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ON $\psi\alpha g$-CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce a new class of set called $\psi\alpha g$-closed sets in topological spaces and also we introduce a new type of functions called $\psi\alpha g$-continuous functions and $\psi\alpha g$-irresolute functions. Further we introduce a $\psi\alpha g$-homeomorphism and studied the group structure of $\psi\alpha g$-homeomorphism in topological spaces.

Keywords: *$\psi\alpha g$-closed set, $\psi\alpha g$-continuous function, $\psi\alpha g$-irresolute function and $\psi\alpha g$-homeomorphism.*

1. INTRODUCTION

O. Njastad [11] introduces the concept of α-closed sets in topological spaces. The notion of ψ-closed sets is introduced by MKRS Veerakumar [17]. In this chapter, we introduce a new class of notion, namely, $\psi\alpha g$-closed sets for topological spaces. As applications of $\psi\alpha g$-closed sets, we introduce and study some new spaces, namely $\psi_{\alpha g}T_{1/2}$ space, $\psi_{\alpha g}T_{\alpha}$ space and $\psi_{\alpha g}T_{s}$ spaces. Further we introduce and study $\psi\alpha g$-continuous and $\psi\alpha g$ - irresolute maps. For a topological Space (X, τ), we define groups $\psi\alpha g$-h(X, τ), $\psi\alpha g$-ch(X, τ) and that contain the group h(X, τ) whose elements are all homeomorphisms from (X, τ) into itself.

2. PRELIMINARIES

In this section we recall some of the basic definitions.

Definition 2.1: A subset A of space (X, τ) is called
- (i) semi open set [9] if A⊆cl(int(A)).
- (ii) semi Pre open set [1] if A⊆cl(int(cl(A))).
- (iii) pre open set [10] if A⊆int(cl(A)).
- (iv) α-open set [11] if A⊆int(cl(int(A))).
- (v) regular open set [12] if A=int(cl(A)).

Definition 2.2: A subset A of space (X, τ) is called
- (i) generalized closed (briefly g-closed) set if cl(A)⊆U whenever A⊆U and U is open set in (X, τ)
- (ii) generalized semi-closed (briefly gs-closed) set if scl(A)⊆U whenever A⊆U and U is open set in (X, τ)
- (iii) α-generalized closed (briefly αg-closed)set if αcl(A)⊆U whenever A⊆U and U is open set in (X, τ)
- (iv) generalized pre-closed (briefly gp-closed)set if pcl(A)⊆U whenever A⊆U and U is open set in (X, τ)
- (v) generalized semi-pre-closed (briefly gsp-closed)set if spcl(A)⊆U whenever A⊆U and U is open set in (X, τ)
- (vi) generalized pre-regular-closed (briefly gpr-closed)set if pcl(A)⊆U whenever A⊆U and U is regular open set in (X, τ)
- (vii) $g^{\#}$-closed set if cl(A)⊆U whenever A⊆U and U is αg-open set in (X, τ)
- (viii) $g^{\#}$s-closed set if scl(A)⊆U whenever A⊆U and U is αg-open set in (X, τ)
- (ix) $\alpha\psi$-closed set if ψcl(A)⊆U whenever A⊆U and U is α-open set in (X, τ)

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Definition 2.3: A space (X, τ) is said to be
(i) α-space if every α-closed set is closed.
(ii) ${}_{s}^{*}T_p$-space if every gsp-closed set is g*p-closed.
(iii) pre-regular $T_{1/2}$-space if every gsp-closed set is g*p-closed.
(iv) $T_b^{\#}$-space if every $g^{\#}s$ -closed set is closed.
(v) semi-$T_{1/3}$-space if every ψ-closed set is semi-closed.
(vi) ${}_{\alpha}\psi T_{1/2}$ – space if every $\alpha\psi$-closed set is closed.

Definition 2.4: A function f: (X, τ)→(Y, σ) is called
(i) semi-continuous if $f^{-1}(V)$ is a semi-closed in (X, τ) for every closed set V of (Y, σ).
(ii) α-continuous if $f^{-1}(V)$ is a α-closed in (X, τ) for every closed set V of (Y, σ).
(iii) gp-continuous if $f^{-1}(V)$ is a gp-closed in (X, τ) for every closed set V of (Y, σ).
(iv) gpr-continuous if $f^{-1}(V)$ is a gpr-closed in (X, τ) for every closed set V of (Y, σ).
(v) gsp-continuous if $f^{-1}(V)$ is a gsp-closed in (X, τ) for every closed set V of (Y, σ).
(vi) $g^{\#}$-continuous if $f^{-1}(V)$ is a $g^{\#}$-closed in (X, τ) for every closed set V of (Y, σ).
(vii) $g^{\#}s$-continuous if $f^{-1}(V)$ is a $g^{\#}s$-closed in (X, τ) for every closed set V of (Y, σ).
(viii) $\alpha\psi$-continuous if $f^{-1}(V)$ is a $\alpha\psi$-closed in (X, τ) for every closed set V of (Y, σ).

3. $\psi\alpha g$-CLOSED SETS IN TOPOLOGICAL SPACES

We introduce the following definition

Definition 3.1: A subset A of a space (X, τ) is $\psi\alpha g$-closed set if ψcl(A)⊆U whenever A⊆U and U is αg-open set in (X, τ).

Theorem 3.2: Every closed set is $\psi\alpha g$-closed set.

Proof: Let A be an closed set in (X, τ), U be an αg-open set containing A. Since A is closed, we have $cl(A) = A$, ψcl(A)⊆ $cl(A) = A \subseteq U$ and hence A is $\psi\alpha g$-closed set.

The converse of the above theorem is need not be true as it can be seen in the following example.

Example 3.3: Let X = {a, b, c} and τ={X, ϕ,{a},{a, b}}. The set {b} is a $\psi\alpha g$-closed set but not an closed set.

Theorem 3.4: Every α-closed set and $g^{\#}$-closed is a $\psi\alpha g$-closed set.

Proof: Obvious.

The converse of the above theorem are need not be true as it can be seen in the following example.

Example 3.5: Let X=Y={a, b, c} with τ={ϕ, X,{b},{c},{b, c}}. A set {b} is $\psi\alpha g$-closed set but not an α-closed and $g^{\#}$-closed set.

Theorem 3.6: Every semi-closed and $g^{\#}s$-closed set is $\psi\alpha g$-closed set.

Proof: Obvious.

The following example shows that the converse of the above theorem are need not be true in general.

Example 3.7: Let X=Y = {a, b, c} with τ={ϕ, X,{b},{b, c}}. A set {b, c} is $\psi\alpha g$-closed set but not an semi-closed and $g^{\#}s$-closed set.

Theorem 3.8: Every $\psi\alpha g$-closed set is gp-closed, gsp-closed and gpr-closed set.

Proof: The proof is obvious. So the class of $\psi\alpha g$-closed set is properly contained in the class of gp-closed (resp. gsp-closed, gpr-closed) set.

The converse of the above theorem are need not be true as it can be seen in the following example.

Example 3.9: Let X=Y= {a, b, c} with τ={ϕ, X,{a},{a,b}}. A set {a, c} is an gp-closed, gsp-closed and gpr-closed set but not an $\psi\alpha g$-closed set.

Theorem 3.10: Every $\psi\alpha g$-closed set is $\alpha\psi$-closed set.

Proof: The converse of the above theorem is need not true in general as it can be seen from the following example.

Example 3.11: Let X=Y={a, b, c} with τ={ϕ,X,{a},{b,c}}. A set {a, b} is an $\alpha\psi$-closed set but not an $\psi\alpha g$-closed set.

4. APPLICATIONS OF $\psi\alpha g$-CLOSED SETS

As applications of $\psi_{\alpha g}T_{1/2}$-closed sets, two new spaces $\psi_{\alpha g}T_{\alpha}$-spaces and $\psi_{\alpha g}T_{s}$-spaces are introduced.

We introduce the following definition.

Definition 4.1: A space (X, τ) is called $\psi_{\alpha g}T_{1/2}$ space if every $\psi\alpha g$-closed set is closed.

Theorem 4.2: Every $\psi_{\alpha g}T_{1/2}$ space is a $T_b^{\#}$-space and α-space but not conversely.

Proof: The space in the following examples are $T_b^{\#}$-space and α-space but not a $\psi_{\alpha g}T_{1/2}$ space.

Example 4.3: Let X= {a, b, c} and τ={X,ϕ,{a},{b,c}}. C(X, τ)= {X,ϕ,{a},{b,c}}, $G^{\#}$SC(X, τ)={X,ϕ,{a},{b,c}} and $\psi\alpha g$-C(X, τ)={ X,ϕ,{a},{b,c},{a,c}}. Thus the space (X, τ) is a $T_b^{\#}$-space but not a $\psi_{\alpha g}T_{1/2}$ space.

Example 4.4: Let X= {a, b, c} and τ={X,ϕ,{a,b}}. C(X, τ) = {X,ϕ,{c}}, α C(X, τ) = {X,ϕ,{c}} and $\psi\alpha g$-C(X, τ)={ X,ϕ,{c},{b,c},{a,c}}. Thus the space (X, τ) is a α-space but not a $\psi_{\alpha g}T_{1/2}$ space.

Theorem 4.5: Every pre regular $T_{1/2}$ space is a $\psi_{\alpha g}T_{1/2}$ space but not conversely.

Proof: A $\psi_{\alpha g}T_{1/2}$ space is need not be $T_{1/2}$ in general as it can be seen from the following example.

Example 4.6: Let X= {a, b, c} and τ={X,ϕ,{a},{a,b},{a,c}}. C(X, τ)= {X,ϕ,{b},{c},{b,c}}, GPRC(X, τ)=P(X) and $\psi\alpha g$-C(X, τ)={ X,ϕ,{b},{c},{b,c}}. Thus the space (X, τ) is a $\psi_{\alpha g}T_{1/2}$ space but not a pre regular $T_{1/2}$ space.

Theorem 4.7: If (X, τ) is a $\psi_{\alpha g}T_{1/2}$ space, then for each x∈X, {x} is either α-closed or open.

Proof: Suppose that (X, τ) is a $\psi_{\alpha g}T_{1/2}$ space. Let x∈X and assume that {x} is not a α-closed set. Then X-{x} is not a α-open set. This implies that X-{x} is a $\psi\alpha g$ -closed set, since X is the only α-open set containing X-{x}. Since (X, τ) is a $\psi_{\alpha g}T_{1/2}$ space. X-{x} is a closed set or equivalently {x} is open.

The converse of the above theorem is need not be true as it can be seen by the following example.

Example 4.8: Let X={a, b, c} and τ={X,ϕ,{a},{a,c}}. The set {a} is an open set of (X, τ). {b} and {c} are α-closed subsets of (X, τ). But (X, τ) is not a $\psi_{\alpha g}T_{1/2}$ space.

Since $\psi\alpha g$-c(X, τ)={ X,ϕ,{b},{c},{b,c}} and C(X, τ)={ X,ϕ,{b},{b,c}.

Definition 4.9: A space (X, τ) is called $\psi_{\alpha g}T_{\alpha}$-space if every $\psi\alpha g$-closed set is α-closed.

Theorem 4.10: Every $\psi_{\alpha g}T_{1/2}$ space is a $\psi_{\alpha g}T_{\alpha}$-space but not conversely.

Proof: The converse of the above theorem is need not be true as it can be seen by the following example.

Example 4.11: Let X={a, b, c} and τ={X,ϕ,{a},{a,c}}. C(X, τ)= {X,ϕ,{b},{b,c}}, αC(X, τ)={X,ϕ,{b},{c},{b,c}} and $\psi\alpha g$-C(X, τ)={ X,ϕ, {b},{c},{b, c}}. Thus the space (X, τ) is a $\psi_{\alpha g}T_{\alpha}$-space but not a $\psi_{\alpha g}T_{1/2}$ space.

Theorem 4.12: Every $\psi_{\alpha g}T_{\alpha}$ space is a α-space and $T_b^{\#}$-space but not conversely.

Proof: A α-space and $T_b^{\#}$-space is need not be $\psi_{\alpha g}T_{\alpha}$ in general as it can be seen from the following example.

Example 4.13: Let X={a, b, c} and τ={X,ϕ,{a,b}}. C(X, τ) = {X,ϕ,{c}}, αC(X, τ)=
{X,ϕ,{c}} and $\psi\alpha g$-C(X, τ)={ X,ϕ,{c},{b,c},{a,c}}. Thus the space (X, τ) is a α-space but not a $\psi_{\alpha g}T_{\alpha}$ space.

Example 4.14: Let X={a, b, c} and τ={X,ϕ,{a},{b,c}}. C(X, τ)= {X,ϕ,{a},{b,c}}, αC(X, τ)={X,ϕ, {a},{b, c}} and $g^{\#}$sC(X, τ)={ X,ϕ,{a},{b,c}} and $\psi\alpha g$-C(X, τ) ={X, ϕ, {a}, {b, c},{a, c}}. Thus the space (X, τ) is a $T_b^{\#}$-space but not a $\psi_{\alpha g}T_{\alpha}$ space.

Theorem 4.15: If (X, τ) is a $\psi_{\alpha g}T_\alpha$ space, then for each x∈X, {x} is either α-closed or α-open.

Proof: Suppose that (X, τ) is a $\psi_{\alpha g}T_\alpha$ space. Let x∈X and assume that {x} is not a α-closed set. Then X-{x} is not a α-open set. This implies that X-{x} is a $\psi\alpha g$ -closed set, since X is the only α-open set containing X-{x}. Since (X, τ) is a $\psi_{\alpha g}T_\alpha$ space. X-{x} is a α-closed set or equivalently {x} is α-open.

The converse of the above theorem is need not be true as it can be seen by the following example.

Example 4.16: Let X={a, b, c} and τ={X,ϕ,{b},{b,c}}. The set {b} is an open set of (X, τ). {a} and {c} are α-closed subsets of (X, τ). But (X, τ) is not a $\psi_{\alpha g}T_{1/2}$ space. Since $\psi\alpha g$-C(X, τ) = {X,ϕ,{a},{c},{a,b},{a,c}} and αC(X, τ)={ X,ϕ,{a},{c}{a,c}}.

Definition 4.17: A space (X, τ) is called $\psi_{\alpha g}T_s$ space if every $\psi\alpha g$-closed set is semi closed.

Theorem 4.18: Every $\psi_{\alpha g}T_{1/2}$ space is a $\psi_{\alpha g}T_s$ space but not conversely.

Proof: The converse of the theorem is need not be true as seen in the following example.

Example 4.19: Let X={a, b, c} and τ={X,ϕ,{a},{a,b}}. C(X, τ)= {X,ϕ,{c},{b,c}}, SC(X, τ)={X,ϕ, {b},{c},{b,c}} and $\psi\alpha g$-C(X, τ)={ X,ϕ,{a},{b,c}} and $\psi\alpha g$-C(X, τ)={ X,ϕ, {b},{c},{b, c}}. Thus the space (X, τ) is a $\psi_{\alpha g}T_s$-space but not a $\psi_{\alpha g}T_{1/2}$ - space.

Theorem 4.20: Every $\psi_{\alpha g}T_s$ space is a $\psi_{\alpha g}T_\alpha$ -space and $T_b^{\#}$-space but not conversely.

Proof: Obvious.

The converse of the theorem is need not be true as seen in the following example.

Example 4.21: Let X={a, b, c} and τ ={X,ϕ,{a}}. α C(X, τ)= {X,ϕ,{b},{c},{b,c}}, SC(X, τ)={X,ϕ, {a},{b},{c},{a,b},{b,c}} and $\psi\alpha g$-C(X, τ)={ X,ϕ, {b},{c},{b,c}}. Thus the space (X, τ) is a $\psi_{\alpha g}T_\alpha$-space but not a $\psi_{\alpha g}T_s$ - space.

Example 4.22: Let X={a, b, c} and τ={X,ϕ,{a},{b,c}}. C(X, τ)= {X,ϕ,{a},{b,c}}, SC(X, τ)={X,ϕ,{a},{b,c}} and $g^{\#}$s-C(X, τ)={ X,ϕ,{a},{b,c}} and $\psi\alpha g$-C(X, τ)={ X,ϕ, {a},{b,c},{a,c}}. Thus the space (X, τ) is a $T_b^{\#}$-space but not a $\psi_{\alpha g}T_s$ - space.

Theorem 4.23: $\psi_{\alpha g}T_s$ ness is independent of ${}_\alpha T_d$ ness.

Proof: This can be proved by the following examples.

Example 4.24: Let X={a, b, c} and τ={X,ϕ,{a},{b,c}}. GC(X, τ)= {X,ϕ,{a},{b,c}}, SC(X, τ)={X,ϕ,{a},{b,c}}, αGC(X, τ)= P(X), and $\psi\alpha g$-C(X, τ)={ X,ϕ, {a},{b,c},{a,c}}. Thus the space (X, τ) is a ${}_\alpha T_d$ -space but not a $\psi_{\alpha g}T_s$ - space.

Example 4.25: Let X={a, b, c} and τ ={X,ϕ,{a},{a,b}}. GC(X, τ) = {X,ϕ,{b},{a,c},{b,c}}, SC(X, τ)={X,ϕ,{b},{c},{b,c}},αGC(X, τ) = {X,ϕ,{b},{c},{b,c},{a,c}} and $\psi\alpha g$-C(X, τ)={ X,ϕ, {b},{c},{b, c}}. Thus the space (X, τ) is a $\psi_{\alpha g}T_s$ -space but not a ${}_\alpha T_d$ - space.

Theorem 4.26: If (X, τ) is a $\psi_{\alpha g}T_s$ space, then for each x∈X, {x} is either α-closed or semi-open.

Proof: Suppose that (X, τ) is a $\psi_{\alpha g}T_s$ space. Let x∈X and assume that {x} is not a α-closed set. Then X-{x} is not a α-open set. This implies that X-{x} is a $\psi\alpha g$ -closed set, since X is the only α-open set containing X-{x}. Since (X, τ) is a $\psi_{\alpha g}T_s$ space. X-{x} is a semi-closed set or equivalently {x} is semi-open.

The converse of the above theorem is need not be true as it can be seen by the following example.

Example 4.27: Let X={a, b, c} and τ={X,ϕ,{b},{b,c}}. The set {b} is an open set of (X, τ). {a} and {c} are α-closed subsets of (X, τ). But (X, τ) is not a $\psi_{\alpha g}T_{s}$- space. Since $\psi\alpha g$-c(X, τ) = {X,ϕ,{a},{c},{a,b},{a,c}} and SC(X, τ) = {X,ϕ,{a},{c},{a,c}}.

5. $\psi\alpha g$-CONTINUOUS MAPS AND $\psi\alpha g$-IRRESOLUTE MAPS

Definition 5.1: A function f: (X, τ)→(Y, σ) is called $\psi\alpha g$-continuous if $f^{-1}(V)$ is a $\psi\alpha g$ -closed in (X, τ) for every closed set V of (Y, σ).

Theorem 5.2: Every $\psi\alpha g$-continuous map is gp-continuous and gsp-continuous.

Proof: Let V be closed set in (Y,σ).Since f is $\psi\alpha g$-continous, $f^{-1}(V)$ is $\psi\alpha g$- closed set in (X, τ),we know that every $\psi\alpha g$ -closed set is gp-closed (resp. gsp-closed), $f^{-1}(V)$ is gp-closed (resp. gsp-closed) set in (X, τ). Therefore f is gp-continous (resp. gsp-closed).

The converse of the above theorem are need not be true as it can be seen in the following example.

Example 5.3: Let X=Y={a,b,c} with τ={ϕ,X,{a},{a,c}} and σ={ ϕ,X,{c}}. Define a function f: (X, τ)→(Y, σ) by f(a)=a, f(b)=b and f(c)=c. Here f is gp-continuous and gsp-continuous but not $\psi\alpha g$ -continuous, since $f^{-1}(\{a,b\}) = \{a,b\}$ is not in $\psi\alpha g$-closed set in (X, τ).

Theorem 5.4: Every $\psi\alpha g$-continuous map is gpr-continuous and $\alpha\psi$-continuous.

Proof: By the Theorem 3.8 and Theorem 3.10, The converse of the above theorem are need not be true as it can be seen in the following example.

Example 5.5: Let X=Y={a, b ,c} with τ={ϕ,X,{a},{b,c}} and σ={ ϕ,X,{a,c}}. Define a function f: (X, τ)→(Y, σ) by f(a)=a, f(b)=b and f(c)=c. Here f is gpr-continuous and $\alpha\psi$ -continuous but not $\psi\alpha g$ -continuous, since $f^{-1}(\{b\}) = \{b\}$ is not in $\psi\alpha g$-closed set in (X, τ).

Theorem 5.6: Every $semi$-continuous and $g^{\#}s$-continuous map is $\psi\alpha g$ -continuous.

Proof: By the Theorem 3.6, The converse of the above theorem are need not be true as it can be seen in the following example.

Example 5.7: Let X=Y={a, b, c} with τ={ϕ,X,{b},{b,c}} and σ={ϕ,X,{a}}. Define a function f: (X, τ)→(Y, σ) by f(a) = a, f(b)=b and f(c)=c. Here f is $\psi\alpha g$ -continuous but not semi-continuous and $g^{\#}s$ -continuous, since $f^{-1}(\{b,c\}) = \{b,c\}$ is not in semi closed and $g^{\#}s$-closed set in (X, τ).

Theorem 5.8: Every α-continuous map is $\psi\alpha g$ -continous.

Proof: By the Theorem 3.4, The converse of the above theorem is need not be true as shown in the following example.

Example 5.9: Let X=Y={a, b, c} with τ={ϕ,X,{a},{a,b}} and σ={ ϕ,X,{c}}. Define a function f: (X, τ)→(Y, σ) by f(a)=a, f(b)=b and f(c)=c. Here f is $\psi\alpha g$-continuous but not α-continuous, since $f^{-1}(\{a,b\}) = \{a,b\}$ is not in α-closed set in (X, τ).

Theorem 5.10: Every $g^{\#}$-continuous map is $\psi\alpha g$-continous.

Proof: By the Theorem 3.4, The converse of the above theorem is need not be true as shown in the following example.

Example 5.11: Let X=Y={a, b, c} with τ={ϕ,X,{a},{b},{a,b}} and σ={ϕ,X,{a,c}}. Define a function f: (X, τ)→(Y, σ) by f(a)=a, f(b)=b and f(c)=c. Here f is $\psi\alpha g$-continuous but not $g^{\#}$-continuous, since $f^{-1}(\{b\}) = \{b\}$ is not in $g^{\#}$-closed set in (X, τ).

Definition 5.12: A function f: (X, τ)→(Y, σ) is called $\psi\alpha g$-irresolute if $f^{-1}(V)$ is a $\psi\alpha g$-closed set of (X, τ) for every $\psi\alpha g$-closed set V of (Y, σ).

Theorem 5.13: Every $\psi\alpha g$-irresolute map is $\psi\alpha g$-continuous.

Proof: Let V be a closed set of (Y, σ) and hence it is $\psi\alpha g$-closed set. Since f is $\psi\alpha g$-irresolute, $f^{-1}(V)$ is a $\psi\alpha g$-closed set of (X, τ). Hence f is a $\psi\alpha g$-continuous map.
The converse of the above theorem is need not be true by the following example.

Example 5.14: Let X=Y={a,b,c} with τ={X,ϕ,{b},{a,b},{b,c}} and σ={X,ϕ,{a},{b}, {a, b}}. Define f: (X, τ)→(Y, σ) by f(a)=a, f(b)=b and f(c)=c. Then f is not $\psi\alpha g$-irresolute , since {a} is a $\psi\alpha g$-closed set in (Y, σ), but $f^{-1}(\{a\}) = \{a\}$ is not a $\psi\alpha g$-closed set of (X, τ). However f is $\psi\alpha g$-continuous.

Theorem 5.15: If f: (X, τ)→(Y, σ) and g: (Y, σ)→(Z, η) are $\psi\alpha g$-irresolute ,then the composition g∘f: (X, τ)→(Z, η) is $\psi\alpha g$-irresolute.

Proof: Let V be a $\psi\alpha g$-closed set in (Z, η). Since g: (Y, σ)→(Z, η) is a $\psi\alpha g$- irresolute function, $g^{-1}(V)$ is a $\psi\alpha g$-closed set in (Y, σ). Since f is an $\psi\alpha g$- irresolute functions, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is an $\psi\alpha g$-closed set in (X, τ). Hence g∘f: (X, τ)→(Z, η) is an $\psi\alpha g$-irresolute functions.

Example 5.16: Let X=Y={a,b,c} with τ={X,ϕ,{a},{a,c}} and σ={X,ϕ,{a,b}}. Define f: (X, τ)→(Y, σ) by f(a)=a, f(b)=b and f(c)=c. Then f is not $\psi\alpha g$-irresolute, since {a, c} is a $\psi\alpha g$-closed set in (Y, σ), but $f^{-1}(\{a,c\}) = \{a,c\}$ is not a $\psi\alpha g$-closed set of (X, τ). However f is $\psi\alpha g$-continuous.

Theorem 5.17: Every $\psi\alpha g$-irresolute function is gp-continous.

Proof: It follows from the fact that the Theorem 3.8. The following example supports that the converse of the above theorem is not true.

Example 5.18: Let X=Y={a,b,c} with τ={X,ϕ,{a}} and σ={X,ϕ,{b}}. Define f: (X, τ)→(Y, σ) by f(a)=a, f(b)=b and f(c)=c. Then f is not $\psi\alpha g$-irresolute , since {a,c} is a $\psi\alpha g$-closed set in (Y, σ), but $f^{-1}(\{a\}) = \{a\}$ is not a gp-closed set of (X, τ). Thus f is not $\psi\alpha g$-irresolute, however f is gp-continuous.

7. $\psi\alpha g$-c-HOMEOMORPHISM AND THEIR GROUP STRUCTURE

Definition 7.1: A function f: (X, τ)→(Y, σ) is said to be
- (i) $\psi\alpha g$-open is the image f(U) is $\psi\alpha g$-open in (Y, σ) for every open set U of (X, τ).
- (ii) $\psi\alpha g$-closed if the image f(U) is $\psi\alpha g$-closed in (Y, σ) for every open set U of (X, τ).
- (iii) $\psi\alpha g$-c-homeomorphism if f is bijective and f and f^{-1} are $\psi\alpha g$-irresolute.
- (iv) $\psi\alpha g$-homeomorphism if f is bijective and f and f^{-1} are $\psi\alpha g$-continuous.

Theorem 7.2:
(i) Suppose that f is a bijection, then the following conditions are equivalent.
- (a) f is a $\psi\alpha g$-homeomorphism.
- (b) f is a $\psi\alpha g$-open and $\psi\alpha g$-continuous.
- (c) f is a $\psi\alpha g$-closed and $\psi\alpha g$-continuous.

(ii) If f is a homeomorphism, then f and f^{-1} are $\psi\alpha g$- irresolute.
(iii) Every $\psi\alpha g$-c-homeomorphism is a $\psi\alpha g$-homeomorphism.

Proof:
- (i) It is obvious.
- (ii) First we prove that f^{-1} is $\psi\alpha g$- irresolute. Let A be a $\psi\alpha g$-closed set of (X, τ). To show $(f^{-1})^{-1} = f(A)$ is $\psi\alpha g$-closed set in (Y, σ). Let U be a $\psi\alpha g$-open set such that f(A)⊆U.
  Then A=($f^{-1}(f(A))$) $\subseteq f^{-1}(U)$and $f^{-1}(U)$ is αg-open. Since A is $\psi\alpha g$-closed, $\psi cl(A) \subseteq f^{-1}(U)$, we have $\psi cl(f(A)) = f(\alpha cl(A)) \subseteq f(f^{-1}(U)) \subseteq U$ and so f(A) is $\psi\alpha g$-closed. Thus f^{-1} is $\psi\alpha g$-irresolute. Since f^{-1} is also a homeomorphism $(f^{-1})^{-1} = f$ is $\psi\alpha g$- irresolute.
- (iii) It is proved by theorem 5.13.

Definition 7.3: For a topological space (X, τ) we define the following three collections of functions.
- (i) $\psi\alpha g$-ch(X, τ) = {f / f: (X, τ)→ (X, τ)} is a $\psi\alpha g$-c-homeomorphism.
- (ii) $\psi\alpha g$-h(X, τ) = {f / f: (X, τ)→ (X, τ)} is a $\psi\alpha g$-homeomorphism.
- (iii) h(X, τ) = {f / f: (X, τ)→ (X, τ)} is a homeomorphism.

Corollary 7.4: For a Topological space. (X, τ) the following properties hold.
- (i) h(X, τ) ⊆ $\psi\alpha g - ch(X,\tau) \subseteq \psi\alpha g$-h(X, τ).
- (ii) The set $\psi\alpha g - ch(X,\tau)$ forms a group under composition of functions.
- (iii) The group h(X, τ) is a subgroup of $\psi\alpha g - ch(X,\tau)$.
- (iv) If f: (X, τ)→(Y, σ) is a $\psi\alpha g - c$-homeomorphism then it induces an isomorphism $f^*: \psi\alpha g - ch$(Y, σ) → $\psi\alpha g - ch$(Y, σ).

Proof:
- (i) It is proved by using theorem 5.9 and theorem 5.13 and a fact that every continuous map is α-continuous.
- (ii) It is proved by using $g \circ f$ is $\psi\alpha g -$irresolute if both f and g are $\psi\alpha g -$irresolute, for any element $a, b \in \psi\alpha g - ch(X,\tau)$ the following binary operation $\omega: \psi\alpha g - ch(X,\tau)\ X\ \psi\alpha g - ch(X,\tau) \to \psi\alpha g - ch(X,\tau)$ is well defined $\omega(a,b) = b \circ a$.

(iii) By (i), h(X, τ) $\subseteq \psi\alpha g - ch(X,\tau)$ and h(X, τ) $\neq \phi$. For any elements $a, b \in h(X,\tau)$ and the binary operation ω in (ii) it is shown that $\omega(a, b^{-1}) = b^{-1} \circ a \in h(X,\tau)$

(iv) We define $f^*: \psi\alpha g - ch$(Y, σ) $\rightarrow$ $\psi\alpha g - ch$(Y, σ) by f*(h)= $f \circ h \circ f^{-1}$.

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