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COMMON FIXED POINT THEOREMS IN PARAMETRIC METRIC SPACES UNDER NONLINEAR TYPE CONTRACTIONS

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ABSTRACT

In this paper, we establish some common fixed point theorems for a class of nonlinear contractive mappings in parametric metric spaces.

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1. INTRODUCTION

The concept of metric spaces has been generalized in many directions. The notion of a b-metric space was studied by Czerwik in [14-15] and a lot of fixed point results for single and multi-valued mappings by many authors have been obtained in (ordered) b-metric spaces (see, e.g., [16-17]).The concept of fuzzy set was introduced by Zadeh [3] in 1965. In 1975, Kramosil and Michalek [4] introduced the notion of fuzzy metric space, which can be regarded as a generalization of the statistical (probabilistic) metric space. This work has provided an important basis for the construction of fixed point theory in fuzzy metric spaces. In 2004, Park introduced the notion of intuitionistic fuzzy metric space [5]. He showed that, for each intuitionistic fuzzy metric space$(X, M, N, *, \diamond)$, the topology generated by the intuitionistic fuzzy metric (M, N) coincides with the topology generated by the fuzzy metric M. For more details on intuitionistic fuzzy metric space and related results we refer the reader to [5-12]. In 2014, Hussain *et. al* [12] defined and studied the concept of parametric metric space and established some fixed point theorems. For more details on parametric metric space and related results we refer the reader to [12-13, 18]. .In this paper, we establish some common fixed point theorems for mapping satisfying nonlinear type contractions in the frame of parametric metric spaces.These results improve and generalize some important known results in [19].

2. PRELIMINARIES

Throughout this paper $\mathbb{R}$ and $\mathbb{R}^+$ will represents the set of real numbers and nonnegative real numbers, respectively.

In 2014, Hussain *et. al* [12] defined and studied the concept of parametric metric space as follows.

Definition 2.1: Let X be a nonempty set and $\mathcal{P} : X \times X \times (0, +\infty) \to [0, +\infty)$ be a function. We say $\mathcal{P}$is a parametric metric on X if,

(1) $\mathcal{P}(x, y, t) = 0$ for all $t > 0$ if and only if $x = y$;
(2) $\mathcal{P}(x, y, t) = \mathcal{P}(y, x, t)$ for all $t > 0$;
(3) $\mathcal{P}(x, y, t) \leq \mathcal{P}(x, z, t) + \mathcal{P}(z, y, t)$ for all $x, y, z \in X$ and all $t > 0$: and one says the pair $(X, \mathcal{P})$ is a parametric metric space.

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The following definitions are required in the sequel which can be found in [12].

Definition 2.2: Let $\{x_n\}_{n=1}^{\infty}$ be a sequence in a parametric metric space$(\mathrm{X}, \mathcal{P})$.

1. $\{x_n\}_{n=1}^{\infty}$is said to be convergent to $x \in X$, written as $\lim_{n\to\infty} x_n = x$, for all $t > 0$, if $\lim_{n\to\infty} \mathcal{P}(x_n, x, t) = 0$.
2. $\{x_n\}_{n=1}^{\infty}$ is said to be a Cauchy sequence in X if for all $t > 0$, if $\lim_{n,m\to\infty} \mathcal{P}(x_n, x_m, t) = 0$.
3. $(\mathrm{X}, \mathcal{P})$is said to be complete if every Cauchy sequence is a convergent sequence.

Definition 2.3: Let $(\mathrm{X}, \mathcal{P})$ be a parametric metric space and $T: X \to X$ be a mapping. We say $T$is a continuous mapping at x in X, if for any sequence $\{x_n\}_{n=1}^{\infty}$ in X such that $\lim_{n\to\infty} x_n = x$, then $\lim_{n\to\infty} Tx_n = Tx$.

Example 2.4: Let X denote the set of all functions$f: (0, +\infty) \to \mathbb{R}$. Define $\mathcal{P} : X \times X \times (0, +\infty) \to [0, +\infty)$ by $\mathcal{P}(f, g, t) = |f(t) - g(t)|\ \forall\, f, g \in X$ and all $t > 0$. Then $\mathcal{P}$ is a parametric metric on X and the pair $(X, \mathcal{P})$is a parametric metric space.

3. MAIN RESULT

The following results are required in the sequel which can be found in [1, 2, and 19].

Definition 3.1: There exists $\phi(\mathrm{t})$ that satisfy the condition ϕ', if one lets $\phi: [0, +\infty) \to [0, +\infty)$ be non-decreasing and non-negative, then $\lim \phi_{\mathrm{n}}(\mathrm{t}) = 0$, for a given $\mathrm{t} > 0$.

Lemma 3.2: If ϕ satisfy the condition ϕ', then $\phi(\mathrm{t}) < t$, for a given $\mathrm{t} > 0$.

Lemma 3.3: Let $\mathcal{F}: \mathbb{R}_+^3 \to \mathbb{R}_+$, and satisfy the condition ϕ'; for all $\mathrm{u}, \mathrm{v} \geq 0$, if $\mathrm{u} \leq \mathcal{F}(\mathrm{v}, \mathrm{v}, \mathrm{u})$ or $\mathrm{u} \leq \mathcal{F}(\mathrm{v}, \mathrm{u}, \mathrm{v})$ or $\mathrm{u} \leq \mathcal{F}(\mathrm{u}, \mathrm{v}, \mathrm{v})$, then $\mathrm{u} \leq \phi(\mathrm{v})$.

Our first main results as follows:

Theorem 3.4: Let $(\mathrm{X}, \mathcal{P})$ be a complete parametric metric space and let S and T be self-mappings on X if

1) either S and T is continuous,
2) there exists $\mathcal{F}$ satisfying the condition ϕ' for all $\mathrm{x}, \mathrm{y} \in \mathrm{X}$ and for all $\mathrm{t} > 0$, such that

(3.1) $$\mathcal{P}(\mathrm{Sx}, \mathrm{Ty}, \mathrm{t}) \leq \mathcal{F}\big(\mathcal{P}(\mathrm{x}, \mathrm{y}, \mathrm{t}), \mathcal{P}(\mathrm{x}, \mathrm{Sx}, \mathrm{a}), \mathcal{P}(\mathrm{y}, \mathrm{Ty}, \mathrm{t})\big).$$

Then S and T have a unique common fixed point.

Proof: Let $\mathrm{x}_0 \in \mathrm{X}$ be arbitrary, T continuous in X, $\{\mathrm{x}_\mathrm{n}\}$ and $\{\mathrm{y}_\mathrm{n}\}$ the sequence of X, and

(3.2) $$\mathrm{x}_\mathrm{n} = (\mathrm{ST})^\mathrm{n}(\mathrm{x}_0) = \mathrm{ST}(\mathrm{x}_{\mathrm{n}-1}),\ \mathrm{y}_\mathrm{n} = \mathrm{T}(\mathrm{ST})^{\mathrm{n}-1}(\mathrm{x}_0),\ \ \forall\, \mathrm{n} \in \mathbb{N}.$$

Obviously,

(3.3) $$\mathrm{y}_\mathrm{n} = \mathrm{T}(\mathrm{x}_{\mathrm{n}-1}),\quad \mathrm{S}(\mathrm{y}_\mathrm{n}) = \mathrm{x}_\mathrm{n},\quad \mathrm{TS}(\mathrm{y}_\mathrm{n}) = \mathrm{T}(\mathrm{x}_\mathrm{n}) = \mathrm{y}_{\mathrm{n}+1},\ \forall\, \mathrm{n} \in \mathbb{N}.$$

From (3.1), we get

(3.4) $$\begin{aligned}\mathcal{P}(\mathrm{x}_{\mathrm{n}+1}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}) &= \mathcal{P}(\mathrm{ST}(\mathrm{x}_\mathrm{n}), \mathrm{T}(\mathrm{x}_\mathrm{n}), \mathrm{t})\\ &\leq \mathcal{F}\big(\mathcal{P}(\mathrm{T}(\mathrm{x}_\mathrm{n}), \mathrm{x}_\mathrm{n}, \mathrm{t}), \mathcal{P}(\mathrm{T}(\mathrm{x}_\mathrm{n}), \mathrm{ST}(\mathrm{x}_\mathrm{n}), \mathrm{t}), \mathcal{P}(\mathrm{x}_\mathrm{n}, \mathrm{T}(\mathrm{x}_\mathrm{n}), \mathrm{t})\big)\\ &= \mathcal{F}\big(\mathcal{P}(\mathrm{y}_{\mathrm{n}+1}, \mathrm{x}_\mathrm{n}, \mathrm{t}), \mathcal{P}(\mathrm{y}_{\mathrm{n}+1}, \mathrm{x}_{\mathrm{n}+1}, \mathrm{t}), \mathcal{P}(\mathrm{x}_\mathrm{n}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t})\big)\end{aligned}$$

Hence by Lemma 3.3, we have

(3.5) $$\mathcal{P}(\mathrm{x}_{\mathrm{n}+1}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}) \leq \phi\big(\mathcal{P}(\mathrm{x}_\mathrm{n}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t})\big)$$

Also,

(3.6) $$\begin{aligned}\mathcal{P}(\mathrm{x}_\mathrm{n}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}) &= \mathcal{P}(\mathrm{x}_\mathrm{n}, \mathrm{T}(\mathrm{x}_\mathrm{n}), \mathrm{t})\\ &= \mathcal{P}(\mathrm{S}(\mathrm{y}_\mathrm{n}), \mathrm{T}(\mathrm{x}_\mathrm{n}), \mathrm{t})\\ &\leq \mathcal{F}\big(\mathcal{P}(\mathrm{y}_\mathrm{n}, \mathrm{x}_\mathrm{n}, \mathrm{t}), \mathcal{P}(\mathrm{y}_\mathrm{n}, \mathrm{S}(\mathrm{y}_\mathrm{n}), \mathrm{t}), \mathcal{P}(\mathrm{x}_\mathrm{n}, \mathrm{T}(\mathrm{x}_\mathrm{n}), \mathrm{t})\big)\\ &= \mathcal{F}\big(\mathcal{P}(\mathrm{y}_\mathrm{n}, \mathrm{S}(\mathrm{y}_\mathrm{n}), \mathrm{t}), \mathcal{P}(\mathrm{y}_\mathrm{n}, \mathrm{S}(\mathrm{y}_\mathrm{n}), \mathrm{t}), \mathcal{P}(\mathrm{x}_\mathrm{n}, \mathrm{T}(\mathrm{x}_\mathrm{n}), \mathrm{t})\big)\end{aligned}$$

Therefore,

$$\mathcal{P}(\mathrm{x}_\mathrm{n}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}) = \mathcal{P}(\mathrm{x}_\mathrm{n}, \mathrm{T}(\mathrm{x}_\mathrm{n}), \mathrm{t}) \leq \phi\big(\mathcal{P}(\mathrm{y}_\mathrm{n}, \mathrm{S}(\mathrm{y}_\mathrm{n}), \mathrm{t})\big)$$

That is,

(3.7) $$\mathcal{P}(\mathrm{x}_\mathrm{n}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}) \leq \emptyset\big(\mathcal{P}(\mathrm{y}_\mathrm{n}, \mathrm{x}_\mathrm{n}, \mathrm{t})\big) = \phi\big(\mathcal{P}(\mathrm{x}_\mathrm{n}, \mathrm{y}_\mathrm{n}, \mathrm{t})\big)$$

From (3.5) and (3.7), we conclude that

(3.8) $$\mathcal{P}(\mathrm{x}_{\mathrm{n}+1}, \mathrm{y}_{\mathrm{n}+1}, \mathrm{t}) \leq \phi^2\big(\mathcal{P}(\mathrm{x}_\mathrm{n}, \mathrm{y}_\mathrm{n}, \mathrm{t})\big)$$

Hence, by induction, for all $n \in \mathbb{N}$, we obtain that

(3.9) $\mathcal{P}(x_{n+1}, y_{n+1}, t) \le \phi^{2n}(\mathcal{P}(x_1, y_1, t)) = \phi^{2n}(\mathcal{P}(x_1, T(x_0), t))$

In similar one obtains that

(3.10) $$\mathcal{P}(y_{n+1}, x_n, t) = \mathcal{P}(x_n, y_{n+1}, t) \le \phi^{2n-1}(\mathcal{P}(x_1, y_1, t)) = \phi^{2n-1}(\mathcal{P}(x_1, T(x_0), t))$$

If $n \ge 2$, we have

(3.11) $$\begin{aligned}\mathcal{P}(x_{n+1}, x_n, t) &\le \mathcal{P}(x_{n+1}, y_{n+1}, t) + \mathcal{P}(y_{n+1}, x_n, t)\\ &= \phi^{2n}(\mathcal{P}(x_1, T(x_0), t)) + \phi^{2n-1}(\mathcal{P}(x_1, T(x_0), t))\\ &\le 2\phi^{2n-1}(\mathcal{P}(x_1, T(x_0), t))\end{aligned}$$

Note that the condition $\emptyset'$ we know, for $n, m \in \mathbb{N}$ such that $m > n$, we have

(3.12) $$\begin{aligned}\mathcal{P}(x_n, x_{n+m}, t) &\le \mathcal{P}(x_n, x_{n+1}, t) + \mathcal{P}(x_{n+1}, x_{n+2}, t) + \cdots \ldots \ldots + \mathcal{P}(x_{n+m-1}, x_{n+m}, t)\\ &\le 2\phi^{2n-1}(\mathcal{P}(x_1, T(x_0), t)) + 2\phi^{2(n+1)-1}(\mathcal{P}(x_1, T(x_0), t))\\ &\qquad + \cdots \ldots \ldots + 2\phi^{2(n+m-1)-1}(\mathcal{P}(x_1, T(x_0), t))\\ &\le \sum_{i=2n-1}^{2(n+m-1)-1} \phi^i (\mathcal{P}(x_1, T(x_0), t))\\ &\le \sum_{i=2n-1}^{\infty} \phi^i (\mathcal{P}(x_1, T(x_0), t)) \to 0.\end{aligned}$$

Hence, $\lim_{m,n\to\infty}(\mathcal{P}(x_n, x_{n+m}, t)) = 0$. Equivalently, $\lim_{m,n\to\infty} \mathcal{P}(x_n, x_{n+m}, t) = 0$. This forces that $\{x_n\}$ is a Cauchy sequence in X. But X is complete parametric metric space, there must exist $x^* \in X$ such that

(3.13) $\lim_{n\to\infty} x_n = x^*$.

By continuity of T, we have

(3.14) $$\lim_{n\to\infty} y_n = \lim_{n\to\infty} T(x_{n-1}) = T\left(\lim_{n\to\infty} x_{n-1}\right) = Tx^*.$$

Now we want to show that $x^* = Sx^* = Tx^*$. First we show that $x^* = Tx^*$. Suppose on the contrary that $x^* \ne Tx^*$, then from (3.1) we get

(3.15) $\mathcal{P}(x_{n+1}, y_{n+1}, t) \le \mathcal{F}(\mathcal{P}(y_{n+1}, x_n, t), \mathcal{P}(y_{n+1}, x_{n+1}, t), \mathcal{P}(x_n, y_{n+1}, t))$

By taking limit $n \to +\infty$ in the above inequality and using (3.13) and (3.14), we obtain that

(3.16) $\mathcal{P}(x^*, Tx^*, t) \le \mathcal{F}(\mathcal{P}(Tx^*, x^*, t), \mathcal{P}(Tx^*, x^*, t), \mathcal{P}(x^*, Tx^*, t))$

Therefore,

(3.17) $\mathcal{P}(x^*, Tx^*, t) \le \phi(\mathcal{P}(x^*, Tx^*, t)) < \mathcal{P}(x^*, Tx^*, t)$.

which is contradiction. Hence $x^* = Tx^*$and so x^* is a fixed point of T. Again from (3.1), we have

(3.18) $$\begin{aligned}\mathcal{P}(Sx^*, x^*, t) &= \mathcal{P}(Sx^*, Tx^*, t)\\ &\le \mathcal{F}(\mathcal{P}(x^*, x^*, t), \mathcal{P}(x^*, Sx^*, t), \mathcal{P}(x^*, Tx^*, t))\\ &= \mathcal{F}(0, \mathcal{P}(x^*, Sx^*, t), 0)\end{aligned}$$

Hence,

$$\mathcal{P}(Sx^*, x^*, t) \le \phi(0) = 0 \Rightarrow \mathcal{P}(Sx^*, x^*, t) = 0 \Rightarrow Sx^* = x^*.$$

So x^* is a common fixed point of S and T. To prove uniqueness, suppose that $x^* \ne x'$, such that $Sx^* = Tx^* = x^*$ and $Sx' = Tx' = x'$. From (3.1), we get

(3.19) $$\begin{aligned}\mathcal{P}(x^*, x', t) &= \mathcal{P}(Sx^*, Tx', t)\\ &\le \mathcal{F}(\mathcal{P}(x^*, x', t), \mathcal{P}(x^*, Sx^*, t), \mathcal{P}(x', Tx', t))\\ &= \mathcal{F}(\mathcal{P}(x^*, x', t), \mathcal{P}(x^*, x^*, t), \mathcal{P}(x', x', t))\\ &= \mathcal{F}(\mathcal{P}(x^*, x', t), 0, 0)\end{aligned}$$

Hence,

$$\mathcal{P}(x^*, x', t) \le \phi(0) = 0 \Rightarrow \mathcal{P}(x^*, x', t) = 0 \Rightarrow x^* = x'.$$

So x^* is a unique common fixed point of S and T.

Theorem 3.5: Let $(X, \mathcal{P})$ be a complete parametric metric space and let S and T be continuous mappings on X if

1) there exists $\mathcal{F}$ satisfying the condition ϕ' for all $x, y \in X$ with $x \ne y$ and $t > 0$, such that

(3.20) $\mathcal{P}(Sx, Ty, t) \le \mathcal{F}(\mathcal{P}(x, y, t), \mathcal{P}(x, Sx, t), \mathcal{P}(y, Ty, t))$.

2) there exists $x_0 \in X$ such that $\{(ST)^n(x_0)\}$ have a condensation point.Then S and T have a unique common fixed point.

Proof: Let $\{x_n\}$, $\{y_n\}$ be the sequence of X, and for all n, $x_n \neq y_n$,

(3.21) $$x_n = (ST)^n(x_0) = ST(x_{n-1}),\ y_n = T(ST)^{n-1}(x_0),\ \ \forall\, n \in \mathbb{N}.$$

Obviously,

(3.22) $$y_n = T(x_{n-1}),\ \ S(y_n) = x_n,\ \ TS(y_n) = T(x_n) = y_{n+1},\ \forall\, n \in \mathbb{N}.$$

Suppose that x^* is the condensation point of $\{x_n\}$; there exists the subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $x_{n_i} \to x^*$. Since T is continuous, $\lim T(x_{n_i}) = T(x^*) = y^*$.

Consider

(3.23) $$\begin{aligned}\mathcal{P}(S(y^*), y^*, t) &= \mathcal{P}(S(y^*), T(x^*), t)\\ &\le \mathcal{F}\big(\mathcal{P}(y^*,\ x^*, t), \mathcal{P}(y^*,\ S(y^*), t), \mathcal{P}(x^*,\ T(x^*), t)\big)\\ &= \mathcal{F}\big(\mathcal{P}(y^*,\ x^*, t), \mathcal{P}(y^*,\ S(y^*), t), \mathcal{P}(x^*,\ y^*, t)\big)\end{aligned}$$

Hence by Lemma 3.3, we have

(3.24) $$\mathcal{P}(S(y^*), y^*, t) \le \phi\big(\mathcal{P}(x^*,\ y^*, t)\big)$$
$$\Rightarrow\ \ \mathcal{P}(S(y^*), y^*, t) \le \mathcal{P}(x^*,\ y^*, t).$$

Also consider

(3.25) $$\begin{aligned}\mathcal{P}(x_{n+1},\ y_{n+1}, t) &= \mathcal{P}(ST(x_n),\ T(x_n), t)\\ &\le \mathcal{F}\big(\mathcal{P}(T(x_n), x_n, t), \mathcal{P}(T(x_n), ST(x_n), t), \mathcal{P}(x_n, T(x_n), t)\big)\\ &= \mathcal{F}\big(\mathcal{P}(y_{n+1}, x_n, t), \mathcal{P}(y_{n+1}, x_{n+1}, t), \mathcal{P}(x_n, y_{n+1}, t)\big)\end{aligned}$$

Hence by Lemma 3.3, we have

(3.26) $$\mathcal{P}(x_{n+1},\ y_{n+1}, t) \le \phi\big(\mathcal{P}(x_n, y_{n+1}, t)\big)$$

Also,

(3.27) $$\begin{aligned}\mathcal{P}(x_n, y_{n+1}, t) &= \mathcal{P}(x_n, T(x_n), t)\\ &= \mathcal{P}(S(y_n), T(x_n), t)\\ &\le \mathcal{F}\big(\mathcal{P}(y_n, x_n, t), \mathcal{P}(y_n, S(y_n), t), \mathcal{P}(x_n, T(x_n), t)\big)\\ &= \mathcal{F}\big(\mathcal{P}(y_n, S(y_n), t), \mathcal{P}(y_n, S(y_n), t), \mathcal{P}(x_n, T(x_n), t)\big)\end{aligned}$$

Therefore,

(3.28) $$\mathcal{P}(x_n, T(x_n), t) = \mathcal{P}(x_n, T(x_n), t) \le \phi\big(\mathcal{P}(y_n, S(y_n), t)\big)$$

That is,

(3.29) $$\mathcal{P}(x_n, y_{n+1}, t) \le \phi\big(\mathcal{P}(y_n, x_n, t)\big) = \phi\big(\mathcal{P}(\ x_n, y_n, t)\big)$$

From (3.26) and (3.27), we conclude that

(3.30) $$\mathcal{P}(x_{n+1},\ y_{n+1}, t) \le \phi^2\big(\mathcal{P}(\ x_n, y_n, t)\big) < \mathcal{P}(\ x_n, y_n, t)$$

Hence, $\{\mathcal{P}(x_{n+1},\ y_{n+1}, t)\}$ is decreasing. Let $\epsilon = \lim\big(\mathcal{P}(x_{n+1},\ y_{n+1}, t)\big)$.

Now,

(3.31) $$\begin{aligned}\lim\Big(\mathcal{P}\big(y_{n_i+1}, x_{n_i+1},\ t\big)\Big) &= \lim\Big(\mathcal{P}\big(T(x_{n_i}), x_{n_i+1},\ t\big)\Big)\\ &= \mathcal{P}(y^*, x^*,\ t)\\ &\le \lim\Big(\mathcal{P}\big(y_{n_i}, x_{n_i},\ t\big)\Big) = \epsilon.\end{aligned}$$

Since $\{x_{n_i}\}$ is the subsequence of $\{x_n\}$, we have

(3.32) $$\begin{aligned}\mathcal{P}(S(y^*), y^*,\ t) &= \lim\Big(\mathcal{P}\big(S(y_{n_i+1}), y_{n_i+1},\ t\big)\Big)\\ &= \lim\Big(\mathcal{P}\big(x_{n_i}, y_{n_i},\ t\big)\Big) = \epsilon.\end{aligned}$$

Hence, from (3.30) and (3.31) we conclude that

(3.33) $$\mathcal{P}(y^*, x^*,\ t) \le \mathcal{P}(S(y^*), y^*,\ t).$$

So $x^* = y^*$, y^* is the fixed point of T. Similarly, y^* is a fixed point of S. To prove uniqueness, suppose that $y^* \neq y'$, such that $Sy^* = Ty^* = y^*$ and $Sy' = Ty' = y'$. From (3.1), we get

(3.34) $$\begin{aligned}\mathcal{P}(y^*, y', t) &= \mathcal{P}(Sy^*, Ty', t)\\ &\le \mathcal{F}\big(\mathcal{P}(y^* y', t), \mathcal{P}(y^*, Sy^*, t), \mathcal{P}(y', Ty', t)\big)\\ &= \mathcal{F}\big(\mathcal{P}(y^*, y',\ t), \mathcal{P}(y^*, y^*, t), \mathcal{P}(y', y', t)\big)\\ &= \mathcal{F}(\mathcal{P}(y^*, y', t), 0,0)\end{aligned}$$

Hence,

$$\mathcal{P}(y^*, y', t) \le \phi(0) = 0 \Rightarrow \mathcal{P}(y^*, y', t) = 0 \Rightarrow y^* = y'.$$

So y^* is a unique common fixed point of S and T.

AUTHOR'S CONTRIBUTION

All authors contributed equally and significantly to writing this paper. All authors read and approved the final paper.

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