.27 [12]: A non – empty subset P of X is a prime ideal of X if and only if the characteristic function χ_P of P is a fuzzy prime ideal of X.

4. CONCLUSIONS

In this paper we have defined the notions of anti fuzzy sublattice, anti fuzzy ideal and anti fuzzy prime ideal of a bounded lattice. We discussed that the complement of an anti fuzzy sublattice of a bounded lattice is a fuzzy sublattice. We also discussed that the union a family of anti fuzzy ideals of a bounded lattice is also an anti fuzzy ideal of a bounded lattice. We have stated how the homomorphic anti images and inverse images of anti fuzzy ideals of a bounded lattice become anti fuzzy ideal of a bounded lattice. We also stated how the anti Cartesian product of anti fuzzy ideals of a bounded lattice becomes anti fuzzy ideal of a bounded lattice. Our future work will focus on studying the intuitionistic anti fuzzy ideals of a bounded lattice.

REFERENCES

1. N. Ajmal and K. V. Thomas, Fuzzy lattices, Information sciences, 79 (1994), 271 - 291.
2. M. Akram, Anti fuzzy Lie ideals of Lie algebras, Quasigroups and Related Systems, 14 (2006), 123 − 132.
3. N. O. Al-Shehri, Anti Fuzzy Implicative Ideals in BCK-Algebras, Punjab University Journal of Mathematics, 43 (2011), 85-91.
4. R. Biswas, Fuzzy subgroups and anti fuzzy subgroups, Fuzzy Sets and Sys, 35 (1990), 121-124.
5. S. K. Datta, On anti fuzzy bi-ideals in rings, Int. J. Pure Appl. Math., 51(2009), 375–382.
6. P. Dheena and G. Mohanraaj, On Intuitionistic Fuzzy K-ideals of Semiring, International Journal of Computational Cognition, 9(2) (2011), 45-50.
7. G. Gratzer, Lattice theory - First concepts and Distributive lattices, Freeman Company, San Francisco (1971).
8. S. M. Hong and Y. B. Jun, Anti fuzzy ideals in *BCK*-algebras, Kyungpook Math. J., 38(1998), 145-150.
9. M. Khan and T. Asif, Characterizations of semigroups by their anti Fuzzy ideals, J. of Mathematics Research., 2 (3) (2010), 134-143.
10. K. H. Kim and Y. B. Jun, Anti fuzzy R-subgroups of near-rings, Scientiae Mathematicae, (2),(1999), 147-153.
11. G. J. Klir and Bo Yuan, Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice Hall of India Pvt. Ltd., New Delhi (1997).
12. B. B. N. Koguep, C. Nkuimi and C. Lele, On Fuzzy Prime Ideals of Lattices, SAMSA Journal of Pure and Applied Mathematics, 3(2008), 1-11.
13. S. Lekkoksung and N. Lekkoksung, On Generalized Anti Fuzzy Bi-Ideals in Ordered Γ-Semigroups, Int. J. Contemp. Math. Sciences, 7(16)(2012), 759 – 764.
14. S. M. Mostafa, A.K. Omar and A. I. Marie, Anti-Fuzzy Sub-Implicative Ideals of BCI-Algebras, Journal of American Science, 7(11)(2011), 274 -282.
15. A. Rosenfeld, Fuzzy Groups, J. Math. Anal. Appl., 35 (1971), 512-517.
16. M. Shabir and Y. Nawaz, Semigroups characterized by the properties of their anti fuzzy ideals, Journal of Advanced Research in Pure Mathematics, 3 (2009), 42-59.
17. T. Srinivas, T. Nagaiah and P. Narasimha Swamy, Anti fuzzy ideals of Γ-near-rings, Annals of Fuzzy Mathematics and Informatics, 3(2) (2012), 255- 266.
18. B. Yuan and W. Wu, Fuzzy ideals on a distributive lattice, Fuzzy sets and systems, 35 (1990), 231-240.
19. L. A. Zadeh, Fuzzy sets, Inform. Control, 8(1965), 338-353.
20. M. Zhou, D. Xiang and J. Zhan, On anti fuzzy ideals of Γ-rings, Ann. Fuzzy Math. Inform, 1 (2011), 197–205.

Source of support: Nil, Conflict of interest: None Declared