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FUZZY MARKOV RENEWAL MODEL FOR THE SECRETION OF CORTICOSTERONE

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ABSTRACT

A mathematical model using fuzzy Markov Renewal was developed and used this model to calculate the mean and variance values of faecal Corticosterone Metabolites secretion in the given time interval. The result shows that the effect of Secretion of Corticosterone is reasonably higher if unconditional probabilities of time interval in the equilibrium probabilities increases.

Keywords: *Fuzzy Markov Renewal model, faecal Corticosterone Metabolites.*

Mathematics Subject Classification 2010: *94D05, 60A86, 62A86, 62E86.*

1. INTRODUCTION

Markov Renewal model has been used to model the stochastic structure of Secretion of Corticosterone. The general class of Markov renewal processes were introduced by Smith [10] and were later studied by Pyke[7], [8] and Cox[2], [3]. An extensive bibliography of theoretical developments and applications of Markov renewal processes is given by Teugels[11]. Markov renewal processes have a flexible dependence structure. It will be seen afterward that Markov chains, Markov processes, renewal processes, and alternating renewal processes [1] are all special cases of the general Markov renewal process. The Markov renewal model put emphasis on the duration times of the consecutive episodes in the patients post treatment experience. Conditionally on the sequence of states visited X, these duration time are independent, whereas the interdependence among the times of the occurrence of successive events is determined by the Markov chain assumption on the X process.

Corticosterone is a steroid hormone, secreted by the adrenal cortex that is involved in regulation of energy, immune reactions and stress response of our body. The level of corticosterone can be investigated by quantifying corticosterone excreted in faeces. Since corticosterone is metabolised in the liver before excretion in the faeces, a more accurate denomination of the measured products is immune reactive corticosterone metabolites [4]. The changes in body weight gain and excreted corticosterone metabolites could be useful measures of preceding stress. It has been shown, that the increase in corticosterone must be substantial to be detected in faeces [6], [9]. The emphasis of the work presented in this paper is on the modeling of fecal Corticosterone Metabolites secretion in the given time duration using fuzzy Markov Renewal model by two parameter exponential distribution.

2. NOTATION

Φ	–	Scale parameter
Ψ	–	Location parameter
$E(X)$	–	Mean of Markov Renewal Model
$Var(X)$	–	Variance of Markov Renewal Model
$E(\overline{X})$	–	Fuzzy Mean of Markov Renewal Model
$Var(\overline{X})$	–	Fuzzy Variance of Markov Renewal Model
$\overline{\phi}\,[\alpha]$	–	alpha cut of scale parameter
$\overline{\Psi}\,[\alpha]$	–	alpha cut of location parameter

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3. FUZZY MARKOV RENEWAL MODEL

For each n∈ N, let a random variable S_n taken values in a countable set of states E = {1,2,3,…} and a random variable T_n take values in $R = [0, +\infty)$ such that $0 = T_0 \le T_1 \le T_2 \le \cdots$.. The stochastic process, $\{S, T\} = \{S_n, T_n\}$, n∈ N is said to be a Markov renewal process with state space E provided that

$P\{S_{n+1} = j, T_{n+1} - T_n \le t | S_0 \dots \dots .. S_n, T_0, \dots \dots .. T_n\} = P\{S_{n+1} = j, T_{n+1} - T_n \le | S_n\}$ for all n∈ N, j∈ E and t ∈ R
Random variable S_n is given the state of an interarrival time and takes on values from the binary set E = {1, 2}. This is a two-state Markov renewal model where the two types of inter arrival time are sampled according to a Markov chain with state space E. Let $< X_i >$ denote the type of the i^{th} inter arrival time, that is $< X_i >$, i = 1, 2. The transition probability matrix of the Markov chain is

$$P = \begin{bmatrix} a_1 & 1 - a_1 \\ 1 - a_2 & a_2 \end{bmatrix}$$

Where $a_j = P\{< X_i > = j | < X_{i-1} > = j\}, j = 1,2$

Associated with the Markov chain are the limit or equilibrium probabilities $\beta_j = \lim_{i\to\infty} P\{< X_i \ge j\}, j = 1,2$ which are the unconditional probabilities of any interval X_i. Note that $\beta_2 = 1 - \beta_1$. From the theory of Markov chain [5] it is known that

$$\beta_1 = \frac{1 - a_2}{2 - a_1 - a_2}$$

If the conditional probabilities $a_1 and\, a_2$ are equal to the unconditional probabilities $\beta_1 and\, \beta_2$ (in that case $a_1 + a_2 = 1$), the process of the types of interarrival times reduces to a renewal process.

The mean and variance function of the inter arrival times are given by

$$E(X) = \frac{\beta1}{\phi} + \frac{\beta2}{\Psi}$$

$$VAR(X) = \frac{\beta1(1 - \Phi)}{\Phi^2} + \frac{\beta2(1 - \Psi)}{\Psi^2} + \beta1\beta2(\frac{1}{\Phi} - \frac{1}{\Psi})^2$$

We consider the two parameter Exponential distribution with fuzzy parameters by replacing the scale parameter ϕ into the fuzzy number $\overline{\phi}$ and the location parameter Ψ into $\overline{\Psi}$ than the corresponding random variable $\overline{T}$ with Fuzzy Renewal Model Mean and Variance E (M, V) has a function

$$E(\overline{X}) = \frac{\beta1}{\overline{\phi}} + \frac{\beta2}{\overline{\Psi}}$$

$$VAR(\overline{X}) = \frac{\beta1}{\overline{\Phi}^2}(1 - \overline{\Phi}) + \frac{\beta2}{\overline{\Psi}^2}(1 - \overline{\Psi}) + \beta1\beta2\ (\frac{1}{\overline{\Phi}} - \frac{1}{\overline{\Psi}})^2$$

so that for α ε[0,1] the α cuts of Fuzzy Renewal Model Mean function is $\overline{P}[\alpha] = \{P1[\alpha], P2[\alpha]\}$,

Where $P1[\alpha] = \inf\left\{\frac{\beta1}{\overline{\Phi}} + \frac{\beta2}{\overline{\Psi}}\right\}, \overline{\Phi} \in \overline{\Phi}[\alpha], \overline{\Psi} \in \overline{\Psi}[\alpha]$

$P2[\alpha] = \sup\left\{\frac{\beta1}{\overline{\Phi}} + \frac{\beta2}{\overline{\Psi}}\right\}, \overline{\Phi} \in \overline{\Phi}[\alpha], \overline{\Psi} \in \overline{\Psi}[\alpha]$

and the α cuts of Fuzzy Renewal Model Variance function is $\overline{P}[\alpha] = \{P3[\alpha], P4[\alpha]\}$,

Where $P3[\alpha] = \inf\{\frac{\beta1}{\overline{\Phi}^2}(1 - \overline{\Phi}) + \frac{\beta2}{\overline{\Psi}^2}(1 - \overline{\Psi}) + \beta1\beta2(\frac{1}{\overline{\Phi}} - \frac{1}{\overline{\Psi}})^2\}, \overline{\Phi} \in \overline{\Phi}[\alpha], \overline{\Psi} \in \overline{\Psi}[\alpha]$

$P4[\alpha] = \sup\{\frac{\beta1}{\overline{\Phi}^2}(1 - \overline{\Phi}) + \frac{\beta2}{\overline{\Psi}^2}(1 - \overline{\Psi}) + \beta1\beta2(\frac{1}{\overline{\Phi}} - \frac{1}{\overline{\Psi}})^2\}, \overline{\Phi} \in \overline{\Phi}[\alpha], \overline{\Psi} \in \overline{\Psi}[\alpha]$

4. APPLICATION

Let us consider an example of secretion of faecal Corticosterone metabolites [5]. The rats are arrived from the breeder. The breeder is located approximately 50 kilometers from the Biomedical Centre, and the duration of the transport was approximately 45 minutes. The day arrival is referred to as day 0. Body weight was recorded and the rats were put in clean cages in groups of three. Faecal sampling started day 1 at 9 am, followed by sampling at 9 am every 24 hours until day 21. Based on data on Corticosterone Metabolite levels obtained, group size used in the present study was determined by power analysis as described in the literature. A statistical power of 80% was considered satisfactory. Difference in daily faecal Corticosterone metabolite secretion was determined (Fig.: 4.1).

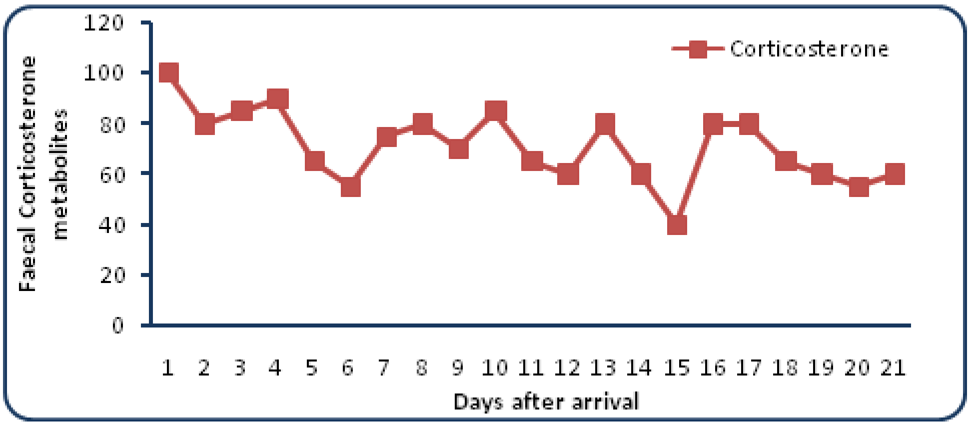

Fig. 4.1: *Secretion of faecal Corticosterone metabolites in µg/24h kg bw*

Body weight changes were analyzed with two parameter Exponential distribution. In some situations the value of the scale and shape parameters of the two parameter Exponential distribution are not known precisely. Therefore we consider triangular numbers for the scale and shape parameter.

The triangular fuzzy number of the scale and the shape parameters respectively are
$\overline{\phi} = [0.05, 0.1, 0.15]$ and $\overline{\Psi} = [0.5, 1, 1.5]$.

The alpha cut of scale and shape parameters respectively are
$\overline{\phi}[\alpha] = [0.05 + 0.05\,\alpha, 0.15 - 0.05\alpha]$ and $\overline{\Psi}[\alpha] = [0.5 + 0.5\alpha, 1.5 - 0.5\alpha]$

Table: 4.1 *Mean of Markov Renewal Model in Excretion pattern of faecal Corticosterone*

α	β1=0.1		β1=0.2		β1=0.3		β1=0.4		β1=0.5	
	P1[α]	P2[α]	P1[α]	P2[α]	P1[α]	P2[α]	P1[α]	P2[α]	P1[α]	P2[α]
0	3.8000	1.2667	5.6000	1.8667	7.4000	2.4667	9.2000	3.0667	11.0000	3.6667
0.1	3.4545	1.3103	5.0909	1.9310	6.7273	2.5517	8.3636	3.1724	10.0000	3.7931
0.2	3.1667	1.3571	4.6667	2.0000	6.1667	2.6429	7.6667	3.2857	9.1667	3.9286
0.3	2.9231	1.4074	4.3077	2.0741	5.6923	2.7407	7.0769	3.4074	8.4615	4.0741
0.4	2.7143	1.4615	4.0000	2.1538	5.2857	2.8462	6.5714	3.5385	7.8571	4.2308
0.5	2.5333	1.5200	3.7333	2.2400	4.9333	2.9600	6.1333	3.6800	7.3333	4.4000
0.6	2.3750	1.5833	3.5000	2.3333	4.6250	3.0833	5.7500	3.8333	6.8750	4.5833
0.7	2.2353	1.6522	3.2941	2.4348	4.3529	3.2174	5.4118	4.0000	6.4706	4.7826
0.8	2.1111	1.7273	3.1111	2.5455	4.1111	3.3636	5.1111	4.1818	6.1111	5.0000
0.9	2.0000	1.8095	2.9474	2.6667	3.8947	3.5238	4.8421	4.3810	5.7895	5.2381
1	1.9000	1.9000	2.8000	2.8000	3.7000	3.7000	4.6000	4.6000	5.5000	5.5000

Table: 4.2 *Variance of Markov Renewal Model in Excretion pattern of faecal Corticosterone*

α	β1=0.1		β1=0.2		β1=0.3		β1=0.4		β1=0.5	
	P3[α]	P4[α]	P3[α]	P4[α]	P3[α]	P4[α]	P3[α]	P4[α]	P3[α]	P4[α]
0	29.7050	2.8750	52.4300	5.5300	68.6750	7.4650	78.4400	8.6800	81.7250	9.1750
0.1	24.5987	3.1478	43.3920	5.9751	56.8299	8.0319	64.9125	9.3181	67.6396	9.8339
0.2	20.7040	3.4454	36.5080	6.4642	47.8120	8.6566	54.6160	10.0224	56.9200	10.5616
0.3	17.6629	3.7715	31.1416	7.0041	40.7859	9.3478	46.5958	10.8027	48.5715	11.3686
0.4	15.2406	4.1306	26.8750	7.6026	35.2033	10.1161	40.2255	11.6710	41.9415	12.2672
0.5	13.2775	4.5281	23.4250	8.2694	30.6925	10.9739	35.0800	12.6416	36.5875	13.2725
0.6	11.6626	4.9705	20.5940	9.0160	26.9941	11.9365	30.8630	13.7320	32.2006	14.4025
0.7	10.3165	5.4658	18.2407	9.8566	23.9228	13.0225	27.3626	14.9634	28.5602	15.6794
0.8	9.1810	6.0238	16.2620	10.8087	21.3430	14.2549	24.4240	16.3621	25.5050	17.1305
0.9	8.2131	6.6567	14.5811	11.8941	19.1541	15.6621	21.9322	17.9607	22.9152	18.7898
1	7.3800	7.3800	13.1400	13.1400	17.2800	17.2800	19.8000	19.8000	20.7000	20.7000

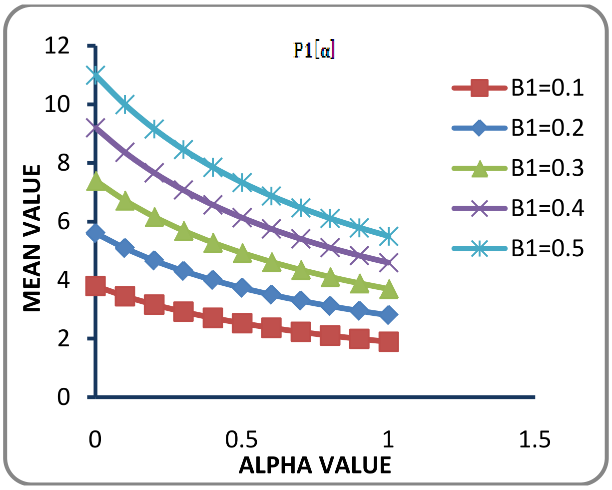

Fig: 4.2 *Lower alpha cut value for Mean*

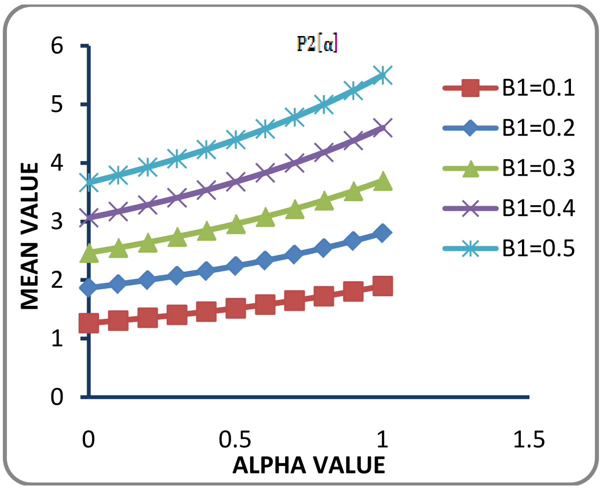

Fig: 4.3 *Upper alpha cut value for Mean*

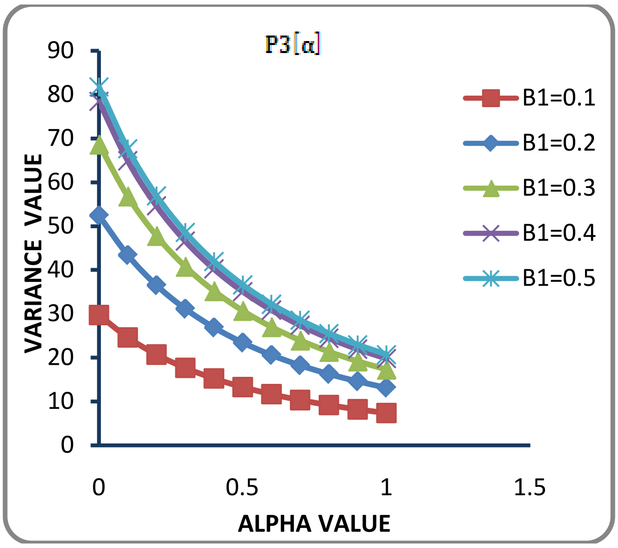

Fig: 4.4 *Lower alpha cut value for Variance*

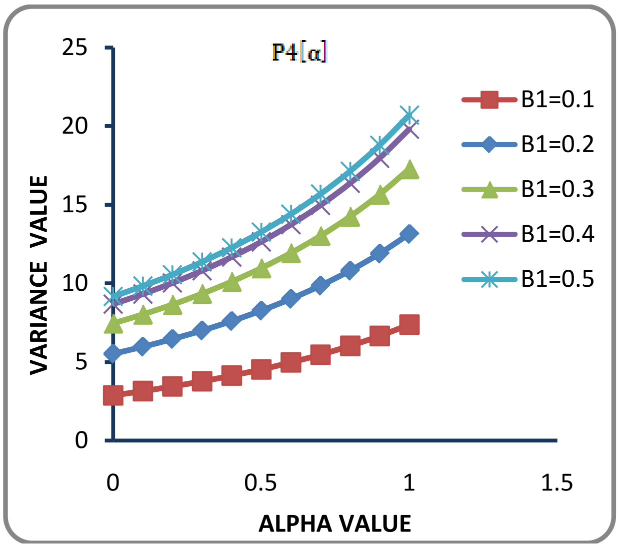

Fig: 4.5 *Upper alpha cut value for Variance*

5. CONCLUSION

In this Study we showed that the fuzzy mean and variance for the faecal Corticosterone Metabolites secretion level is decreased in the lower α cuts and they are increased in the upper α cuts by using fuzzy Markov Renewal model. The result shows that the effect of Secretion of Corticosterone is comparatively higher if Unconditional Probability of time interval in the equilibrium probabilities increases.

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