International Journal of Mathematical Archive-7(1), 2016, 159-165
Available online through www.ijma.info ISSN 2229 – 5046

ON $^{\#}g\alpha$-QUOTIENT MAPPINGS IN TOPOLOGICAL SPACES

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(Received On: 31-01-16; Revised & Accepted On: 08-01-16)

ABSTRACT

The aim of this paper is to introduce $^{\#}g\alpha$ - quotient mappings using $^{\#}g\alpha$ -closed sets and characterize their basic properties. We also derive the relation between the stronger forms of $^{\#}g\alpha$ - quotient mappings.

Keywords: *$^{\#}g\alpha$ -closed sets, $^{\#}g\alpha$ -open sets, $^{\#}g\alpha$ - continuous map, $^{\#}g\alpha$ - irresolute map.*

1. INTRODUCTION

The topological notions of semi - open sets and semi - continuity and pre - open sets and pre - continuity were introduced by N. Levine [2] and A. S. Mashhouret.al., [6] respectively. Generalized closed sets, briefly g - closed sets in topological spaces were introduced by N. Levine [2] in order to extend some important properties of closed sets to a larger family of sets. M. LellisThivagar [10] introduced the concepts of α - quotient mappings and α* - quotient mappings in topological spaces. K. Nono [9] introduced the concept of $g^{\#}\alpha$ - closed sets to investigate some topological properties. In 2009, R. Devi [1] introduced the notion of $^{\#}g\alpha$ - closed sets in topological spaces. In this paper, we introduce the $^{\#}g\alpha$ - quotient functions. Several characterizations and its properties have been established for this functions.

2. PRELIMINARIES

Throughout this dissertation (X, τ) and (Y, σ) (or X and Y) represent non - empty topological spaces on which no separation axioms are assumed unless otherwise explicitly stated. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure and the interior of A in (X, τ) respectively. We list some definitions which are useful in the following sections.

Definition 2.1: A subset A of a topological space (X, τ) is called

(a) a pre - open set [6] if $A \subseteq int(cl(A))$,
(b) a semi - open set [2] if $A \subseteq cl(int(A))$,
(c) a generalized closed set (briefly g - closed) [3] if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ),
(d) a$g^{\#}\alpha$ - closed set (briefly $g^{\#}\alpha$ - closed) [9] if $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is g - open in (X, τ).

The complement of a semi - closed set (respectively α - closed set, $g^{\#}\alpha$ - closed set) of (X, τ) is called a semi - open set (respectively α - open set, $g^{\#}\alpha$ - open set) of (X, τ). It is evident that a subset B of X is $g^{\#}\alpha$ - open in (X, τ) if and only if $F \subseteq \alpha cl(B)$, whenever $F \subseteq B$ and F is g - closed set in (X, τ); a subset B of X is α - closed in (X, τ) if and only if $cl(int(cl(B))) \subseteq B$ holds; a subset B of X is semi – closed in (X, τ) if and only if $int(cl(B)) \subseteq B$ holds.

Definition 2.2: A subset A of a topological space (X, τ) is called a $^{\#}g\alpha$ - closed set (briefly $^{\#}g\alpha$ - closed) [1] if $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is $g^{\#}\alpha$ - open in (X, τ). Let $^{\#}g\alpha O(X)$ denote the collection of $^{\#}g\alpha$ - open sets of X and $^{\#}g\alpha C(X)$ denote the collection of $^{\#}g\alpha$ - closed sets of X.

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Definition 2.3: A function $f: (X, \tau) \to (Y, \sigma)$ is called

(a) α - continuous [7] if $f^{-1}(V)$ is α - closed in (X, τ) for every closed set V of (Y, σ),
(b) g - continuous [4] if $f^{-1}(V)$ is g - closed in (X, τ) for every closed set V of (Y, σ),
(c) $g^{\#}\alpha$ - continuous [9] if $f^{-1}(V)$ is $g^{\#}\alpha$ - closed in (X, τ) for every closed set V of (Y, σ),
(d) $^{\#}g\alpha$- continuous [1] if $f^{-1}(V)$ is $^{\#}g\alpha$- closed in (X, τ) for every closed set V of (Y, σ),
(e) $^{\#}g\alpha$ - irresolute [1] if $f^{-1}(V)$ is $^{\#}g\alpha$- closed in (X, τ) for every $^{\#}g\alpha$ - closed set V of (Y, σ),
(f) strongly $^{\#}g\alpha$ - irresolute [16] if $f^{-1}(V)$ is closed in (X, τ) for every $^{\#}g\alpha$ - closed set V of (Y, σ),
(g) α - irresolute [10] if $f^{-1}(V)$ is α - closed in (X, τ) for every α - closed set V of (Y, σ),
(h) $^{\#}g\alpha$ - open [1] if the image $f(U)$ is $^{\#}g\alpha$ - open in (Y, σ) for every open set U of (X, τ),
(i) $^{\#}g\alpha$ - closed [1] if the image $f(U)$ is $^{\#}g\alpha$ - closed in (Y, σ) for every closed set U of (X, τ),
(j) a quotient map [8], provided a subset V of (Y, σ) is open if and only if $f^{-1}(V)$ is open in (X, τ),
(k) anα - quotient map [8], if f is α - continuous and $f^{-1}(V)$ is open in (X, τ) implies V is α - open in (Y, σ),
(l) anα^{*} - quotient map [8], if f is α - irresolute and $f^{-1}(V)$ is α - open in (X, τ) implies V is open in (Y, σ).
(m) agc - homeomorphism [5] if both f and f^{-1} are g - continuous,
(n) $^{\#}g\alpha c$ - homeomorphism [1] if f and f^{-1} are $^{\#}g\alpha$ - irresolute,
(o) $^{\#}g\alpha$ - homeomorphism [1] if f and f^{-1} are $^{\#}g\alpha$ - continuous.

3. $^{\#}g\alpha$ - QUOTIENT MAP

Definition 3.1: A surjective function $f: (X, \tau) \to (Y, \sigma)$ is said to be a $^{\#}g\alpha$ - quotient map if f is $^{\#}g\alpha$ - continuous and $f^{-1}(V)$ is open in (X, τ) implies V is a $^{\#}g\alpha$ - open set in (Y, σ).

Example 3.2: Let X = {a, b, c} = Y with$\tau = \{\phi, X, \{a, b\}\}$ and$\sigma = \{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$.

Here $^{\#}g\alpha O(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $^{\#}g\alpha O(Y) = \{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$.

Let $f: (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = a$, $f(b) = b$, $f(c) = c$. Then f is $^{\#}g\alpha$ - continuous and $f^{-1}(V)$ is open in (X, τ) implies V is a $^{\#}g\alpha$ - open set in (Y, σ).

Definition 3.3: A map $f: (X, \tau) \to (Y, \sigma)$ is said to be strongly $^{\#}g\alpha$ - open if $f(U)$ is $^{\#}g\alpha$ - open set in (Y, σ) for each $^{\#}g\alpha$ - open set U in (X, τ).

Theorem 3.4: If a map $f: (X, \tau) \to (Y, \sigma)$ is surjective, $^{\#}g\alpha$ - continuous and $^{\#}g\alpha$ - open, then f is a$^{\#}g\alpha$ - quotient map.

Proof: It is enough to prove that $f^{-1}(V)$ is open in (X, τ) implies V is a $^{\#}g\alpha$ - open set in (Y, σ). Let$f^{-1}(V)$ be open in (X, τ). Then $f(f^{-1}(V))$ is a $^{\#}g\alpha$ - open set, since f is $^{\#}g\alpha$ - open. Hence V is a $^{\#}g\alpha$ - open set, as f is surjective, $f(f^{-1}(V)) = V$. Thus, f is a $^{\#}g\alpha$ - quotient map.

Theorem 3.5: If a map $f: (X, \tau) \to (Y, \sigma)$ is $^{\#}g\alpha$ - homeomorphism, then f is a $^{\#}g\alpha$ - quotient map.

Proof: Since f is $^{\#}g\alpha$ - homeomorphism, f is bijective and f is $^{\#}g\alpha$ - continuous. Let $f^{-1}(V)$ be open in X. Since $f^{-1}(V)$ is $^{\#}g\alpha$ - continuous, $f(f^{-1}(V)) = V$ is $^{\#}g\alpha$ - open in Y. Hence f is a $^{\#}g\alpha$ - quotient map.

Theorem 3.6: If $f: (X, \tau^{\#g\alpha}) \to (Y, \sigma^{\#g\alpha})$ be a quotient map, then $f: (X, \tau) \to (Y, \sigma)$ is a$^{\#}g\alpha$ - quotient map.

Proof: Let V be any open set in (Y, σ), then V is a $^{\#}g\alpha$ - open set in (Y, σ) and $V \in \sigma^{\#g\alpha}$. Then $f^{-1}(V)$ is open in (X, τ). Since f is a quotient map, that is, $f^{-1}(V)$ is a $^{\#}g\alpha$ - open set in (X, τ). Suppose $f^{-1}(V)$ is open in (X, τ), that is, $f^{-1}(V) \in \tau^{\#g\alpha}$. Since f is a quotient map, $V \in \tau^{\#g\alpha}$ and V is a $^{\#}g\alpha$ open set in (Y, σ). This shows that $f: (X, \tau) \to (Y, \sigma)$ is a $^{\#}g\alpha$ - quotient map.

4. STRONGER FORM OF $^{\#}g\alpha$ - QUOTIENT MAP

Definition 4.1: Let $f: (X, \tau) \to (Y, \sigma)$ be a surjective map. Then f is called strongly $^{\#}g\alpha$ - quotient map provided a set U of Y is open in (Y, σ) if and only if $f^{-1}(U)$ is a $^{\#}g\alpha$ - open set in (X,τ).

Example 4.2: Let X = {p, q, r, s} with$\tau = \{ \phi, X, \{p\}, \{q, r\}, \{p, q, r\}\}$ and Y = {a, b, c} with$\sigma = \{ \phi, Y, \{a\}, \{b\}, \{a, b\}\}$. Here $^{\#}g\alpha O(X) = \{ \phi, X, \{p\}, \{q\}, \{r\}, \{p, r\}, \{q, r\}, \{p, q, r\}\}$ and $^{\#}g\alpha O(Y) = \{ \phi, Y, \{a\}, \{b\}, \{a, b\}\}$.

The function $f: (X, \tau) \to (Y, \sigma)$ is defined by $f(p) = a = (q)$, $f(r) = b$, $(d) = c$. Then, clearly $f^{-1}(U)$ is a $^{\#}g\alpha$ - open set in (X, τ) if and only if U is open in (Y, σ). Hence f is strongly $^{\#}g\alpha$ - quotient map.

Theorem 4.3: Every strongly #gα - quotient map is #gα - open.

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a strongly #gα - quotient map. Let V be an open set in (X, τ). Since every open set is #gα - open [1] and hence V is #gα - open in (X, τ). That is $f(f^{-1}(V))$ is #gα - open in (X, τ). Since f is strongly #gα - quotient, $f(V)$ is open and hence #gα - open in (Y, σ). This shows that f is a #gα - open.

The converse of the above theorem is not true by the following example.

Example 4.4: Let X = {a, b, c} = Y with τ = {ϕ, X, {a, b}} and σ = {ϕ, Y, {a},{b},{a, b},{a, c}}.

Here #gαO(X) = {ϕ, X, {a}, {b}, {a, b}} and #gαO(Y) = {ϕ, Y, {a}, {b}, {a, b}, {a, c}}.

The function f: $(X, \tau) \rightarrow (Y, \sigma)$ is defined by f (a) = a, f (b) = b, f(c) = c. Then, clearly f is #gα - open but not strongly #gα - quotient map, since f^{-1}({a, c}) = {a, c} is not #gα - open in (X, τ).

Theorem 4.5: Every strongly #gα - quotient map is strongly #gα - open.

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a strongly #gα - quotient map. Let V be a #gα -open set in (X, τ). That is $f^{-1}((V))$ is #gα - open in (X, τ). Since f is strongly #gα -quotient, f (V) is open and hence #gα - open in (Y, σ). This shows that f is strongly #gα - open.

The converse need not be true which can be seen from the following example.

Example 4.6: Let X = {a, b, c, d} withτ = {ϕ, X, {a}, {b, c}, {a, b, c}} and Y = {a, b, c} withσ = {ϕ, Y, {a}, {b}, {a, b}, {a, c}}.

Here #gαO(X) = { ϕ, X, {a}, {b}, {c}, {a, c}, {b, c}, {a, b, c}} and #gαO(Y) = { ϕ, Y, {a}, {b}, {a, b}, {a, c}}.

The function f: $(X, \tau) \rightarrow (Y, \sigma)$ is defined as f (a) = a = (b), f(c) = b,(d) = c. Then, the function f is strongly #gα - open but not strongly #gα - quotient map, since {a, c} is open in (Y, σ) but not #gα - open in (X, τ).

Theorem 4.7: Every strongly #gα - quotient map is #gα - quotient.

Proof: It is obvious.

The converse of the above theorem is not true which can be seen from the following example.

Example 4.8: Let X = {a, b, c} = Y with τ = {ϕ, X, {a}, {b}, {a, b}} andσ= {ϕ, Y, {a, b}}.

Here #gαO(X) = {ϕ, X, {a}, {b}, {a, b}} and #gαO(Y) = {ϕ, Y, {a}, {b}, {a, b}}.

The function f: $X \rightarrow Y$ is defined as f (a) = a, (b) = b, f(c) = c. Then, the function f is #gα - quotient map but not strongly #gα - quotient, since {a} is #gα - open in (X, τ) but not open in Y.

Definition 4.9: A surjective function f: $(X, \tau) \rightarrow (Y, \sigma)$ is said to be a #gα* - quotient map if f is#gα - irresolute and f^{-1}(V) is #gα - open set in (X, τ) implies V is open in (Y, σ).

Example 4.10: Let X = {a, b, c, d} withτ = {ϕ, X, {a}, {b, c}, {a, b, c}} and Y = {p, q, r} withσ = {ϕ, Y, {p}, {q}, {p, q}}.

Here #gαO(X) = {ϕ, X, {a}, {b}, {c}, {a, c}, {b, c}, {a, b, c}} and #gαO(Y) = {ϕ, Y, {p}, {q}, {p, q}}.

Define a map f: $(X, \tau) \rightarrow (Y, \sigma)$ by f (a) = p = f(c), (b) = q, (d) = r. Then clearly, f is #gα - irrresolute and f^{-1}(V) is #gα - open in (X, τ) implies V is a open set in (Y, σ).

Theorem 4.11: Every #gα* - quotient map is #gα - irresolute.

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a #gα* - quotient map. Let V be a #gα - open set in X. That is $f^{-1}(f(V))$ is #gα - open in X. Since f is #gα* - quotient map, thus f (V) is open in Y and hence #gα - open in Y. Therefore, f is #gα - irresolute.

The converse of the above theorem need not be true which can be seen from the following example.

Example 4.12: Let X = {a, b, c} = Y withτ = {ϕ, X, {a}, {b}, {a, b}, {a, c}} and σ = {ϕ, Y, {a}, {b}, {a, b}}. Here #gαO(X) = {ϕ, X, {a}, {b}, {a, b}, {a, c}} and #gαO(Y) = {ϕ, Y, {a}, {b}, {a, b}}.

The function f is defined as f (a) = a, f (b) = b, f(c) = c. Therefore, the function f is #gα - irresolute but not #gα* - quotient map. Since f^{-1}({a, c}) = {a, c} is #gα - open in (X, τ) but {a, c} is not open in (Y, σ).

Definition 4.13: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a surjective map. If a set U is #gα - open in Y if and only if f^{-1}(U) is #gα - open in X, then f is called strongly #gα* - quotient map.

Example 4.14: Let X = {a, b, c, d} with τ = {ϕ, X, {a}, {b, c}, {a, b, c}} and Y = {p, q, r} withσ = {ϕ, Y, {p}, {q}, {p, q}}.

Here #gαO(X) = {ϕ, X, {a}, {b}, {c}, {a, c}, {b, c}, {a, b, c}} and #gαO(Y) = {ϕ, Y, {p}, {q}, {p, q}}.

Define a map f: $(X, \tau) \rightarrow (Y, \sigma)$ by f (a) = p = f (b), f(c) = q, f (d) = r. Then clearly, f is #gα - open in X if and only if f^{-1}(U) is #gα - open in Y. Hence f is strongly #gα* - quotient map.

Theorem 4.15: Every #gα* - quotient map is strongly #gα*- quotient.

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be #gα* - quotient map. Let U be a #gα - open set in Y. Since f is #gα - irresolute, f^{-1}(U) is #gα - open in X. Since f is #gα* - quotient, it follows that U is open. Hence U is #gα - open and $f$is strongly #gα* - quotient map.

The converse of the above theorem need not be true which can be seen from the following example.

Example 4.16: Let X = {a, b, c} with τ = {ϕ, X, {a}, {a, b}, {a, c}} and Y = {p, q, r} withσ = {ϕ, Y, {p}}. Here #gαO(X) = { ϕ, X, {a}, {a, b}, {a, c}} and #gαO(Y) = { ϕ, Y, {p}, {p, q}, {p, r}}.

The function f: $(X, \tau) \rightarrow (Y, \sigma)$ is defined as f (a) = p, f (b) = q, f(c) = r. Thus, the function f is strongly #gα*- quotient but not #gα* - quotient map. Since f^{-1}({p, q}) = {a, b} is #gα - open in X but not open in Y.

Theorem 4.17: Every strongly #gα* - quotient map is #gα - quotient.

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be strongly #gα* - quotient map. Let V be an open set in Y. Then V is a #gα - open set.

Since f is strongly #gα* - quotient, f^{-1}(V) is #gα - open. Hence f is #gα - continuous. Let f^{-1}(V) be an open set in X. Then f^{-1}(V) is #gα - open in X. Hence V is #gα - open and f is a #gα - quotient map

The converse of the above theorem is not true as seen from the following example.

Example 4.18: Let X = {a, b, c} = Y withτ = {ϕ, X, {a}} and σ = {ϕ, Y, {a, b}}.

Here #gαO(X) = {ϕ, X, {a}, {a, b}, {a, c}} and #gαO(Y) = {ϕ, Y, {a}, {b}, {a, b}}.

The function f is defined by f (a) = a, f (b) = b, f(c) = c. Then, the function f is #gα - quotient map but not strongly #gα* - quotient map, since {b} is #gα - open in Y but not #gα - open in X.

Definition 4.19: A space (X, τ) is $T_{\#g\alpha}$ - space if every #gα - closed set is closed.

Theorem 4.20: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be strongly #gα* - quotient map and Y is $T_{\#g\alpha}$- space. Then f is strongly#gα - quotient map.

Proof: Let U be an open set in Y. Then U is #gα - open in Y. Since f is strongly #gα* - quotient, f^{-1}(U) is #gα - open. Let f^{-1}(U) be #gα - open in X. Then U is #gα -open in Y. Since Y is $T_{\#g\alpha}$- Space, U is open in Y and hence f is a strongly #gα -quotient map.

5. COMPARISONS

Theorem 5.1:

(i) Every quotient map is a #gα-quotient map.
(ii) Every α-quotient map is a #gα-quotient map.

Proof: Since every continuous and α - continuous map is #gα - continuous. Also every open set and α - open set is #gα - open [1] and hence the proof follows from the definitions ([3.1], [2.3]).

The converse of the above theorem need not be true which can be seen from the following example.

Example 5.2: (a) Let X = {a, b, c} = Y with τ = {ϕ, X, {a, b}} andσ = {ϕ, Y, {a}, {b}, {a, b}}.

Here #gαO(X) = {ϕ, X, {a}, {b}, {c}, {a, c}, {b, c}, {a, b, c}} and #gαO(Y) = {ϕ, Y, {a}, {b}, {a, b}, {a, c}}.

The function f: (X, τ) → (Y, σ) is defined by f (a) = a, f (b) = b, f(c) = c. Then, the function f is #gα - quotient map but not quotient map, since f^{-1}({b}) = {b} is open in (Y, σ) but {b} is not open in (X, τ).

(b) Let X = {p, q, r} and Y = {a, b, c} with topologies τ = {ϕ, X, {p}, {q}, {p, q}} andσ = {ϕ, Y, {a, b}}.

Here #gαO(X) = {ϕ, X, {p}, {q}, {p, q}}, #gαO(Y) = {ϕ, Y, {a}, {b}, {a, b}} and αO(Y) = {ϕ, Y, {a, b}}.

The function f is defined by f (p) = a, f (q) = b, f(r) = c. Then, the function f is strongly #gα - quotient map but not α-quotient map, since f^{-1}({a}) = {p} is open in (X, τ) but {a} is not α - open in (Y, σ).

Theorem 5.3: Every α* - quotient map is a #gα* - quotient map.

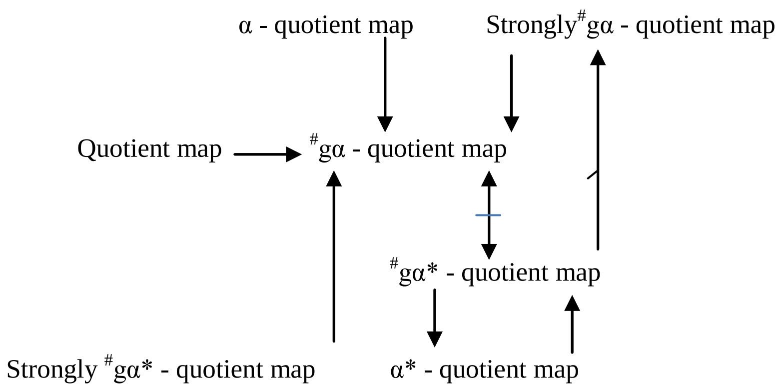
Proof: Let f be an α* - quotient map then f is surjective, α - irresolute and f^{-1}(U) is an α - open set in (X, τ) implies U is an open set in (Y, σ). Since every α - irresolute map is #gα - irresolute, f^{-1}(U) is α - open set which is a #gα - open set. Hence f is a #gα* -quotient map.

The converse of the above theorem need not be true which can be seen from the following example.

Example 5.4: Let X = {a, b, c, d} with τ = {ϕ, X, {a}, {b, c}, {a, b, c}} and Y = {p, q, r} withσ = {ϕ, Y, {p}, {q}, {p, q}}.

Here #gαO(X) = {ϕ, X, {a}, {b}, {c}, {a, c}, {b, c}, {a, b, c}}, #gαO(Y) = {ϕ, Y, {p}, {q}, {p, q}} and αO(X) = {ϕ, X, {a}, {b, c}, {a, b, c}}. Define a function f by f (a) = p = f(c), (b) = q, f (d) = r. Then, the function f is #gα* - quotient map but not α* - quotient map, since f^{-1}({p, q}) = {a, b} is α - open in (X, τ) but {p, q} is not open in (Y, σ).

Remark 5.5: From the above results we obtain the following implication diagram.



6. APPLICATONS

Theorem 6.1: Let f: (X, τ) → (Y, σ) be an open, surjective#gα - irresolute map and g: (Y, σ) → (Z, η) be a #gα - quotient map. Then their composition $g \circ f$: (X, τ) → (Z, η) is a #gα - quotient map.

Proof: Let V be any open set in (Z, η). Since g is #gα - quotient, g^{-1}(V) is #gα -open in Y. Since f is #gα - irresolute, f^{-1}(g^{-1}(V)) = $(g \circ f)^{-1}$(V) is a #gα - open set in X. This implies that $(g \circ f)^{-1}$(V) is #gα - open. This shows that $g \circ f$ is a #gα - continuous map. Also, assume that $(g \circ f)^{-1}$(V) is open in (X, τ) for V ⊆ Z. That is, f^{-1}(g^{-1}(V)) is open in (X, τ). Since f is an open map, (f^{-1}(g^{-1}(V))) is an open set in Y. It follows that g^{-1}(V) is open in Y. Since g is #gα - quotient map, V is #gα - open set in (Z, η) and hence $g \circ f$: (X, τ) → (Z, η) is a #gα - quotient map.

Theorem 6.2: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a $^{\#}$gα - open, surjective and $^{\#}$gα - irresolute map and g: $(Y, \sigma) \rightarrow (Z, \eta)$ be a strongly $^{\#}$gα - quotient map. Then $g\circ f$: $(X, \tau) \rightarrow (Z, \eta)$ is a strongly $^{\#}$gα - quotient map.

Proof: Let U be an open set in Z. Then U is $^{\#}$gα - open. Since g is a strongly $^{\#}$gα -quotient map, g^{-1}(U) is $^{\#}$gα - open in Y. Then $f^{-1}(g^{-1}$(U)) is $^{\#}$gα - open in X (Since f is $^{\#}$gα - irresolute). Hence $(g\circ f)^{-1}$(U) is $^{\#}$gα - open in Y. Let $(g\circ f)^{-1}$(U) is $^{\#}$gα - open in X. That is, $f^{-1}(g^{-1}$(U)) is $^{\#}$gα - open in X. Since f is a $^{\#}$gα - open map, $(f^{-1}(g^{-1}$(U))) is $^{\#}$gα - open in Y and hence g^{-1}(U) is $^{\#}$gα - open in Y. Since g is a strongly $^{\#}$gα - quotient map, U is open in Z and therefore $g\circ f$: $(X, \tau) \rightarrow (Z, \eta)$ is a strongly $^{\#}$gα - quotient map.

Theorem 6.3: If h: $(X, \tau) \rightarrow (Y, \sigma)$ is a $^{\#}$gα - quotient map and g: $(X, \tau) \rightarrow (Z, \eta)$ is a continuous map that is constant on each set h^{-1}(y), for y ∈ Y, then g induces a $^{\#}$gα -continuous map f: $(Y, \sigma) \rightarrow (Z, \eta)$ such that $f\circ h = g$.

Proof: The set g(h^{-1}(y)) is a one point set in (Z, η), since g is constant on h^{-1}(y), for each y ∈ Y. If f(y) denotes this point, then it is clear that f is well defined and for each x ∈ X, f(h(x)) = g(x). We claim that f is $^{\#}$gα - continuous. For if, let U be any open set in (Z, η). Since g is continuous, g^{-1}(U) is open in X. But g^{-1}(U) =$h^{-1}(f^{-1}$(U)) is open in X. Since h is $^{\#}$gα - quotient map, f^{-1}(U) is $^{\#}$gα - open and hence f is $^{\#}$gα - continuous.

Theorem 6.4: Let p: $(X, \tau) \rightarrow (Y, \sigma)$ be a $^{\#}$gα - quotient map where X and Y are $T_{\#g\alpha}$-spaces. Then: $(Y, \sigma) \rightarrow (Z, \eta)$ is strongly $^{\#}$gα - irresolute if and only if the composite map $f\circ$ p: $(X, \tau) \rightarrow (Z, \eta)$ is strongly $^{\#}$gα - irresolute.

Proof: Let f: $(Y, \sigma) \rightarrow (Z, \eta)$ be strongly $^{\#}$gα - irresolute and U be a $^{\#}$gα - open set in (Z, η). Since f is strongly $^{\#}$gα - irresolute, f^{-1}(U) is open in Y. Then $(f\circ p)^{-1}$(U) = $p^{-1}(f^{-1}$(U)) is $^{\#}$gα - open in X (since p is $^{\#}$gα - quotient). Since X is $T_{\#g\alpha}$ - space, $p^{-1}(f^{-1}$(U)) is open in X and hence the composite map $f\circ$p is strongly $^{\#}$gα - irresolute.

Conversely, suppose that the composite function $f\circ$p is strongly $^{\#}$gα - irresolute. Let U be a $^{\#}$gα - open set in Z, $p^{-1}(f^{-1}$(U)) is open in X. Since p is $^{\#}$gα - quotient map, it implies that, f^{-1}(U) is $^{\#}$gα - open in (Y, σ). Since Y is $T_{\#g\alpha}$- space, it implies that f^{-1}(U) is open in Y. Hence, f is strongly $^{\#}$gα - irresolute.

Theorem 6.5: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be surjective, strongly $^{\#}$gα - open and $^{\#}$gα - irresolute map and: $(Y, \sigma) \rightarrow (Z, \eta)$ be a $^{\#}$gα* - quotient map then $g\circ f$: $(X, \tau) \rightarrow (Z, \eta)$ is $^{\#}$gα* - quotient map.

Proof: Let V be a $^{\#}$gα - open set in Z. Then g^{-1}(V) is $^{\#}$gα - open in Y because g is $^{\#}$gα* - quotient map. Since f is $^{\#}$gα - irresolute, $f^{-1}(g^{-1}$(V)) is $^{\#}$gα - open in (X, τ). Then $g\circ f$ is $^{\#}$gα - irresolute in X. Hence $(g\circ f)$ is $^{\#}$gα - irresolute. Suppose $(g\circ f)^{-1}$(V) is a $^{\#}$gα - open set in X for a subset V ⊆ Z. That is, $f^{1}(g^{-1}$(V)) is $^{\#}$gα - open in X. Since f is strongly $^{\#}$gα - open map, $(f^{-1}(g^{-1}$(V))) =g^{-1}(V) is $^{\#}$gα - open in Y. Since g is a $^{\#}$gα* - quotient map, V is open set in (Y, σ). Hence $g\circ f$ is $^{\#}$gα* - quotient map.

Theorem 6.6: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a strongly $^{\#}$gα - quotient map and g: $(Y, \sigma) \rightarrow (Z, \eta)$ be a$^{\#}$gα* - quotient map and Y be a $T_{\#g\alpha}$ - space. Then $g\circ f$: $(X, \tau) \rightarrow (Z, \eta)$ is a $^{\#}$gα* - quotient map.

Proof: Let V be a $^{\#}$gα - open set in Z. Then g^{-1}(V) is $^{\#}$gα - open in Y (since g is $^{\#}$gα* - quotient map). Since Y is a $T_{\#g\alpha}$ - space, g^{-1}(V) is an open set in Y. Since f is strongly $^{\#}$gα - quotient, $f^{-1}(g^{-1}$(V)) is $^{\#}$gα - open in X. That is, $(g\circ f)^{-1}$(V) is $^{\#}$gα -open in X and hence $g\circ f$: $(X, \tau) \rightarrow (Z, \eta)$ is $^{\#}$gα - irresolute. Let $(g\circ f)^{-1}$(V) be a $^{\#}$gα - open set in X. That is, $f^{-1}(g^{-1}$(V)) is $^{\#}$gα - open in X. This implies g^{-1}(V) is open in Y. Hence g^{-1}(V) is a $^{\#}$gα - open set. Since g is $^{\#}$gα* - quotient, V is open and hence $g\circ f$: $(X, \tau) \rightarrow (Z, \eta)$ is a $^{\#}$gα* - quotient map.

Theorem 6.7: The composition of two $^{\#}$gα* - quotient maps is also a $^{\#}$gα* - quotient map.

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ and g: $(Y, \sigma) \rightarrow (Z, \eta)$ be two $^{\#}$gα* - quotient maps. Let U be a $^{\#}$gα - open set in Z. Then g^{-1}(U) is a $^{\#}$gα - open set in Y. Since f is a $^{\#}$gα* - quotient map, $f^{-1}(g^{-1}$(U)) is a $^{\#}$gα - open set in X. That is, $(g\circ f)^{-1}$(U) is $^{\#}$gα -open in X. Hence $g\circ f$: $(X, \tau) \rightarrow (Z, \eta)$ is $^{\#}$gα - irresolute. Let $(g\circ f)^{-1}$ (V) be a $^{\#}$gα -open set in X. Then $f^{-1}(g^{-1}$(V)) is $^{\#}$gα - open in X. This implies g^{-1}(V) is open in Y and hence g^{-1}(V) is $^{\#}$gα - open. Since g is a $^{\#}$gα* - quotient map, V is open. Hence $g\circ f$ is a $^{\#}$gα* - quotient map.

REFERENCES

1. R. Devi, H. Maki, V. Kokilavani, The group structure of $^{\#}g\alpha$ - closed sets in topological spaces, International Journal of General Topology, Vol. 2, No. 1, (June 2009), pp. 21 - 30.
2. N. Levine, "Semi-open sets and semi - continuity in topological spaces, Amer. Math. Monthly 70 (1963), 36- 41.
3. N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo, 19(2): 89: 96, 1970.
4. H. Maki, P. Sundaram and K. Balachandran, "On generalized continuous maps in topological Spaces", Mem. Fac. Sci. Kochi Univ. Ser - A Math., 12 (1991), 5 - 13.

5. H. Maki, P. Sundaram and K. Balachandran, "On generalized homeomorphisms in topological Spaces", Bull. Fukuoka Univ. Ed. Part III, 40 (1991), 13 - 21.
6. A. S. Mashour, I. A. Hasanein and S. N. EI.Deeb, "A note on semi - continuity and pre - Continuity", Indian. J. Pure Appl. Math., 13 (1982), 213 - 218.
7. A. S. Mashour, I. A. Hasanein and S. N. EI.Deeb, "α - continuous and α - open mappings, Acta Math. Hung, 41(3-4), (1983), 213 - 218
8. J. R. Munkers, Topology, A first course, Fourteenth Indian Reprint.
9. K. Nono, R. Devi, M. Devipriya, K. Muthukumraswamy and H. Maki, Ong$^{\#}\alpha$ - closed sets and the digital plane, Bull. Fukuoka Univ. Ed. Part III, 53 (2004), 15 - 24.
10. M. L. Thivagar, A Note on quotient mappings, Bull. Malaysian Math. Soc. 14 (1991), 21-30.

Source of support: Nil, Conflict of interest: None Declared