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SUM OF ANNIHILATOR NEAR-FIELD SPACES OVER NEAR-RING IS ANNIHILATOR NEAR-FIELD SPACE (SA-NFS-ONR-ANFS)

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ABSTRACT

We call a near-field space N over a near-ring R a right SA-near-field space if for any sub near-field spaces I and J of N there is an ideal K of N such that r(I) + r(J) = r(K). This class of near-field spaces is exactly the class of near-field spaces for which the lattice of right annihilator near-field spaces is a sub-lattice of the lattice of near-field spaces. The class of right SA-near-field spaces includes all quasi-Baer (hence all Baer) near-field spaces and all right IN-near-field spaces (hence all right self-injective near-field spaces). This class is closed under direct products, full and upper triangular matrix near-field spaces over near-rings, certain polynomial near-field spaces over near-rings, and two-sided near-field spaces over near-rings of quotients. The right SA-near-field space over near-ring property is a Morita invariant. For a semi-prime near-field space over near-ring R, it is shown that R is a right SA-near-field space over near-ring if and only if R is a quasi-Baer near-ring if and only if r(I) + r(J) = r(K) = r(I ∩ J) for all near-field spaces I and J of N if and only if Spec(N) is extremally disconnected. Examples are provided to illustrate and delimit our results.

Key Words: *Annihilator-near-field space; Extremally disconnected near-field space; IN-near-field space over near-ring; Quasi-Baer near-field space over near-ring; SA-near-field space over near-ring.*

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INTRODUCTION

Throughout this paper, N denotes near-field space over near-ring R and R denotes a nonzero associative near-ring with identity. In this paper, we introduce and investigate the concept of a right SA-near-field space over near-ring. We call N a right SA-near-field space over near-ring, if for any sub near-field spaces I and J of N over near-ring R there is a sub near-field space K of N over a near-ring R such that r(I) + r(J) = r(K), where r(I) (resp., l(J)) denotes the right annihilator sub near-field space (resp., left annihilator sub near-field space) of I.

Section 1: Introduction

Throughout this paper, N denotes a nonzero associative near-field over a near-ring R with identity. In this paper, I introduce and investigate the concept of a *right SA*-near-field space over near-*ring*. We call N a *right SA*-near-field space over near-*ring R*, if for any near-field sub spaces I and J of N there is an near-field sub space K of N such that r(I) + r(J) = r(K), where r(I) (resp., l(J) denotes the right annihilator near-field space (respectively left Annihilator near-field space) of I.

In Section 2, we show that all quasi-Baer near-field spaces over regular δ-near-rings and all left IN-near-field spaces over regular δ-near-rings are right SA- near-field spaces over regular δ-near-rings. Moreover, I provide examples of right SA- near-field spaces over regular δ-near-rings which are neither quasi-Baer nor left IN- near-field spaces over regular δ-near-rings. Theorem 2.6 yields that the right SA- near-field spaces over regular δ-near-rings condition is exactly the condition which ensures that the lattice of right annihilator near-field spaces over regular δ-near-rings is a sub-ϕ lattice of the lattice of near-field spaces over regular δ-near-rings of a near-ring R. Also in this theorem, we prove that N is a right SA- near-field spaces over regular δ-near-rings if and only if rlI + rlJ = rlI ∩ lJ for all ideals I and J of N. The section concludes with the result that the class of right SA- near-field spaces over regular δ-near-rings is closed under direct products.

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In Section 3, we consider the closure of the class of right SA- near-field spaces over regular δ-near-rings with respect to various near-field space extensions including matrix, polynomial, and dense near-field space extensions. The right SA- near-field spaces over regular δ-near-rings is shown to be a Mortia invariant in theorem 3.4.

Semi-prime right SA- near-field spaces over regular δ-near-rings are the focus of **Section 4,** in theorem 4.4, for a semi-prime near-field space N, we show that N is a right SA- near-field spaces over regular δ-near-rings if and only if N is a quasi – Baer near-field spaces over regular δ-near-rings of prime ideals. Spec (N), is extremally disconnected. Various corollaries and examples illustrating this result are provided.

Let $\phi \neq X \subseteq N$, then $X \leq N$ and $X \trianglelefteq N$ denote that X is a right ideal of near-field spaces over regular δ-near-ring and X is an ideal respectively. For subset S of N, l(S) and r(S) denote left annihilator near-field space and the right annihilator near-field space of S in N a near-field space over regular δ-near-ring.

An independent of e of N is a left (or right) semi-central idempotent annihilator near-field space if Ne = eNe (eN = eNe), and we use $S_l(N)$ $(S_r(N))$ to denote the set of left (or right) semi-central idempotent annihilator near-field spaces of N. The annihilator of near-field space of n × n (upper triangular) matrices over N is denoted by $(T_n(N))M_n(N)$.

A near-field space N is called a right lkeda-nakayama or a right IN-near-field space if the left annihilator of the intersection of any two ideals is the sum of the left annihilators. i.e., if $l(I \cap J) = l(I) + l(J)$ for all $I, J \leq N$; and we say N is an IN- annihilator near-field space if N is a left and a right IN- annihilator near-field space.

A near-field space N is called a quasi–Baer near-field space if the left annihilator of every (ideal) non-empty near-field space of N is generated, as a left ideal, by an idempotent. The quasi – Baer near-field space if and only if $M_n(N)$ is quasi-Baer near-field space if and only if $T_n(N)$ is a quasi –Baer near-field space.

Section 2: Preliminary Results and Examples

An ideal I of N is a right (or left) annihilator near-field space ideal if r(l(I)) = I l(r(I)) = I; equivalently, $l(I) \subseteq l(x)$ $(r(I) \subseteq r(x))$ and $x \in N, \Rightarrow x \in l$.

Definition 2.1: N be a right SA-near-field space over regular δ-near-ring. If for any two $I, J \trianglelefteq N$ there is a $K \trianglelefteq N$ such that r(I) + r(J) = r(K) = r(K). Since r(X) = r(RX) for all $X \leq N$, N is a right SA ⇔ for all $X, Y \leq N$ there exists $V \leq N$ such that r(X) + r(Y) = r(V).

Definition 2.1(a): A sub-near-field space S of a near-field space N is called right intrinsic extension of N if every non-zero right sub-near-field space of S has non-zero intersection with N.

Definition 2.1(b): If S is an essential over sub-near-field space of a near-field space N i.e., $N_N \leq^{ess} S_N$, then S is a right intrinsic extension of Z, but it is not an essential over sub-near-field space of Z.

Proposition 2.2: The following statements hold good (i) A left IN-near-firld space is a right SA-near-field space. (ii) A quasi – Baer near-field space is a right SA-near-field space.

Proof: To prove (i): Assume N is a left IN-near-field space and $I, J \trianglelefteq N$. Then $r(I) + r(J) = r(I \cap J)$, by definition. Therefore, N is a right SA-near-field space. Proved (i).

To prove (ii): Let $I, J \trianglelefteq N$. Then there exists $e, f \in S_l(N)$ such that r(I) = eN and r(J) = *f*N and by known [5, proposition 1.3(ix, x)] r(I) + r(J) = eN + *f*N = (e + *f* - e*f*)N and $e + f - ef \in S_l(N)$. Let c = e + *f* – e*f*. Then r(N(1 – c)) = cN. Also $N(1-c) \trianglelefteq N$. Therefore, N is a right SA – near-field space. Hence proved (ii).
This completes the proof of the proposition.

Example 2.3: Let N be a commutative universal near-field space which is not a domain ($N = Z_p^n$, where n > 1 and p is prime). Then N is an IN-near-field space and hence a SA-sub near-field space of N but N is not a quasi-Baer near-field space. By corollary 3.6, $T_n(N)$, where n > 1, is a right (or a left) SA-sub-near-field which is neither a left nor a right IN-near-field space, and is not a quasi-Baer near-field space of N.

Example 2.4: Let *F* be a set of all near-field spaces over a Baer-ideals. $N = \begin{pmatrix} F & F \oplus F \\ 0 & F \end{pmatrix}$, $I = \begin{pmatrix} 0 & F \oplus 0 \\ 0 & 0 \end{pmatrix}$, and $J = \begin{pmatrix} 0 & 0 \oplus F \\ 0 & 0 \end{pmatrix}$. Then $I, J \trianglelefteq N$ and $r(I) + r(J) = \begin{pmatrix} F & F \oplus F \\ 0 & 0 \end{pmatrix} \neq r(I \cap J)$.Moreover, $l(I) + l(J) = \begin{pmatrix} F & F \oplus F \\ 0 & F \end{pmatrix} \neq N = l(I \cap J)$ .Hence N is neither left nor right IN-near-field space. N is a quasi – Baer near-field space.

Example 2.5: Every left self-injective near-filed space is a left IN-near-field space. A left self injective near-field space is a right SA-sub-near-field space. Thus any QF-near-field space is a right SA-near-field space. However, any QF-near-field space which is not semi-prime is not quasi – Baer near-field space.

Note 2.6: A near-field space N the set of right annihilator near-field spaces of N. {r(I) : I $\trianglelefteq$ N}, partially ordered by set inclusion. Forms a lattice with inf (r(I), r(J)) = r(I) ∩ r(J), sup (r(I), r(J)) = rl((I) + r(J)) for all ideals of I, J of a near-field space N.

Note 2.7: In general, this lattice is not a sub lattice of the lattice of near-field spaces of a near-field space N.

Note 2.8: the following result shows that right SA-sub-near-field space of a near-field space N condition is the exactly the condition needed to ensure that the lattice of right annihilator near-field spaces is a sub lattice of the lattice of near-field spaces over a near-field space N.

Theorem 2.9: The following conditions are equivalent:

(a) N is a right SA-near-field space
(b) $\forall\ I, J \trianglelefteq N$, r(I) + r(J) = r(l(r(I)) ∩ l(r(J)))
(c) The lattice of right annihilator near-field spaces is a sub lattice of the lattice of near-field spaces of N.
(d) $\forall\ X, Y \trianglelefteq N$, r(I(X)) + r(J(Y)) = r(I(X) ∩ J(Y)).

Proof: we prove this theorem by the method of cyclic.

To prove (a) ⇒ (b):

Given N is a right SA-near-field space.

Let $\forall\ I, J \trianglelefteq N$, r(I) + r(J) = r(K) for some $K \trianglelefteq N$.

Now r(K) = r(l(r(K))) = r(l(r(I)) ∩ l(r(J))) = r(l(r(I)) ∩ l(r(J))). Hence (a) ⇒ (b).

To prove (b) ⇔ (c): Given $\forall\ I, J \trianglelefteq N$, r(I) + r(J) = r(l(r(I)) ∩ l(r(J))).

⇒ This equivalence follows from the comment immediately that the lattice of right annihilator near-field spaces is a sub lattice of the lattice of near-field spaces of N. Hence (b) ⇔ (c).

To prove (b) ⇒ (d): Given that $\forall\ I, J \trianglelefteq N$, r(I) + r(J) = r(l(r(I)) ∩ l(r(J))).

Let $X, Y \trianglelefteq N$, in (b) take I = l(X) and J = l(Y).

Then r(I(X)) + r(J(Y)) = r(I) + r(J).
= r(l(r(I)) ∩ l(r(J)))
= r(l(r(I(X))) ∩ l(r(l(J(Y))))
= r(I(X) ∩ J(Y)). Hence (b) ⇒ (d).

To prove (d)⇒ (a): Given $\forall\ X, Y \trianglelefteq N$, r(I(X)) + r(J(Y)) = r(I(X) ∩ J(Y)).

Let $X, Y \trianglelefteq N$. By assumption, r(l(r(X))) + r(l(r(Y))) = r(l(r(X)) ∩ l(r(Y))).

Take K = l(r(X)) ∩ l(r(Y)). Since, r(X) + r(Y) = r(l(r(X))) + r(l(r(Y))) . r(X) + r(Y) = r(K). Therefore, N is a right SA-near-field space. Hence (d)⇒ (a).

This completes the proof of the theorem.

Note 2.10: N is a right SA-near-field space if and only if for any two left annihilator near-field spaces I and J of N, r(I ∩ J) = r(I) + r(J).

Note 2.11: Direct product of near-field spaces the right annihilator near-field spaces are products of right annihilator near-field spaces of each of the components in the product.

Section 3: Extensions of right SA-near-field spaces

In this section, I, Dr N V Nagendram investigate the behaviour of the right SA-near-field space property with respect to various extensions including matrix, polynomial and dense near-field space extensions.

I construct this behaviour with that of left IN-near-field spaces. Here I show that the right SA-near-field space property is a Morita invariant whereas this is not so far the right (or left) IN-near-field space property.

Lemma 3.1: Let N be a near-field space and T = $M_n(N)$.

(i) Then $I \trianglelefteq T$ if and only if I = $M_n(J)$ for some $J \trianglelefteq N$.
(ii) If $J \trianglelefteq N$, then $r_3(M_n(J)) = M_n(r_N(J))$.

Proof: Proof is obvious and routine.

Lemma 3.2: If N is a right SA-near-field space and e is an idempotent of N, then eNe is a right SA-near-field space.

Proof: Let J = eJe and I = eIe be two right sub near-filed spaces of eNe. Then JN and IN are two right sub near-field spaces of N, so there is a right sub-near-field space K of N such that r(JN) + r(IN) = r(K). We know that eNe = {n ∈ N : n = en = ne}. Firstly, we show that $r_{eNe}(J) = {}_{er}(JN)_e$, and then we prove that $r_{eNe}(J) + r_{eNe}(I) = r_{eNe}(eKe)$. Assume that x ∈ $r_{eNe}(J)$. Hence we have JNx = JeNex ⊆ eJex = 0 so x ∈ r(JN). Now let x ∈ er(JN)e and y ∈ J. Therefore, we have $r_{eNe}(J) + r_{eNe}(I) = {}_{er}(JN)_e + {}_{er}(IN)_e =$ e(r(JN) + r(IN))e = ${}_{er}(K)_e = r_{eNr}(rKe)$. This completes the proof of the lemma.

Theorem 3.3: N is a right SA-near-field space if and only if $M_n(N)$ is a right SA-near-field space.

Proof: Let $J,\ I \trianglelefteq M_n(N)$. Then, by lemma 3.1, there are J_1, $I_1 \trianglelefteq$ N such that J = $M_n(J_1)$, I = $M_n(I_1)$. Hence by hypothesis and lemma 3.1, there is $K \trianglelefteq N$ such that

$$r(J) + r(I) = r(M_n(J_1)) + r(M_n(I_1)) = M_n(r(J_1))+M_n(r(I_1))= M_n(r(J_1)+M_n(r(I_1)) = M_n(r(K)) = r(M_n(K)).$$

Conversely, let $M_n(N)$ is a right SA-near-field space. Clear that $EM_n(N)E \cong N$, where in matrix E, E_{11} =1 and for each i ≠ 1 and j ≠ 1 E_{ij} = 0 so by lemma 3.2,N is a right SA-near-field space. This completes the proof of the theorem.

Theorem 3.4: The right SA-near-field space property is a Morita invariant.

Proof: This result is a consequence of lemma 3.2 and theorem 3.3. Obvious.

Theorem 3.5: the following conditions are equivalent:

(i) N is a right SA-near-field space
(ii) $S_m(N)$ is a right SA-near-field space, for some +ve integer m;
(iii) $S_m(N)$ is a right SA-near-field space, for every +ve integer m;

Proof: We prove this theorem by cyclic method of proof as below:

To Prove (iii) ⇒ (ii): this implication is obvious.

To prove (ii) ⇒ (i):

Let e ∈ $S_m(N)$ be matrix with 1 in (1,1) – position and 0 elsewhere.

Then $eS_m(N)e$ is a near-field space isomorphism to N.

By lemma 3.2, N is a right SA-near-field space. Proved (ii) $\Rightarrow$ (i).

To prove (i) $\Rightarrow$ (iii):

Let e$\in X, Y \trianglelefteq S_m(N)$. Then X = $\begin{pmatrix} X_{11} & X_{12} & . & . & X_{1m} \\ 0 & X_{22} & . & . & X_{2m} \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & . & . & X_{mm} \end{pmatrix}$ where $\forall X_{ij} \trianglelefteq N$, $X_{ij} = \{0\}$ $\forall$ i > j, $X_{ij} \subseteq X_{ik}$ $\forall$ k $\geq$ j,

and $X_{hj} \subseteq X_{ij}$ $\forall$ h $\geq$ i. Similarly, Y has such a matrix form.

Let S = $S_m(N)$. Then $r_S(X) = r_S \begin{pmatrix} X_{11} & X_{12} & . & . & X_{1m} \\ 0 & 0 & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & . & . & 0 \end{pmatrix} = \begin{pmatrix} r_N(X_{11}) & r_N(X_{11}) & . & . & r_N(X_{11}) \\ 0 & r_N(X_{12}) & . & . & r_N(X_{12}) \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & . & . & r_N(X_{1m}) \end{pmatrix}$

Similarly, $r_S(Y)$ has such a matrix form. Then $r_N(X_{1j}) + r_N(Y_{1j}) = r_N(K_{1j})$ for some K_{1j} and $\forall$ j = 1,2,...,m.

So $r_S(X) + r_S(Y) = r_S(K)$, where K = $\begin{pmatrix} K_{11} & K_{12} & . & . & K_{1m} \\ 0 & 0 & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & . & . & 0 \end{pmatrix}$.

Proved (i) $\Rightarrow$ (iii).

This completes the proof of the theorem.

Corollary 3.6: Let m be a positive integer and S = $S_m(N)$.
(i) For m > 1, $S_m(N)$ is neither a left nor a right IN-near-field space.
(ii) For every m, $S_m(N)$ is a quasi-Baer near-field space if and only if N is a quasi-Baer near-field space.
(iii) For every m, $S_m(N)$ is a right SA-near-field space if and only if N is right SA- near-field space.

Proof: To prove (i):

Let J = cS, where c $\in$ S with 1 in the (m, m) – position and 0 elsewhere. Let I = eS, where e $\in$ S with 1 in the (1, j) – position for j = 1,2,3,...,m and 0 elsewhere. Then l(J) + l(I) $\subseteq \begin{pmatrix} N & N & . & . & N \\ 0 & N & . & . & N \\ . & . & . & . & . \\ . & . & . & . & N \\ 0 & 0 & . & . & 0 \end{pmatrix} \neq$ S = l(J $\cap$ I).

Hence S is not a right IN-near-field space. Similarly, S is not a left IN-near-field space.

To prove (ii): is obvious and refer [20, Prop.9, 16] or [7, Th.3.2]

To prove (iii): refer theorem 3.5.

This completes the proof of the corollary.

Here I extended the Nagendram's near-field space upon regular delta near-rings under ring theory.

Definition 3.7: A near-field space N is called an Nagendram's near-field space if whenever polynomial near-field spaces $f(x) = a_0 + a_1x + \text{------} + a_mx^m$, $g(x) = b_0 + b_1x + \text{------} + b_nx^n \in N[x]$ satisfy $f(x)g(x) = 0$. Then $a_ib_j = 0$ for each i, j. It is clear that if N is an Nagendram near-field space, and N[x] is a right IN-near-field space. Then N is a right IN-near-field space.

Note 3.8(i): Apart from the definition of Nagendram near-field space or Armendariz near-field space we prove the following result.

Note 3.8(ii): If N is an IN-near-field space, $S_n(N)$ need not be an IN-near-field space. For example, if F is a near-field space and $N = \begin{pmatrix} F & F \\ 0 & F \end{pmatrix}$. Then N is not a left IN-near-field space. But, we see that N is right SA-near-field space. So the class of all right SA-near-field spaces behaves better than the class of left IN-near-field spaces, for triangular matrix near-field space extensions.

Proposition 3.9: If N[x] is a right IN-near-field space, then N is a right IN-near-field space.

Proof: let J and I be two right sub-near-field spaces of a near-field space N. Then *J*[x] and I[x] are two right sub-near-field spaces of N[x], so by hypothesis, $l_{N[x]}(J[x]) + l_{N[x]}(I[x]) = l_{N[x]}(J[x]) \cap (I[x])$. We know that $l(J) + l(I) \subseteq l(J \cap I)$. Now let $t \in l(J \cap I)$. Then $t \in l_{N[x]}(J[x]) \cap (I[x])$. Hence there is $f(x) \in l_{N[x]}(J[x])$ and $g(x) \in l_{N[x]}(I[x])$, such that $t = f(x) + g(x)$. i.e., $t = f_0 + g_0$. It can be seen that $f_0 \in l(J)$ and $g_0 \in l(I)$. Thus $t \in l(J) + l(I)$. This completes the proof of the proposition.

Note 3.10: Let N be a trivial near-field space extension of Z and the Z – module $Z_2^{\propto}$. Then N is a Nagendram, right IN-near-field space, but N[x] is not a right IN-near-field space.

Note 3.11: Let N be a reduced near-field space. Then N is a right SA-near-field space if and only if N[x] is a right SA-near-field space.

Definition 3.12: A sub-near-field space module N of N-module M is said to be an essential sub-near-field space module if for every sub-near-field space module H of M, $H \cap N = \{0\} \Rightarrow H = \{0\}$.

Definition 3.13: A sub-near-field space module N of right N-module M is said to be a dense sub-near-field space module if for every x and y in M with $x \neq 0$, there exists an element $n \in N$ such that $xn \neq 0$ and $yn \in N$. for $X, Y \leq N$, $X \leq^{den} Y$ ($X \leq^{den} Y$) denotes that X is essential (dense) in Y as right N-near-field space modules.

Theorem 3.14: The following statements hold:

(i) If N[x] is a right SA-near-field space, then N is a right SA-near-field space.
(ii) If N is a Nagendram near-field space, then N is a right SA-near-field space if and only if N[x] is a right SA-near-field space.

Proof: To prove (i): Let $J, I \trianglelefteq N$. Then $f(x), f[x] \trianglelefteq N[x]$.

So there is $K \trianglelefteq N[x] \ni r_N[x]\ )l[x]) + r_N[x]\ (f[x]) = r_{N[x]}(K)$.

Now let $K_0 = \cup_{f \in K} C_j$, where C_j denotes the set of co-efficients of $f(x)$.

Then it follows that $K_0 \trianglelefteq N$. We prove that $r(J) + r(I) = r(K_0)$.

For that Suppose that $b \in r(J)$ and $c \in r(I)$.

Then $b \in r_{N[x]}\ (l[x])$ and $c \in r_N[x]\ (f[x])$. So $b + c \in r_{N[x]}\ (K)$.

Now let $a \in K_0$.

Then there is $f(x) = a_0 + a_1x$ +------+a_mx^k + ----+$a_nx^n \in K$ such that $a = a_m$, where m is some integer such that $0 \leq m \leq n$.

Then $(b + c)f(x) = 0$, so $(b + c)a = 0$ and hence $b + c \in r(K_0)$. Then therefore $r_N(J) + r_N(I) \subseteq r(K_0)$.

Now let $d \in r(K_0)$.Then $d \in r_N(K)$. So there are $h(x) \in r_N\ (f[x])$ and $g(x) \in r_N\ (f[x])$ such that $d = h_0 + g_0$.

Then $h_0 \in r_N\ (J)$ and $g_0 \in r_N\ (I)$. So $d \in r_N\ (J) + r_N\ (I)$. Hence $r_N(K_0) \subseteq r_N(J) + r_N(I)$. Therefore N is a right SA-near-field space. Hence proved(i).

To prove (ii):
The necessary is evident by (i). Now let N be a nagendram right SA-near-field space and $J, I \trianglelefteq N[x]$. Then $J_0 = \bigcup_{f \in J} C_j$, $I_0 = \bigcup_{f \in I} C_i \trianglelefteq N$.

So there is a sub-near-field space K $\trianglelefteq$ N such that $r(J_0) + r(I_0) = r(K)$.

Now we prove that $r_{N[x]}(J) + r_{N[x]}(I) = r_{N[x]}(K[x])$.

Let $f(x) = f_0 + f_1x + ---- + f_n x^n \in r_{N[x]}(J)$, $g(x) = g_0 + g_1x + ---- + g_k x^k \in r_{N[x]}(I)$ and $a \in J_o$.

Then there is $h(x) \in J$ such that $a \in C_h$, i.e., $h_j = a$ for some j and $h(x)f(x) = 0$.

Hence by hypothesis, $af_{i=,}$ $\forall\ 0 \le i \le n$. So $f_i \in r(J_0)$, $\forall\ 0 \le i \le n$ and similarly, $g_i \in r(I_0)$ $\forall\ 0 \le i \le k$. So $f_i + g_i \in r(K)$.

Thus $f(x)+ g(x) \in r_{N[x]}(K[x])$. Now let $h(x) = h_0 + h_1x + ---- + h_k x^k \in r_{N[x]}(K[x])$.

Thus then $h_i \in r(K)$, $\forall\ 1 \le i \le k$.

Therefore $\forall\ 1 \le i \le k$, there are $a_i \in r(J_0)$, $b_i \in r(J_0)$ such that $h_i = a_i + b_i$, so $h(x) = f(x) + g(x)$, where $f(x) = a_0 + a_1x$ +------$+a_mx^k + ----+a_nx^n \in r_{N[x]}(J)$ and $g(x) = g_0 + g_1x + ---- + g_k x^k \in r_{N[x]}(I)$.

Hence proved (ii).

This completes the proof of the theorem.

Proposition 3.15: Let N be a sub-near-field space of a near-field space S, is a sub-near-field space of the maximal right near-field space of quotients of N. If N is a right SA-near-field space and S is also a sub-near-field space of the maximal left near-field spaces of quotients of N, then S is a right SA-near-field space.

Proof: Let $J, I \trianglelefteq S$. Then there is $K \trianglelefteq N$ such that $r_N(J \cap N) + r_N(I \cap N) = r_N(K)$.

We claim that $r_\gamma(J) + r_\gamma(I) \subseteq r_\gamma(KN)$.

To see this, let $a \in r_\gamma(J)$ such that there exists $k \in K$ and $t \in S$ with $kta \neq 0$.

Since N is dense in S, there exists $x \in N$ such that $ktax \neq 0$ and $tax \in N$.

Hence $tax \in N \cap r_\gamma(J) = r_N(J)$.

Since S is also a left near-field space of quotient near-field spaces, yields that $r_N(J) = r_N(J \cap N)$.

Then $tax \in r_N(J \cap N) \subseteq r_N(K)$, which is a contradiction. So, $r_\gamma(J) \subseteq r_\gamma(KS)$.

Similiarly, $r_\gamma(I) \subseteq r_\gamma(KS)$.

Now we claim that $r_\gamma(KS) \subseteq r_\gamma(J) + r_\gamma(I)$.

To see this, assume that $b \in r_\gamma(KS)$ and there exists $j \in J$ such that $jb \neq 0$.

Since N is dense in S a sub-near-field space of a near-field space N, there exists $t \in N$ such that $jbt \neq 0$ and $bt \in N$.

Then $bt \in r_N(K)$. So $bt \in r_N(J \cap I)$. $R_N(J \cap I) = r_N(J)$.

So $bt \in r_N(J)$, which is a contradiction. Hence the claim is proved. Therefore, S is a right SA-near-field space.

This completes the proof of the proposition.

Section 4: Semi-prime SA-Near-field spaces

In this section, we show that for a semi-prime near-field space N the right SA-near-field space condition is equivalent to various other well-known near-field space conditions including the quasi-Baer near-field space condition, the condition that $r(J) + r(I) = r(J \cap I)$ $\forall$ J, I $\trianglelefteq$ N and the condition that the set of prime sub-near-field spaces of N i.e., Spec(N) with the hull-kernel topology is extremally disconnected near-field space. When N is reduced near-field space i.e., N has no non-zero nilpotent elements the condition becomes much stronger.

A topological near-field space X is called an extremally disconnected near-Field space, if the closure of any open sub-near-field space is open equivalently, the interior of any closed sub-near-field space of a near-field space is closed.

We deonote by Int(A) the interior points of a sub-near-field space A of topological near-field space X (the largest open sub-near-field space in A) and by clA we mean the closure points of A the smallest closed sub-near- field space containing A. For $a \in N$, let supp(a) = {P ∈ Spec(N) / a ∉ P}. ∀ near-field space N, {supp(a) / a ∈ N} forms a basis of open near-field spaces on Spec(N).

This topology is called the extension to hull-kernel topology. We use V(J)(V(a)) to denote the set of P ∈ Spec(N), where $J \subseteq P (a \in P)$.

Note 4.1: $V(J) = \cap_{a \in J} V(a)$ and V(a) = Spec(N)\supp(a).

Lemma 4.2: Let N be a semi-prime near-field space with $J, I \trianglelefteq N$.

(i) If $J_N \leq^{ess} I_N$, then r(J) = r(I)
(ii) $J_N \leq^{ess} I_N$ if and only if $J_N \leq^{den} I_N$.

Proof: Obvious.

Lemma 4.3: Let N be a semi-prime near-field space.

(i) ∀ a ∈ N and any sub-near-field space J of a near-field space N, supp(a) ∩ supp(J) = supp(a)
(ii) If J and I are two sub-near-field spaces of a near-field space N, then $r(J) \subseteq r(I) \Leftrightarrow intV(J) \subseteq intV(I)$.
(iii) $A \subseteq Spec(N)$ is a clopen sub-near-field space ⇔ ∃ an idempotent e ∈ N such that A = V(e).

Proof: To prove
(i): Let us oberve that P ∈ Supp(a) ∩ Supp(J) ⇔ a ∉ P and J ⊄ P ⇔ Ja ⊄ P. Thus Supp(a) ∩ Supp(J) = Supp(Ja).

(ii)Let J, I be two sub-near-field spaces of a near-field space N and P ∈ intV(J). Then there is a ∈ N such that $P \in Supp(a) \subseteq V(J)$. Hence supp(Ja) = supp(J) ∩ supp(a) = ϕ.
⇒ Ix = 0, i.e., x ∈ r(I).

(iii) Let A be a clopen sub-near-field space, $J = 0_A := \{x \in N: A \subseteq V(x)\}$ and $I = O_x := \{x \in N: A^c \subseteq V(x)\}$. Then A = clA = $V(O_A)$ = V(J) and $A^c = V(O_A{}^c)$ = V(I). Hence V(J + I) = V(J) ∩ V(I) = ϕ, so there are a ∈ J and b ∈ I such that J = a+b. But V(a) ∪ V(b) = Spec(N), and thus we have ab = 0, which implies that $a = a^2$ and V(J) = V(a). The converse is evident.

Hence completes the proof of the lemma.

Theorem 4.4: Let N be a semi-prime near-field space. Then $(r(J) + r(I))_N \leq^{den} r(J \cap I)_N$ ∀ J, I $\trianglelefteq$ N.

Proof: There exists X $\trianglelefteq$ N such that (r(J) + r(I)) ∩ X = 0 and $[(r(J) + r(I)) \oplus X]_N \leq^{ess} r(J \cap I)_N$.

Then $X \subseteq l(r(J)) = l(l(J))$ and $X \subseteq l(l(J))$. ([10], 2.2(i), Lemma 2.3). $(X \cap J)_N \leq^{ess} X$. If X ≠ 0, then $0 \neq X \cap J \cap I \subseteq (J \cap I) \cap r(J \cap I)$.

On the contrary to N being semi-prime near-field space.

Hence $(r(J) + r(I))_N \leq^{den} r(J \cap I)_N$ ∀ J, I $\trianglelefteq$ N.

Theorem 4.5: Let N be a semi-prime near-field space. The following conditions are equivalent:

(i) N is a quasi-Baer near-field space.
(ii) N is a FJ-extending near-field space.
(iii) N is an JJLAS-near-field space.
(iv) N is a right SA-near-field space.
(v) ∀ J, I $\trianglelefteq$ N, r(J) + r(I) = r(J ∩ I).
(vi) The near-field space of all prime sub-near-field spaces, Spec(N) is extremally disconnected near-field space.

Proof: We prove this by the method of cyclic By ([2], Th. 2.2) extended and follows implication that (i) ⇔(ii) ⇔(iii). Proposition 2.2 ⇒ (i) ⇔(iv).

To prove (iv) ⇒ (v): Let sub-near-field spaces J, I $\trianglelefteq$ N. Since N is a right SA-near-field space, there exists K $\trianglelefteq$ N such that r(J) + r(I) = r(K). But r(K) is essentially closed sub-near-field space i.e., r(K) has no essential extension in N. From theorem 4.4, r(J) + r(I) = r(J ∩ I). Hence proved (iv) ⇒ (v).

To prove (v) ⇒ (i): Let J $\trianglelefteq$ N. Since N is semi-prime near-field space. J ∩ r(J) = 0 = r(J) ∩ r(r(J)). Then r(J) ⊕ r(r(J)) = r(J ∩ r(J)) = r(0) = N. Therefore, r(J) = eN for some $e = e^2 \in N$. So N is a quasi-Baer near-field space.Proved (v) ⇒(i).

To prove (vi) ⇒ (i): Let J $\trianglelefteq$ N. By hypothesis, intV(J) is closed near-field space. By lemma 4.3(iii), there is an idempotent e ∈ N such that intV(J) = V(e). And also by Lemma 4.3(ii), r(J) = r(e) = (1 – e)N. Hence N is quasi-Baer near-field space. Proved (vi) ⇒ (i).

To prove (i) ⇒ (vi): Let A be a closed sub-near-field space of Spec(N). Since {V(a) : a ∈ N} is a base for closed sub-near-field spaces in Spec(N), a near-field space of N. Then there exists T ⊆ N such that $A = \cap_{a \in S} V(a)$. J = NTN. Then A = V(J). By hypothesis, there exists $e = e^2 \in N$ such that r(J) = eN. From lemma 4.3(ii), int(A) = int(V(J)) = V(e) is closed near-filed space. Thus Spec(N) is an essentially disconnected near-field space of N. proved (i) ⇒ (vi).

This completes the proof of the theorem.

Corollary 4.6: let N be a reduced near-field space. Consider the following statements:

(i) N is a Baer near-field space.
(ii) N is a JLAS-near-field space.
(iii) N is a right SA-near-field space.
(iv) The near-field space of all prime sub-near-field spaces, Spec(N) is extremally disconnected near-field space.
(v) N is an IN-near-field space.
(vi) N is a quasi-continuous near-field space.

Then (i) ⇔(ii) ⇔(iii) ⇔(iv) and (v) ⇒ (i). If N is also a duo near-field space

i.e., every one-sided sub-near-field space is two sided sub-near-field space of a near-field space N, then (v) ⇔ (vi) ⇔ (i).

Proof: We prove this by cyclic method of proof that (i) ⇔(ii) ⇔(iii) ⇔ (iv).

These equivalences are a consequences of theorem 4.5 and the fact that in a reduced near-field space if X is a non-empty near-field space of N, then r(X) = *l*(X) $\trianglelefteq$ N.

(iv) ⇔ (vi) [19, Th. 6.32] yields this implication extended and is follows.

(vi) ⇔ (i) [13, Th. 2.1] yields this implication extended and is follows.

If N is also a duo near-field space then theorem 4.5 yields that (v) ⇔ (vi)⇔ (i). This completed the proof of the corollary.

Corollary 4.7: Let N be a semi-prime right SA-near-field space and S is a right intrinsic extension of a near-field space N. Then S is a semi-prime right SA-near-field space.

Proof: [9, Th. (3.3, 3.15)] and Theorem 4.5, N is a quasi-Baer near-field space. Then S is a semi-prime quasi-Baer near-field space. Therefore, S is a right SA-near-field space. This completes the proof of the corollary.

Example 4.8(i): Let $N[C_2]$ be the N-group near-field of the cyclic N-group of order two over a commutative near-field domain N such that char(N) ≠ 2 is not invertible. Then $N(C_2)$ is a commutative reduced near-field space that is not a Baer near-field space so that is not a right SA-near-field space. To observe this, note that r(1 + g) = (1 – g)$N[C_2]$ is not generated by an idempotent where C_2 = {1,g} with g of order 2. If *F* is the near-field of fractions of N, then $F[C_2]$ is the maximal near-field of quotients of n and is a Baer near-field space. Thus the right SA-near-field space condition does not transfer from an essential over near-field space to its base near-field space. Since a reduced near-field space is a sub direct product of sub-near-field spaces of domains it cannot be extended to sub direct products. Moreover, this leads to that cannot be replace $\leq^{den}$ in theorem 4.4.

Example 4.8(ii): Every semi-prime near-field space has a semi-prime quasi-Baer hull, and the local multiplier algebra of each C^*-algebras is a semi-prime quasi-Baer near-field space whereas all commutative AW^*-algebras are examples of reduced Baer C^*-algebras thereby SA-near-field spaces.

Example 4.8(iii): If Q(N) i.e., the maximal right near-field space of all quotients of N is a right IN-near-field space, then N need not be a right IN-near-field space. For that let N = $M_2(Z)$. Then Q(N) = $M_2(Q)$ we have Q(N) is an IN-near-field space but N is not a right IN-near-field space.

Note 4.9: If N is a near-field space such that $J \cap I = 0$ implies that r(J) + r(I) = N for left annihilator sub-near-field spaces J, I of a near-field space N, must be a right SA-near-field space can be the affirmative for semi-prime near-field spaces in final result as we derived.

Lemma 4.10: Let N be a semi-prime near-field space. Then N is a right SA-near-field space if and only if $J \cap I = 0$ implies that r(J) + r(I) = N for left annihilator sub-near-field spaces J, I of a near-field space N.

Proof: (IF PART⇒) this implication follows from that N is a right SA-near-field space if and only if for any two left annihilator near-field spaces J and I of N, $r(J \cap I) = r(J) + r(I)$.

(⇐IFF PART) Let J, I $\trianglelefteq$ N. Since N is a semi-prime near-field space. R(J) = l(J). Hence, r(J) and r(r(J)) are both left annihilator sub-near-field spaces of a near-field space N.

Also, by using that N is a semi-prime near-field space, $r(J) \cap r(r(J)) = 0$. Now, assuming that, N = r(r(r(J))) + r(r(J)) = r(J) + r(r(J)) is generated sub-near-field space of a near-field space N by an idempotent, So N is a quasi-Baer near-field space. Hence N is a right SA-near-field space. This completes the proof of the lemma.

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