

FLOW OF A NON-NEWTONIAN FLUID SANDWICHED BETWEEN TWO NEWTONIAN FLUIDS

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ABSTRACT

Laminar flow of a couple-stress fluid in contact with a Newtonian fluid in a channel bounded by permeable beds has been studied. The flow in the permeable beds is governed by Darcy's law. The expressions for the velocity and temperature fields are obtained. The effects of flow parameters on the velocity and temperature are discussed in detail through graphs. It is observed that the velocity and temperature are increases with increasing Darcy number and Couple stress parameter.

Key words: Couple-stress fluid, Newtonian fluid, Channel, Permeable beds.

1. INTRODUCTION

The flow of non-Newtonian fluids through and past porous media have wider applications in many branches like petroleum engineering, biomedical engineering, chemical engineering and such other important fields. There are several non-Newtonian/Newtonian fluid models available in literature [1-17]. Some of these models deal with theory of polar fluids. The theory of polar fluids was originally developed from a statistical mechanics model that assumed non-central forces of interaction between particles. If the inter particle forces are not central forces in a particle interaction, there is an inter particle couple as well as an inter particle force. Due to the action of this couple, the fluid particles will have a tendency to rotate relative to their neighbours. The idea of a polar fluid is obtained by introducing a kinematic variable to model the forces that balance the action of the couple. The polar fluid theory is different from the other fluid theories in the angular momentum effects such as couple-stresses and symmetry of the symmetric stress tensor. The theory of fluids with couple stresses is obtained if the polar fluid is constrained so that the cosseratte triad rotates with the underlying medium, but remains rigid. Both polar and di-polar fluids reduce to the theory of fluids with couple stresses introduced by Stokes.

The couple stress fluid model is very useful to explain the behaviour of some slurries and some physiological fluids. Initial studies of experimental data on blood suggest that some deviations from Newtonian behaviour of blood may be explained through couple stress fluid model. Based on the couple – stress theory of Stokes [18], Valanis and Sun [19], chaturani and kaloni [20], Chaturani *et al.* [21,22] have proposed some theoretical models for blood flow through narrow tubes with non-zero couple stress boundary condition at interface.

Further, Umavathi and Malashetty [23] studied the effects of couple stresses on the free convective flow in a vertical channel. Free convection flow of an electrically conducting couple-stress fluid for the radiating medium in a vertical channel has been studied by Umavathi [24]. Umavathi *et al.* [25] made a detailed study on the flow of heat transfer of a couple-stress fluid in contact with a Newtonian fluid.

Umavathi *et al.* [26] investigated the flow and heat transfer of a Couple stress fluid and viscous fluids in a vertical channel. Iyengar and Punnamchandrar Bitla [27] studied the pulsating flow of an incompressible couple stress fluid between permeable beds. Devakar *et al.* [28] determined the analytical solutions of couple stress fluid flows with slip boundary conditions.

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In view of the several practical applications fully developed flow and heat transfer in a horizontal channel consisting of a couple-stress fluid sandwiched between two Newtonian fluids bounded by permeable beds is analysed. The flow in the permeable beds is governed by Darcy's law. The closed form solutions are obtained for the velocity and temperature fields in the channel. The effect of permeability on the velocity and temperature is discussed.

2. NOMENCLATURE

x, y	Cartesian Coordinates
U_1, U_2, U_3	Velocities in region I, region II, region III,
U_{B1}	Slip velocity at the upper permeable bed.
U_{B2}	Slip velocity at the lower permeable bed.
k_1, k_2	permeabilities of the upper and lower beds.
Da_1	Darcy number at the upper permeable bed, $\left(\frac{k_1}{h^2}\right)$
Da_2	Darcy number at the lower permeable bed, $\left(\frac{k_2}{h^2}\right)$
T_{B1}	Temperature at $y= 2h$
T_{B2}	Temperature at $y = -h$
α	Slip parameter
a	Couple stress parameter, $\left(\frac{\mu_1 h^2}{\eta}\right)$
C_p	Specific heat at constant pressure.
E_c	Eckert number
h	Height of the regions I, II, III
K	ratio of thermal conductivities, $\left(\frac{K_1}{K_2}\right)$
K_1	Thermal conductivity of the fluid in regions 1 and 3
K_2	Thermal conductivity of the fluid in region 2
m	ratio of viscosities , $\left(\frac{\mu_1}{\mu_2}\right)$
P	Non-dimensional pressure gradient, $\left(-Re \frac{\partial p}{\partial x}\right)$
T_1, T_2, T_3	Temperatures in region I, II and III.
Pr	Prandtl number
H	Biot number
U	Velocity in x -direction
η	material constant
θ	non dimensional temperature , $\left(\frac{T - T_{B2}}{T_{B1} - T_{B2}}\right)$

3. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider the flow of a couple stress fluid in contact with a Newtonian fluid in a channel bounded by permeable beds. The flow and heat transfer in a system consists of a couple-stress fluid layer sandwiched between two viscous fluid layers. (as shown in fig.1). The regions - $h \leq y \leq 0$ and $h \leq y \leq 2h$ are filled with a Newtonian fluid of viscosity μ_1 and thermal conductivity K_1 and the region $0 \leq y \leq h$ is occupied by a couple stress fluid having viscosity μ_2 and thermal conductivity K_2 .

We make the following assumptions and derive the basic equations of motion

1. The flow is steady, viscous, incompressible and fully developed.
2. The flow is in X-direction. All the physical quantities except the pressure are functions of y only.

3. The motion is caused by a constant pressure gradient $\frac{\partial P}{\partial x}$.
4. The permeabilities of the upper and lower beds are k_1 and k_2 .

In view of the above assumptions, the equations of motion for velocity take the following form

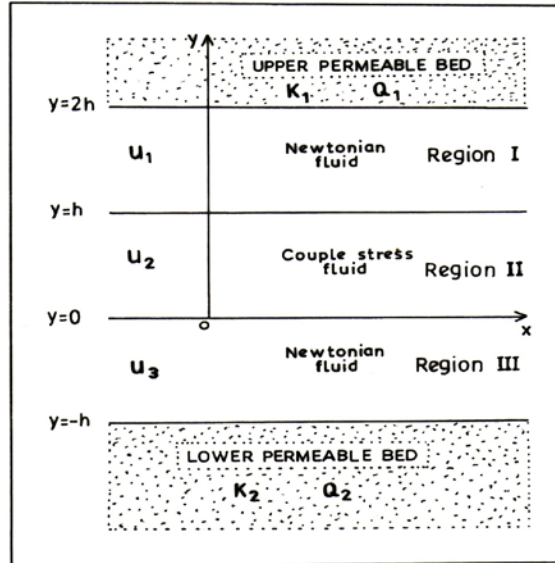


Fig.1: Physical Model

Basic equations

$$\frac{\partial U}{\partial x} = 0 \tag{1}$$

(i) Flow in the channel

Region 1:

$$\mu_1 \frac{\partial^2 U_1}{\partial y^2} - \frac{\partial p}{\partial x} = 0 \quad (h \leq y \leq 2h) \tag{2}$$

Region 2:

$$\mu_2 \frac{\partial^2 U_2}{\partial y^2} - \eta \frac{\partial^4 U_2}{\partial y^4} - \frac{\partial p}{\partial x} = 0 \quad (0 \leq y \leq h) \tag{3}$$

Region 3:

$$\mu_3 \frac{\partial^2 U_3}{\partial y^2} - \frac{\partial p}{\partial x} = 0 \quad (-h \leq y \leq 0) \tag{4}$$

Similarly, the equations of motion for temperature in three regions take the following form

Region 1:

$$\frac{\partial^2 T_1}{\partial y^2} + \frac{\mu_1}{K_1} \left(\frac{\partial U_1}{\partial y} \right)^2 = 0 \quad (h \leq y \leq 2h) \tag{5}$$

Region 2:

$$\frac{\partial^2 T_2}{\partial y^2} + \frac{\mu_2}{K_2} \left(\frac{\partial U_2}{\partial y} \right)^2 = 0 \quad (0 \leq y \leq h) \tag{6}$$

Region 3:

$$\frac{\partial^2 T_3}{\partial y^2} + \frac{\mu_3}{K_1} \left(\frac{\partial U_3}{\partial y} \right)^2 = 0 \quad (-h \leq y \leq 0) \quad (7)$$

(ii) Flow in the porous medium

The flows in the upper and lower permeable beds are governed by Darcy's law and is given by

$$Q_i = \frac{K_i}{\mu} \frac{\partial p}{\partial x} \quad (i = 1 \text{ for upper permeable bed, } i = 2 \text{ for lower permeable bed})$$

Boundary Conditions

$$1) \quad U_1 = U_{B_1}, \quad \frac{\partial U_1}{\partial y} = \frac{-\alpha}{\sqrt{k_1}} (U_{B_1} - \bar{Q}_1) \quad \text{at } y = 2h \quad (8)$$

$$2) \quad U_1 = U_2 \quad \text{at } y = h \quad (9)$$

$$3) \quad U_2 = U_3 \quad \text{at } y = 0 \quad (10)$$

$$4) \quad U_3 = U_{B_2}, \quad \frac{\partial U_3}{\partial y} = \frac{\alpha}{\sqrt{k_2}} (U_{B_2} - \bar{Q}_2) \quad \text{at } y = -h \quad (11)$$

$$5) \quad \mu_1 \frac{\partial U_1}{\partial y} = \mu_2 \frac{\partial U_2}{\partial y} - \eta \frac{\partial^3 U_2}{\partial y^3} \quad \text{at } y = h \quad (12)$$

$$6) \quad \mu_2 \frac{\partial U_2}{\partial y} - \eta \frac{\partial^3 U_2}{\partial y^3} = \mu_1 \frac{\partial U_3}{\partial y} \quad \text{at } y = 0 \quad (13)$$

$$7) \quad \frac{\partial^2 U_2}{\partial y^2} = 0 \quad \text{at } y = h \quad (14)$$

$$8) \quad \frac{\partial^2 U_2}{\partial y^2} = 0 \quad \text{at } y = 0 \quad (15)$$

$$9) \quad T_1 = T_{B_1}; \quad \frac{\partial T}{\partial y} = \frac{-H}{\sqrt{k_1}} (T_{B_1} - T_0) \quad \text{at } y = 2h \quad (16)$$

$$10) \quad T_1 = T_2 \quad \text{at } y = h \quad (17)$$

$$11) \quad T_2 = T_3 \quad \text{at } y = 0 \quad (18)$$

$$12) \quad T_3 = T_{B_2}; \quad \frac{\partial T}{\partial y} = \frac{H}{\sqrt{k_2}} (T_{B_2} - T_0) \quad \text{at } y = -h \quad (19)$$

$$13) \quad K_1 \frac{\partial T_1}{\partial y} = K_2 \frac{\partial T_2}{\partial y} \quad \text{at } y = h \quad (20)$$

$$14) \quad K_2 \frac{\partial T_2}{\partial y} = K_1 \frac{\partial T_3}{\partial y} \quad \text{at } y = 0 \quad (21)$$

4. NON DIMENSIONALISATION OF THE FLOW QUANTITIES

The following non-dimensional quantities are introduced to make the basic equations and the boundary conditions dimensionless

$$x^* = \frac{x}{h}; \quad y^* = \frac{y}{h}; \quad U_i^* = \frac{\bar{U}_i}{U}; \quad Q_i^* = \frac{\bar{Q}_i}{U_1}$$

$$\theta = \frac{T - T_{B_2}}{T_{B_1} - T_{B_2}}; \quad m = \frac{\mu_1}{\mu_2}; \quad p^* = \frac{p}{\rho U^2}$$

In view of the above dimensionless quantities the basic equations and the boundary conditions for the velocity takes the following form

Basic equations

Region 1:

$$\mu_1 \frac{d^2 U_1}{dy^2} - \frac{\partial p}{dx} = 0 \quad (h \leq y \leq 2h) \tag{22}$$

Region 2:

$$\mu_2 \frac{d^2 U_2}{dy^2} - \eta \frac{d^4 U_2}{dy^4} - \frac{\partial P}{dx} = 0 \quad (0 \leq y \leq h) \tag{23}$$

Region 3:

$$\mu_3 \frac{d^2 U_3}{dy^2} - \frac{\partial p}{dx} = 0 \quad (-h \leq y \leq 0) \tag{24}$$

Boundary conditions

$$U_1 = U_{B_1} \text{ at } y = 2 \tag{25}$$

$$\frac{dU_1}{dy} = \frac{-\alpha}{\sqrt{Da_1}} (U_{B_1} - Q_1) \text{ at } y = 2 \tag{26}$$

$$U_1 = U_2 \text{ at } y = 1 \tag{27}$$

$$U_2 = U_3 \text{ at } y = 0 \tag{28}$$

$$U_3 = U_{B_2} \text{ at } y = -1 \tag{29}$$

$$\frac{dU_3}{dy} = \frac{\alpha}{\sqrt{Da_2}} (U_{B_2} - Q_2) \text{ at } y = -1 \tag{30}$$

$$\frac{dU_1}{dy} = \frac{1}{m} \frac{dU_2}{dy} - \frac{1}{a^2} \frac{d^3 U_2}{dy^3} \text{ at } y = 1 \tag{31}$$

$$\frac{dU_2}{dy} - \frac{m}{a^2} \frac{d^3 U_2}{dy^3} = m \frac{dU_3}{dy} \text{ at } y = 0 \tag{32}$$

$$\frac{d^2 U_2}{dy^2} = 0 \text{ at } y = 1 \tag{33}$$

$$\frac{d^2 U_2}{dy^2} = 0 \text{ at } y = 0 \tag{34}$$

Similarly the basic equations and the boundary conditions for the temperature takes the following form:

Basic equations

Region 1:

$$\frac{d^2 \theta_1}{dy^2} + Ec \Pr \left(\frac{dU_1}{dy} \right)^2 = 0 \tag{35}$$

Region 2:

$$\frac{d^2 \theta_2}{dy^2} + \frac{K}{m} Ec \Pr \left(\frac{dU_2}{dy} \right)^2 = 0 \tag{36}$$

Region 3:

$$\frac{d^2 \theta_3}{dy^2} + Ec \Pr \left(\frac{dU_3}{dy} \right)^2 = 0 \tag{37}$$

Boundary conditions

$$\frac{d\theta_1}{dy} = \frac{-H}{\sqrt{Da_1}} \theta_{B_1} \text{ at } y = 2 \tag{38}$$

$$\theta_1 = \theta_2 \text{ at } y = 1 \tag{39}$$

$$\theta_2 = \theta_3 \text{ at } y = 0 \tag{40}$$

$$\frac{d\theta_3}{dy} = \frac{H}{\sqrt{Da_2}} \theta_{B_2} \text{ at } y = -1 \tag{41}$$

$$\frac{d\theta_1}{dy} = \frac{1}{K} \frac{d\theta_2}{dy} \text{ at } y = 1 \tag{42}$$

$$\frac{d\theta_2}{dy} = K \frac{d\theta_3}{dy} \text{ at } y = 0 \tag{43}$$

5. SOLUTION OF THE PROBLEM

a) Solving the equations (22) - (24) subject to the boundary conditions (25) –(34) we obtain the velocity field as

$$U_1 = \frac{Py^2}{2} + C_1y + C_2; \text{ where } P = -\text{Re} \frac{\partial p}{\partial x}, \quad 1 \leq y \leq 2 \tag{44}$$

$$U_2 = C_3 + C_4y + C_5 \cosh(A_4y) + C_6 \sinh(A_4y) + A_5y^2, \quad 0 \leq y \leq 1 \tag{45}$$

$$U_3 = \frac{Py^2}{2} + C_7Y + C_8 \quad -1 \leq y \leq 0 \tag{46}$$

Where $A_5 = \frac{-G_2}{2A_2}$, $A_2 = a^2 / m$

$$A_4 = \sqrt{A_2} \quad G_2 = -a^2P; C_1 = \frac{mC_7 + 2A_5 - mP}{m}; C_2 = \frac{mB_3 - B_4(mC_7 + 2A_5 - mP)}{m}$$

$$C_3 = C_8 - C_5; C_4 = mC_7; C_5 = \frac{-2A_5}{A_4^2}; C_6 = -C_5D_1; C_7 = \frac{m^2C_7 + mD_2 - mB_1 - mC_5 - B_5}{m(1 - B_2 - B_4)};$$

$$C_8 = B_2C_7 - B_1; D_1 = \frac{\cosh A_4 - 1}{\sinh A_4}$$

$$B_1 = \left(\frac{1}{2} + \frac{\sqrt{Da_2}}{\alpha} \right) P - Q_2; B_2 = \left(1 + \frac{\sqrt{Da_2}}{\alpha} \right); B_3 = Q_1 - 2P \left(1 + \frac{\sqrt{Da_1}}{\alpha} \right)$$

$$B_4 = \left(2 + \frac{\sqrt{Da_1}}{\alpha} \right); B_5 = 2A_5 - mP + mB_3 + mB_4P - 2B_4A_5$$

$$D_2 = C_5 \cosh A_4 + C_6 \sinh A_4 + 2A_5 - P / 2$$

The slip velocity at the upper permeable bed is obtained as

$$U_{B_1} = 2C_1 + C_2 + 2P$$

The slip velocity at the lower permeable bed is obtained as

$$U_{B_2} = -C_7 + C_8 + P / 2$$

b) Solving the equations (35) – (37) subject to the boundary conditions (38) – (43) we obtain the temperature as

$$\theta_1 = r_1y^4 + r_2y^3 + r_3y^2 + B_1y + B_2; \quad 1 \leq y \leq 2 \tag{47}$$

$$\begin{aligned} \theta_2 = & r_{14} \cosh(2A_4 y) + r_{15} \sinh(2A_4 y) + r_{16} y \cosh(A_4 y) \\ & + r_{17} y \sinh(A_4 y) + r_{18} \cosh(A_4 y) + r_{19} \sinh(A_4 y) \quad ; 0 \leq y \leq 1 \end{aligned} \quad (48)$$

$$+ r_{20} y^4 + r_{21} y^3 + r_{22} y^2 + B_3 y + B_4$$

$$\theta_3 = r_{23} y^4 + r_{24} y^3 + r_{25} y^2 + B_5 y + B_6 \quad -1 \leq y \leq 0 \quad (49)$$

$$r_1 = \frac{-Ec \text{ Pr } \rho^2}{12}; r_2 = \frac{-Ec \text{ Pr } \rho C_1}{3}; r_3 = \frac{-Ec \text{ Pr } \rho C_1^2}{2}; r_4 = A_6 A_4^2 C_5^2;$$

$$r_5 = A_6 A_4^2 C_6^2; r_6 = A_6 C_5 C_6 A_4^2; r_7 = 4 A_6 A_4 A_5 C_6; r_8 = 4 A_6 A_5 A_4 C_5;$$

$$r_9 = 2 A_6 C_4 C_6 A_4; r_{10} = 2 A_6 C_4 C_5 A_4; r_{11} = 4 A_6 A_5^2; r_{12} = 4 A_6 A_5 C_4;$$

$$r_{13} = A_6 C_4^2; r_{14} = \frac{(r_4 + r_5)}{8A_4^2}; r_{15} = \frac{r_6}{4A_4^2}; r_{16} = \frac{r_7}{A_4^2}; r_{17} = \frac{r_8}{A_4^2}; r_{18} = \left(\frac{-2r_8}{A_4^3} + \frac{r_9}{A_4^2} \right);$$

$$r_{19} = \left(\frac{-2r_7}{A_4^3} + \frac{r_{10}}{A_4^2} \right); r_{20} = \frac{r_{11}}{12}; r_{21} = \frac{r_{12}}{6}; r_{22} = \frac{2r_{13} + r_5 - r_4}{4}; r_{23} = \frac{-Ec \text{ Pr } \rho^2}{12};$$

$$r_{24} = \frac{-Ec \text{ Pr } \rho c_7}{3}; r_{25} = \frac{-Ec \text{ Pr } c_7^2}{2}; T_{B_2} = -B_5 + B_6 + e_2; e_1 = -4r_{23} + 3r_{24} - 2r_{25}$$

$$e_2 = r_{23} - r_{24} + r_{25}; B_6 = \left(\frac{\sqrt{Da_2}}{H} + 1 \right) B_5 + \frac{\sqrt{Da_2}}{H} e_1 - e_2;$$

$$T_{B_1} = \frac{-\sqrt{Da_1}}{H} (32r_1 + 12r_2 + 4r_3 + B_1)$$

$$e_3 = 32r_1 + 12r_2 + 4r_3; e_4 = 16r_1 + 8r_2 + 4r_3; e_5 = 2A_4 r_{15} + r_{16} + A_4 r_{19}$$

$$e_6 = 2A_4 r_{14} \sinh 2A_4 + 2A_4 r_{15} \cosh 2A_4 + A_4 r_{16} \sinh A_4 + A_4 r_{18} \sinh A_4$$

$$+ A_4 r_{17} \cosh A_4 + A_4 r_{19} \cosh A_4 + r_{16} \cosh A_4 + r_{17} \sinh A_4 + 4r_{20} + 3r_{21} + 2r_{22}$$

$$e_7 = 4r_1 + 3r_2 + 2r_3; e_8 = r_{14} + r_{18}$$

$$e_9 = r_{14} \cosh 2A_4 + r_{15} \sinh 2A_4 + r_{16} \cosh A_4 +$$

$$r_{18} \cosh A_4 + r_{17} \sinh A_4 + r_{19} \sinh A_4 + r_{20} + r_{21} + r_{22} - r_1 - r_2 - r_3$$

6. RESULTS AND DISCUSSIONS

From equations (2) – (4) we have calculated velocity U (i.e U_1 for region I, U_2 for region II, and U_3 for region III) as a function of y for different values of Darcy number Da and is shown in figure 2. We observe that the velocity increases with the increase in y initially and attains a maximum point after that it decreases with the increment in y. For a given y, we notice that the velocity increases with increasing Darcy number.

The variation of velocity U with y, are calculated from equations (44) - (46) for different values of couple stress parameter a and is shown in figure 3. We notice that for a given y, the velocity increases with increasing couple stress parameter a . From figure 4, we observe that for a given y, the velocity decreases with the increasing in the viscosity ratio m .

From equations (47) – (49) we have calculated temperature θ (θ_1 for Region I, θ_2 for Region II and θ_3 for region III) as a function of y for different values of Darcy number Da and is shown in figure 5. We observe that the temperature increases with the increasing y and reaches a peak further it decreases with the increment in y . For a given y , we notice that the temperature increases with an increase in the Darcy number.

Figure 6 depicts that for a given y , the temperature decreases with increasing viscosity ratio m . From figure 7 we observe that for a given y , the temperature increases with increasing couple stress parameter a .

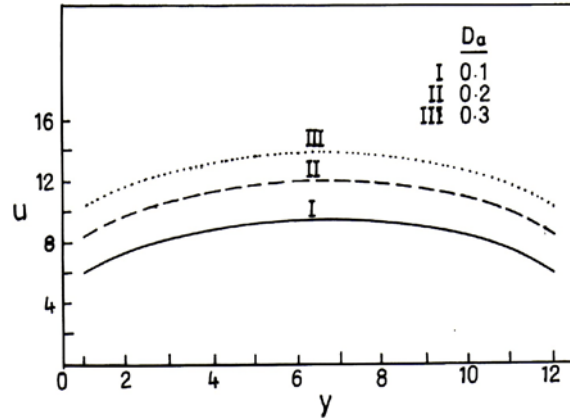


Fig. 2: Velocity profiles for different values of Da with fixed values of $P = -5, a = 1, m = 2, \alpha = 0.3$

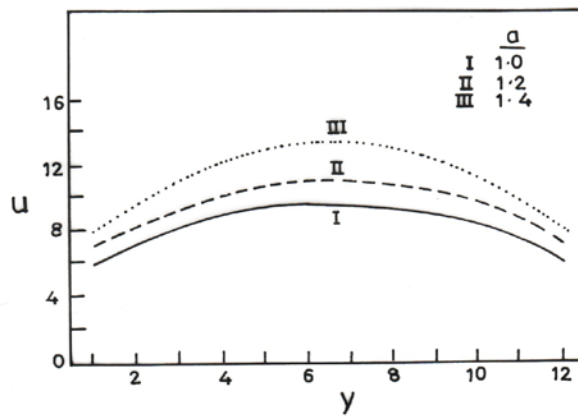


Fig. 3: Velocity profiles for different values of a with fixed values of $P = -5, m = 2, \alpha = 0.3, Da = 0.1$

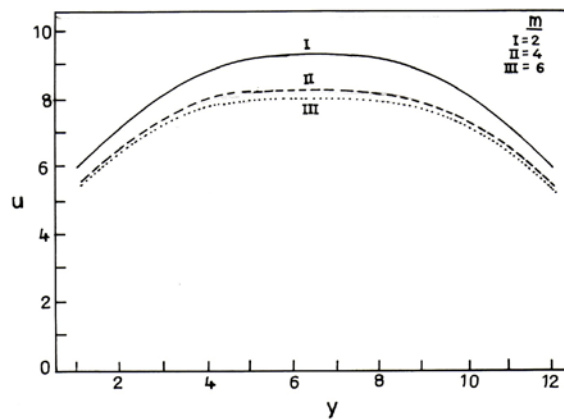


Fig. 4: Velocity profiles for different values of m with fixed values of $P = -5, a = 1, \alpha = 0.3, Da = 0.1$

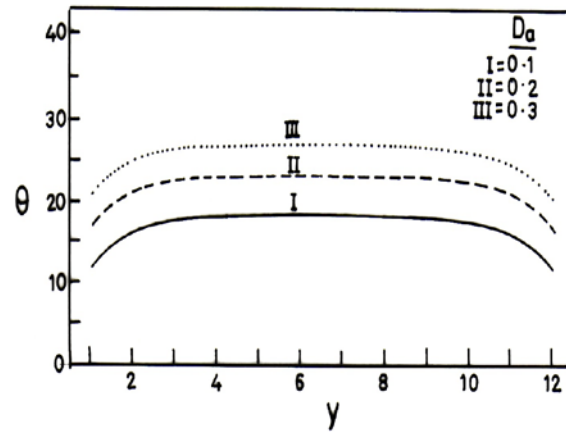


Fig. 5: Temperature profiles for different values of Da with fixed values of $P = -5, a = 1, m = 2, \alpha = 0.3, E = 1, H = 0.6, K = 1$

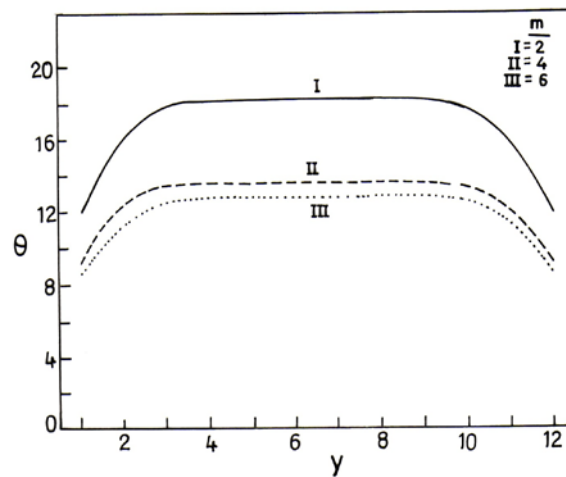


Fig.6: Temperature profiles for different values m with fixed values of $P = -5, a = 1, \alpha = 0.3, E = 1, H = 0.6, K = 1, Da = 0.1$

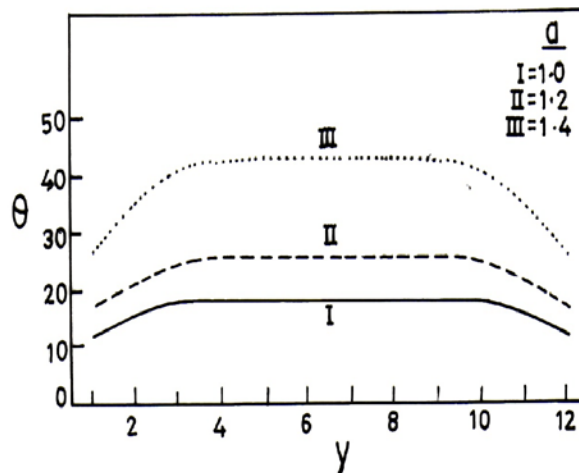


Fig.7: Temperature profiles for different values a , with fixed values of $P = -5, m = 2, \alpha = 0.3, E = 1, H = 0.6, K = 1, Da = 0.1$

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