International Journal of Mathematical Archive-7(1), 2016, 110-115
Available online through www.ijma.info ISSN 2229 – 5046

CONNECTED TWO OUT DEGREE EQUITABLE DOMINATION POLYNOMIAL OF A GRAPH

M. S. MAHESH*
Department of Mathematics,
Francis Xavier Engineering College, Tirunelveli-627003, Tamil Nadu, India.

P. NAMASIVAYAM
PG and Research Department Of Mathematics,
The M.D.T Hindu College, Tirunelveli-627010, Tamil Nadu, India.

(Received On: 11-12-15; Revised & Accepted On: 25-01-16)

ABSTRACT

Let G= (V, E) be a connected graph. A dominating set D of G is called connected two out degree equitable dominating set if for any u,v∈ D, such that $|od_D(u) - od_D(v)| \leq 2$ and the induced sub graph<D> is connected. The minimum cardinality of connected two out degree equitable dominating set is called connected two out degree equitable domination number and it is denoted by $\gamma_{c2oe}(G)$.In this paper we introduce the connected two out degree equitable domination polynomial of G. The connected two out degree equitable domination polynomial of connected graph G of order p is the polynomial $D_{c2oe}(G,x) = \sum_{i=\gamma_{c2oe}}^{p} d_{c2oe}(G,i)x^i$, where $d_{c2oe}(G,i)$ is the number of connected two out degree equitable dominating set of G of size i and $\gamma_{c2oe}(G)$ is the connected two out degree equitable domination number and we compute this polynomial and its roots for some standard graphs.

Key words: *Connected, two out degree, equitable, polynomial.*

Mathematics Subject Classification: *05C69.*

1. INTRODUCTION

Throughout this paper we will consider only a simple connected graph finite and undirected without loops and multiple edges. In general, we use <X> to denoted the sub graph induced by the set of vertices of N(v) denote the open neighborhood of a vertex. A set D of vertices in a graph G is a dominating set if every vertex in $V - D$ is adjacent to some vertices in D. The minimum cardinality of such dominating set is called domination number and it is denoted by γ(G). For more detail about domination number and related parameter we refer [7], [8]. A dominating set D of G is called a connected dominating set if the induced sub graph <D> is connected. The minimum cardinality of such dominating set is called domination number and it is denoted by $\gamma_c(G)$[3].

Let G be a graph, and v be a vertex in D. The out degree of v with respect to D is denoted by $od_D(v) = |N(v) \cap V - D|$. A dominating set D of G is called two out degree equitable dominating set if for any u,v∈ D, such that $|od_D(u) - od_D(v)| \leq 2$. The minimum cardinality of two out degree equitable dominating set is called two out degree equitable domination number and it is denoted by $\gamma_{2oe}(G)$[1]. A two out degree equitable dominating set D of G is called a connected two out degree equitable dominating set if the induced sub graph <D> is connected. The minimum cardinality of such connected two out degree equitable dominating set is called connected two out degree equitable domination number and it is denoted by $\gamma_{c2oe}(G)$[5]. The domination polynomial of a graph is introduced by Saeid Alikhani and Yee-hock Peng[6]. The connected domination polynomial is introduced by B.V. Dhanajaya Murthy, G.Deepak and N.D.Soner[2].The connected domination polynomial of connected graph G of order p is the polynomial $D_c(G,x) = \sum_{i=\gamma_c}^{p} d_c(G,i)x^i$, where $d_c(G,i)$ is the number of connected dominating set of G of size i and $\gamma_{c2oe}(G)$ is the connected domination number.

In this paper motivated by connected domination polynomial of a graph [2]. We introduce connected two out degree equitable domination polynomial of G. we compute the connected two out degree equitable domination polynomial and its roots form some standard and special graphs.

Corresponding Author: M. S. Mahesh, Department of Mathematics,*
Franics Xavier Engineering College, Tirunelveli-627003, Tamil Nadu, India.

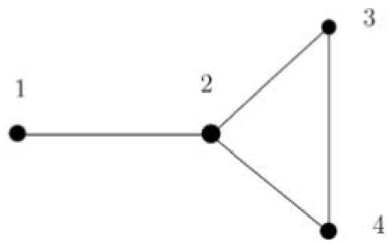
2. CONNECTED TWO OUT DEGREE EQUITABLE DOMINATION POLYNOMIAL OF A GRAPH

Definition: 2.1 Let G be a connected graph of order p. Let $d_{c2oe}(G,i)$ denoted the number of connected dominated set with cardinality i. Then the connected two out degree equitable domination polynomial $D_{c2oe}(G,x)$ of G is defined as $D_{c2oe}(G,x) = \sum_{i=\gamma_{c2oe}(\mathrm{G})}^{p} d_{c2oe}(G,i)x^i$ where $d_{c2oe}(G,i)$ is the number connected two out degree equitable dominating set of G of size i and γ_{c2oe}(G) is connected two-out degree equitable domination number of G. The roots of the connected two out degree equitable domination polynomial are called the connected domination roots of G and denoted by Z($D_{c2oe}(G,x)$).

Example: 2.2 let G be the graph in the Figure 1. V(G)={1,2,3,4}. Then the connected two out degree equitable domination number is two and the connected two out degree equitable domination number of size two are [1,2},{2,3}, {2,4}. The connected two out degree equitable dominating set of size three are {1,2,3},{1,2,4},{2,3,4} and connected two out degree equitable dominating set of size four is {1,2,3,4}.Hence

$$D_{c2oe}(G,x) = x^4 + 3x^3 + 3x^2$$
$$= x^2(x^2 + 3x + 3)$$

And the connected two out degree equitable dominating roots of G are 0 with 2 multiplicities and $\frac{-3\pm i\sqrt{3}}{2}$



Theorem: 2.3 For any path P_p, p ≥ 3 vertices $D_{c2oe}(P_p,x) = x^p + 2x^{p-1} + x^{p-2}$ and the connected two out degree equitable dominating roots 0 with multiplicity $p-2$ and -1 with multiplicity 2.

Proof: Let G be path P_p with p≥3 and let $P_p = \{v_1, v_2, v_3 \dots\dots\dots v_p\}$.

The connected two out degree equitable domination number of P_p is $p-2$ and there is only one connected two out degree equitable domination set of order p-2. That means $d_{c2oe}(P_p, p-2) = 1$.

Also there are only two connected two out degree equitable dominating set of order $p-1$ namely $\{v_2, v_3 \dots\dots\dots v_p\}$ and $\{v_1, v_2 \dots\dots\dots v_{p-1}\}$.

Therefore $d_{c2oe}(P_p, p-1) = 2$.

There is only one connected dominating set of order n. Then $d_{c2oe}(P_p, p) = 1$

Hence $D_{c2oe}(G,x) = x^p + 2x^{p-1} + x^{p-2}$ and it is clear that the roots of the polynomial $x^p + 2x^{p-1} + x^{p-2}$ are 0 with multiplicity $p-2$ and $p-1$ with multiplicity 2.

Theorem: 2.4 For an cycle $C_p$with p vertices $D_{c2oe}(C_p,x) = x^{p-2}(x^2 + px + p)$And the connected two out degree equitable dominating roots are 0 with multiplicity $p-2$and $\frac{-p+\sqrt{p^2-4p}}{2}$, $\frac{-p-\sqrt{p^2-4p}}{2}$.

Proof: Let G be cycle C_p with p and let $C_p = \{v_1, v_2, v_3 \dots\dots\dots v_p, v_1\}$.

The connected two out degree equitable domination number of C_p is $p-2$ and there are p possibilities connected two out degree equitable domination set of order $p-2$. That means $d_{c2oe}(C_p, p-2) = p$.

Also there are only two connected two out degree equitable dominating set of order $p-1$namely $\{v_2, v_3 \dots\dots\dots v_p\}$, $\{v_2, v_3, v_4 \dots\dots\dots v_{p-1}\}$,… … …$\{v_1, v_2, v_3 \dots\dots\dots v_{p-1}\}$

Therefore $d_{c2oe}(C_p, p-1) = p$.

There is only one connected dominating set of order p. Then $d_{c2oe}(C_p, p) = 1$

© 2016, IJMA. All Rights Reserved

Hence $D_{c2oe}(C_p, x) = x^p + px^{p-1} + px^{p-2} = x^{p-2}(x^2 + px + p)$.
it is clear that the roots of the polynomial $x^{p-2}(x^2 + px + p)$ are 0 with multiplicity p-2 and $\frac{-p+\sqrt{p^2-4p}}{2}, \frac{-p-\sqrt{p^2-4p}}{2}$.

Theorem: 2.5 For any star graph $K_{1,p}$ with $p + 1$ vertices, where p≥ 2, then

$$D_{c2oe}(K_{1,p}, x) = x\left[\binom{p}{2}x^{p-2} + px^{p-1} + x^p\right]$$

Proof: Let G be a star graph of size $p + 1$ and p≥ 2. By labeling the vertices of G as v_0, v_1, v_2 ……… $v_{p,}$ where v_0 is the vertex of degree p, and the connected two out degree equitable domination number of $K_{1,p}$ is $p - 1$.

For the connected two out degree equitable dominating set of size n-1 we need to select the vertex v_0 and $p - 2$ vertices from the set of vertices $\{v_1, v_2 \dots\dots\dots v_{p,}\}$.

That means there are $\binom{p}{p-2}$ connected two out degree equitable dominating set of size n-1.

Similarly for the connected two out degree equitable dominating set of size p we need to select the vertex v_0 and $p - 1$ vertices from the set of vertices $\{v_1, v_2 \dots\dots\dots v_{p,}\}$.

That means there are $\binom{p}{p-1}$ connected two out degree equitable dominating set of size p.

For the connected two out degree equitable dominating set of size n+1 we need to select the vertex v_0 and n vertices from the set of vertices $\{v_1, v_2 \dots\dots\dots v_{p,}\}$.

That means there are $\binom{p}{p}$ connected two out degree equitable dominating set of size $p + 1$.

$$\begin{aligned}
\text{Hence } D_{c2oe}(K_{1,p}, x) &= \binom{p}{p-2}x^{p-1} + \binom{p}{p-1}x^p + \binom{p}{p}x^{p+1} \\
&= x\left[\binom{p}{p-2}x^{p-2} + \binom{p}{p-1}x^{p-1} + \binom{p}{p}x^p\right] \\
&= x\left[\binom{p}{2}x^{p-2} + \binom{p}{1}x^{p-1} + \binom{p}{0}x^p\right] \\
&= x\left[\binom{p}{2}x^{p-2} + px^{p-1} + x^p\right]
\end{aligned}$$

Theorem: 2.6 For any complete graph K_p of p vertices $D_{c2oe}(K_p, x) = (1 + x)^p - 1 - px$

Proof: Let G be a complete graph K_p. Then for any 2$\leq i \leq p$, it is easy to see that $d_{c2oe}(K_p, i) = \binom{p}{i}$

$$\begin{aligned}
D_{c2oe}(K_p, x) &= \textstyle\sum_{i=2}^{p}\binom{p}{i}x^i \\
&= \binom{p}{2}x^2 + \binom{p}{3}x^3 + \binom{p}{4}x^4 + \dots\dots\dots \binom{p}{p}x^p \\
&= \left[\textstyle\sum_{i=0}^{p}\binom{p}{i}x^i\right] - 1 - px \\
&= (x + 1)^p - 1 - px
\end{aligned}$$

Theorem: 2.7 Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs of orders p_1 and p_2 respectively. Then
$D_{c2oe}((G_1 + G_2), x) = D_{c2oe}(G_1, x) + D_{c2oe}(G_2, x) + [(1 + x)^{p_1} - 1][(1 + x)^{p_2} - 1]$

Proof: From the definition of $G_1 + G_2$, if the D_1 be any connected two out degree equitable dominating set of G_1,then D_1 be any connected two out degree equitable dominating set of $G_1 + G_2$. Similarly if the D_2 be any connected two out degree equitable dominating set of G_2,then D_2 be any connected two out degree equitable dominating set of $G_1 + G_2$ Then

$$\begin{aligned}
&D_{c2oe}((G_1 + G_2), x) \\
&\quad = D_{c2oe}(G_1, x) + D_{c2oe}(G_2, x) + \binom{p_1}{1}\binom{p_2}{1}x^2 + \binom{p_1}{1}\binom{p_2}{2}x^3 + \binom{p_1}{2}\binom{p_2}{1}x^3 + \binom{p_1}{1}\binom{p_2}{3}x^4 + \binom{p_1}{2}\binom{p_2}{2}x^4 \\
&\qquad + \binom{p_1}{3}\binom{p_2}{1}x^4 + \binom{p_1}{1}\binom{p_2}{4}x^5 + \binom{p_1}{2}\binom{p_2}{3}x^5 + \binom{p_1}{3}\binom{p_2}{2}x^5 + \binom{p_1}{4}\binom{p_2}{1}x^5 + \cdots\dots\dots \\
&\qquad + \left[\binom{p_1}{1}\binom{p_2}{p_1 + p_2 - 1} + \dots\dots\dots \binom{p_1}{p_1 + p_2 - 1}\binom{p_2}{1}\right]x^{p_1+p_2} \\
&= D_{c2oe}(G_1, x) + D_{c2oe}(G_2, x) + \left[\binom{p_1}{1}x + \binom{p_1}{2}x^2 + \dots\dots\dots \binom{p_1}{p_1}x^{n_1}.\binom{p_2}{1}x + \binom{p_2}{2}x^2 + \dots\dots\dots \binom{p_2}{p_2}x^{p_2}\right]
\end{aligned}$$

$= D_{c2oe}(G_1, x) + D_{c2oe}(G_2, x) + \sum_{i=1}^{p_1} \binom{p_1}{i} x^i \sum_{i=1}^{p_2} \binom{p_2}{i} x^i$
$= D_{c2oe}(G_1, x) + D_{c2oe}(G_2, x) + [(1+x)^{p_1} - 1][(1+x)^{p_2} - 1]$

Theorem: 2.8 For any complete bipartite graph $K_{r,t}$,where $1 < r < t$ and $|r - t| \leq 2$ of n vertices

$$D_{c2oe}(K_{r,t}, x) = [(1+x)^r - 1][(1+x)^t - 1]$$

Proof: If G is complete bipartite graph then $\gamma_{c2oe}(K_{r,t}) = 2$ with partite set $V_1$and V_2, then any connected two out degree equitable dominating set of G contains at least one vertex from $V_1$and at least one vertex from V_2.

So we have

$$D_{c2oe}(K_{r,t}, x) = \binom{r}{1}\binom{t}{1} x^2 + \binom{r}{1}\binom{t}{2} x^3 + \binom{r}{1}\binom{t}{2} x^3 + \cdots \ldots \ldots \left[\binom{r}{1}\binom{t}{r+t-1} + \ldots \ldots \ldots \binom{r}{t+r-1}\binom{r}{1}\right] x^{r+t}$$
$$= \left[\binom{r}{1} x + \binom{r}{2} x^2 + \ldots \ldots \ldots \binom{r}{r} x^r . \binom{t}{1} x + \binom{t}{2} x^2 + \ldots \ldots \ldots \binom{t}{t} x^t\right]$$
$$= \sum_{i=1}^{r} \binom{r}{i} x^i \sum_{i=1}^{t} \binom{t}{i} x^i$$
$$= [(1+x)^r - 1][(1+x)^t - 1]$$

Theorem: 2.9 For any pathP_p of p$\geq$ 2, vertices $D_{c2oe}(P_2 \times P_n, x) = (x^2 + x)^p$

Proof: The vertices of $P_2 \times P_p$is V($P_2 \times P_p$)= {(1,1),(1,2), (1,3)…(1,p) (2,1),(2,2),(2,3)…(2,p)} containing 2p vertices and $\gamma_{c2oe}(P_2 \times P_p)$=p for p ≥ 2

There is only one connected two out degree equitable domination number p is {(1,1),(1,2), (1,3)…(1,p)} or (2,1),(2,2),(2,3)…(2,p)}

To find connected two out degree equitable domination of size p+1 we need to select {(1,1),(1,2), (1,3)-------(1,p)} or (2,1),(2,2),(2,3)--------(2,p)} and select one vertex from the set of vertices {(2,1),(2,2),(2,3)--------(2,p)} or {(1,1),(1,2), (1,3)-------(1,p)}

That means there are $\binom{p}{1}$ connected two out degree equitable dominating set of size p

Similarly there are $\binom{p}{i}$ possibilities to extend the connected two out degree equitable dominating set of size $p + i$.

Hence $D_{c2oe}(P_2 \times P_p, x) = x^p + \binom{p}{1} x^{p+1} + \binom{p}{2} x^{p+2} + \ldots \ldots \ldots + \binom{p}{p} x^{p+p}$

$$= x^p + \binom{p}{1} x^{p+1} + \binom{p}{2} x^{p+2} + \ldots \ldots \ldots + x^{2p}$$
$$= x^p \left(1 + \binom{p}{1} x^1 + \binom{p}{2} x^2 + \ldots \ldots \ldots + x^p\right)$$
$$= x^p \sum_{k=0}^{p} \binom{p}{k} x^k$$
$$= (x^2 + x)^p$$

Theorem: 2.10 For any connected graph G, with p vertices $D_{c2oe}(\mathrm{G} \times \mathrm{K}_1, x) = (x^2 + x)^p$

Proof: Let H= G $\times$ K_1. Then clearly γ_{c2oe}(H) =p and it is easy to see that there are $\binom{p}{1}$ connected two out degree equitable dominating set of size p

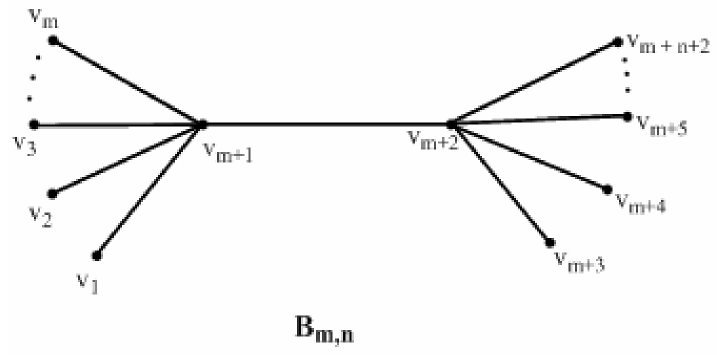
Similarly there are $\binom{p}{i}$ possibilities to extend the connected two out degree equitable dominating set of size $p + i$.

Hence $D_{c2oe}(\mathrm{G \ o \ K}_1, x) = x^p + \binom{p}{1} x^{p+1} + \binom{p}{2} x^{p+2} + \ldots \ldots \ldots + x^{2p}$

$$= x^p \left(1 + \binom{p}{1} x^1 + \binom{p}{2} x^2 + \ldots \ldots \ldots + x^p\right)$$
$$= x^p \left[\sum_{k=0}^{p} \binom{p}{k} x^k\right]$$
$$= x^p \sum_{k=0}^{p} \binom{p}{k} x^k$$
$$= (x^2 + x)^p$$

© 2016, IJMA. All Rights Reserved

Theorem: 2.11 Let G be a bi-star graph B(r, t) as in Figure 2 Then $D_{c2oe}(\mathrm{B(r,t)}, x)$=$x^2(1+x)^{r+t}$ if $|r-t| \le 2$



Proof: Let G be a bi-star and with labeling as figure 2. Then $\gamma_{c2oe}(B(r,t)) = 2$ if $|r-t| \le 2$ namely the set {u,v} is only unique minimum connected two out degree equitable dominating set.

Hence $d_{c2oe}(\mathrm{B(r,t)}, \gamma_{c2oe}) = 1$, and it is obvious that any other connected two out degree equitable dominating set must contain the two vertices u and.

Hence there are $\binom{r+t}{1}$ possibilities to extend the connected two out degree equitable dominating set of size 3, and there are $\binom{r+t}{2}$ possibilities to extend the connected two out degree equitable dominating set into connected two out degree equitable dominating set of size 4.

In general it is easy to see that there are $\binom{r+t}{i-2}$ possibilities to extend the connected two out degree equitable dominating set into connected two out degree equitable dominating set of size i.

Therefore $D_{c2oe}(\mathrm{B(r,t)}, x) = x^2 + \binom{r+t}{1}x^3 + \binom{r+t}{2}x^4 + \cdots \ldots\ldots + \binom{r+t}{r+t}x^{r+t+2}$

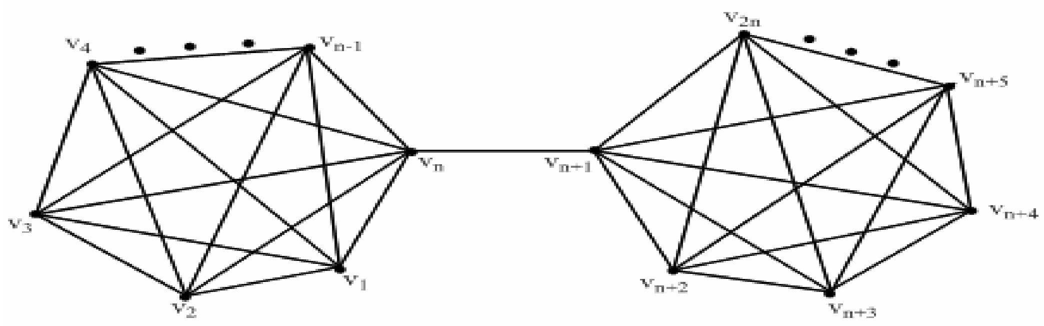
$$= x^2 + \binom{r+t}{1}x^3 + \binom{r+t}{2}x^4 + \ldots\ldots\ldots + x^{r+t+2}$$

$$= x^2\left[1 + \binom{r+t}{1}x + \binom{r+t}{2}x^2 + \ldots\ldots\ldots + x^{r+t}\right]$$

Then $D_{c2oe}(\mathrm{B(r,t)}, x)$=$x^2(1+x)^{r+t}$ if $|r-t| \le 2$

Theorem: 2.12 For a Barbell graph B_n with 2n vertices the connected two out degree equitable domination polynomial is $D_{c2oe}(B_n, x)$=$x^2(1+x)^{2n-2}$

Proof; Let B_n be a Barbell graph with 2n vertices. Label the vertices of B_n as $v_1, v_2, \ldots\ldots\ldots, v_n, v_{n+1} \ldots\ldots\ldots v_{2n}$ is given in figure



Then the set $\{v_n, v_{n+1}\}$ is the unique minimum connected two out degree equitable dominating set of cardinality of 2.

Therefore $\gamma_{c2oe}(B_n)$ = 2 and $D_{c2oe}(B_n, 2) = 1$

It is obvious that the other connected two out degree equitable dominating set must contain two vertices v_n *and* v_{n+1}

Hence these are $\binom{2n-2}{1}$ connected two out degree equitable dominating set of cardinality 3 and $\binom{2n-2}{2}$connected two out degree equitable dominating set of cardinality 4.

Proceeding like this we obtain $\binom{2n-2}{2n-2}$ connected two out degree equitable dominating set of cardinality 2n vertices

Hence $D_{c2oe}(B_n, x) = x^2 + \binom{2n-2}{1}x^3 + \binom{2n-2}{2}x^4 + \binom{2n-2}{3}x^5 \ldots\ldots\ldots \binom{2n-2}{2n-2}x^{2n}$

$$= x^2\left[1 + \binom{2n-2}{1}x + \binom{2n-2}{2}x^2 + \binom{2n-2}{3}x^3 \ldots\ldots\ldots \binom{2n-2}{2n-2}x^{2n-2}\right]$$

$$= x^2(1+x)^{2n-2}$$

REFERENCE

1. A.Sahal and V.Mathad" Two out degree equitable domination in graphs" Transaction and combinatories, vol 2, No 3 (2013) 13-19
2. B.V.Dhanaajaya Murthy, G.Deepak and N.D. Soner" Connected domination polynomial of graph" International Journal of Mathematical Archive" Vol 4, No-11(2013) 90-96.
3. E.Sampath Kumar and H.B.Walikar, The connected domination number of a graph" J. Math. Physics. Sci13 (1979)607-613.
4. F.Harray, Graph theory, Addision-Wesley, Reading Mass, 1969
5. M.S.Mahesh and P.Namasivayam "Connected two out degree equitable domination number in different graphs" International Journal of Mathematical Archive" Vol 6, No-1(2014) 67-74
6. Saeid Alikhani and Yee-hock Peng" Introduction to domination of graph" arXiv: 0905,225 V1(2209)
7. T.W.Haynes, S.T.Hedetniemi and P.J.Slater" Domination in graphs-Advanced topics", Marcel Dekkar, Inc, New York (1998).
8. T.W.Haynes, S.T.Hedetniemi and P.J.Slater" Fundamentals of domination in graphs, Marcel Dekkar, Inc, New York (1989).

Source of support: Nil, Conflict of interest: None Declared

[Copy right © 2016. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]