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ON PROPERTIES OF αc-INTERIOR AND αc-CLOSURE OF SETS

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ABSTRACT

In this paper new concepts namely αc-Neighbourhood, αc-Interior, αc-Limit point, and αc-Closure of sets are introduced and their properties are analyzed. Also αc-continuous mappings are defined and their properties are characterized.

Keywords: *αc-Interior, αc-Closure, αc-compact, αc-continuous.*

AMS Subject classification: *54A05, 54H05, 54C05, 54D30.*

1. INTRODUCTION

The notion of alpha open sets(briefly α-open) was introduced by Njastad [9] in 1965. As an extension of this class, J.S.I Mary and Sindhu[10] developed a new class open sets namely αc-open sets and its topological properties are initialized. Followed by the class of α-open sets, several other related classes such as αg-open sets and $g\alpha$-open sets were defined by Maki, *et.al* [7].

The class of b-open sets is defined and studied by Andrijevic [2] in 1984. As an extension of this class, Hariwan Ibrahim [3] introduced Bc-open sets, and concepts such as Bc-interior, Bc-limit points, and Bc-closure of sets. In this paper we define topological properties namely αc-neighbourhood, αc-interior, αc-closure, and αc-compact of a set. Levine [6] introduced the concept of a semi-open sets and semi-continuous functions. As the extension of this function Alias Khalaf, *et.al* [1] initialized sc-open sets and sc-continuous function and further properties are analyzed. In this paper we introduce and investigate the concept of αc-continuous functions.

2. PRELIMINARIES

Throughout this paper, (X,τ) denote a topological space with topology τ. For a subset A of X the interior of A and closure of A are denoted by $int(A)$ and $cl(A)$ respectively.

Definition 2.1: A subset A of a topological space (X,τ) is called

1. α-$open\ set$ if $A \subset Int(Cl(int(A))$ and α-$closed\ set$ if $Int(Cl(int(A)) \subset A$ [9].
2. $Semi$-$open\ set$ if $A \subset Cl(int(A))$ and $Semi$-$closed\ set$ if $Cl(int(A)) \subset A$[6] .
3. b-$open\ set$ if $A \subset (Int\ Cl(A)) \cup (Cl\ int(A))$ and b-$closed\ set$ if $\left(Int\ Cl(A)\right) \cup \left(Cl\ int(A)\right) \subset A$ [2].
4. θ-$open\ set$ if for each $x \in A$, there exists an open set G such that $x \in G \subset cl(G) \subset$ A [11].

Definition 2.2:

1. The intersection of all semi-closed sets containing A is called the $semi$-$closure\ of\ A$ denoted by $sCl(A)$ [6].
2. The intersection of all α -closed sets containing A is called α-$closure\ of\ A$ denoted by $\alpha Cl(A)$ [9].
3. The intersection of all b-closed sets containing A is called the b-$closure\ of\ A$ denoted by $bCl(A)$ [2].

Definition 2.3: The family of all open sets, semi-open sets, α-open sets, θ-open sets are denoted by $O(X), SO(X), \alpha O(X),\ \theta O(X)$ respectively.

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Definition 2.4: [10] A subset A of a topological space X is called $\boldsymbol{\alpha c}$***-open set*** if for each $x \in A \in \alpha O(X)$, there exists a closed set F, such that $x \in F \subset A$. The family of all αc-open subsets of a topological space (X, τ) is denoted by αcO(X).

Definition 2.5: [9] Let A be a subset of a topological space (X, τ).

1. A point $x \in X$ is said to be α*-interior point of* A, if there exists an α-open set U such that $x \in U \subset A$. The set of all α-interior points of A is called α-interior of A and is denoted by $\alpha Int(A)$.
2. A subset A of X is said to be α*-neighbourhood* of x, if there exists a α-open set U in X such that $x \in U \subset A$.
3. A point $x \in X$ is said to be α*-limit point of* A if for each α-open set U containing x, $U \cap (A \backslash \{x\}) \neq \emptyset$. The set of all α-limit points of A is called α-Derived set of A denoted by $(\alpha D(A))$.

Definition 2.6: [9] A topological space (X,τ) is $\boldsymbol{\alpha}$***-compact*** if for every α-open cover $\{V_\alpha : \alpha \in \Delta\}$ of X, there exist a finite subset Δ_0 *of* Δ such that $X = \cup \{V_\alpha : \alpha \in \Delta_0\}$.

Definition 2.7: [5] The space X is **Hausdorff** if for each pair u, v of distinct points of X, there exists disjoint neighbourhoods U and V containing u and v respectively [15].

Definition 2.8: A topological space X is said to be:

1. ***Locally indiscrete***, if every open subset of X is closed
2. ***Regular*** if for each $x \in X$ and for each open set A containing x, there exists an open set G containing x such that $x \in G \subset cl(G) \subset A$.
3. $\boldsymbol{T_1}$***- space*** if to each pair of distinct points x, y of X there exist a pair of open sets, one containing x but not y and other containing y but not x, as well as is T_1 if and only if for any point $x \in X$, the singleton set $\{x\}$ is closed.

Definition 2.9: A mapping f: $X \to Y$ is said to be

1. ***always*** $\boldsymbol{\alpha}$***-open*** if the image of every α-open set of X is an α-open set in Y.
2. $\boldsymbol{\alpha}$***-open*** if the image of every open set of X is an α-open set in Y.
3. $\boldsymbol{\alpha}$***-continuous*** if the inverse image of every open subset in Y is an α-open set in X[8] .
4. ***clopen-continuous*** if the inverse image of every open subset in Y is an *clopen* set in X [1].
5. $\boldsymbol{\theta}$***-continuous*** if the inverse image of every open subset in Y is an θ-open set in X [11].

Theorem 2.10: [10] Let (X,τ) be a topological space and $\{A_j : j \in \Delta\}$ be a collection of αc-open sets in X. Then $\cup \{A_j : j \in \Delta\}$ is αc-open.

Theorem 2.11: [10] The set A is αc-open in the space (X,τ) if and only if for each $x \in A$, there exists a αc-open set B such that $x \in B \subset A$.

Theorem 2.12: [10] Let $\{B_j : j \in \Delta\}$ be a collection of αc-closed sets in a topological space X. Then $\cap \{B_j : j \in \Delta\}$ is αc-closed set.

Theorem 2.13: [10] Every open set is αc-open set in X, if one of the following holds.

(i) (X,τ) is Locally indiscrete.
(ii) X is Regular.

Theorem 2.14: [10] Every θ-open set of a space X is αc-open.

Theorem 2.15: [10] Let X and Y be two topological spaces and $X \times Y$ be the product topology. If $A \in \alpha cO(X)$ and $B \in \alpha cO(Y)$. Then $A \times B \in \alpha cO(X \times Y)$.

3. *ON αc-INTERIOR AND αc*-CLOSURE OF SETS

In this section, we define and study topological properties of αc-Neighborhood, αc-Interior, αc-Closure and αc-derived of a set using the concept of αc-open sets.

3.1 αc-Neighborhood:

Definition 3.1: Let (X, τ) be a topological space and $x \in X$, then a subset N of X is said to be $\boldsymbol{\alpha c}$***-neighborhood*** of x, if there exists a αc-open set U in X such that $x \in U \subset N$.

The following Theorem gives a characterization of αc-open set with respect to the αc-neighbourhood of each of its points.

Theorem 3.1.1: In a topological space (X, τ), a subset A of X is αc-open set if and only if it is aαc-neighbourhood of each of its points.

Proof: Let A be a αc-open set. By Definition (2.4), for every $x \in A$, $x \in A \subset A$.

Hence A is αc-neighbourhood of each of its points.

Conversely, Let A be a αc-neighbourhood of each of its points. Then for each $x \in A$, there exists an $B_x \in \alpha cO(X)$ such that $x \in B_x \subset A$. Then A=$\cup \{B_x : x \in A\}$ where B_x- is $\alpha cO(X)$. By Theorem (2.10), Since the union of αc-open set is αc-open set. We have A is αc-open set.

Remark 3.1:

1. For any two subsets A and B of a topological space (X, τ) and $A \subset B$, if A is αc-neighborhood of the point $x \in X$, then B is also a αc-neighbourhood of the same point x.
2. Every αc-neighbourhood of a point is α-neighbourhood. It follows from the fact that every αc-open set is α-open.

3.2 αc - Interior points:

In this section we introduce the definition of αc-interior point of a set A as further study of αc - open sets.

Definition 3.2.1: Let A be a subset of a topological space (X, τ). A point $x \in X$ is said to be $\boldsymbol{\alpha c}$***-interior point of*** $\boldsymbol{A}$, if there exists an αc-open set U such that $x \in U \subset A$. The set of all αc-interior points of A is called αc-interior of A is denoted by $\alpha cInt(A)$.

The following Theorem gives the properties of αc-interior of a set.

Theorem 3.2.1: For subsets A, B of a space X, the following statements hold:

(i) $\alpha cInt(A)$ is the union of all αc-open sets contained in A.
(ii) $\alpha cInt(A)$ is an αc-open set in X.
(iii) A is αc-open set if and only if $A = \alpha cInt(A)$.
(iv) $\alpha cInt(\alpha cIntA)) = \alpha cInt(A)$.
(v) $\alpha cInt(\emptyset) = \emptyset$ and $\alpha cInt(X) = X$
(vi) $\alpha cInt(A) \subset A$.
(vii) $If\ A \subset B$, then $\alpha cInt(A) \subset \alpha cInt(B)$.
$(viii)$ $\alpha cInt(A) \cup \alpha cInt(B) \subset \alpha cInt(A \cup B)$
(ix) $\alpha cInt(A \cap B) \subset \alpha cInt(A) \cap \alpha cInt(B)$.
(x) $\alpha cInt(A) \subset \alpha Int(A)$.

Proof:

$(\boldsymbol{i})$ Let $\cup B_i$ be the union of all αc-open sets B_i contained in A. Let $x \in \alpha cInt(A)$, then there exists an αc-open set V such that $x \in V \subset A$. Then for some i , $B_i = V$ implies $x \in \cup B_i$. Thus $\alpha cInt(A) \subset \cup B_i$.

Conversely, Let $x \in \cup B_i$ where B_i's are αc-open set contained in A. Then there exists some i such that $x \in B_i \subset A$, implies $x \in \alpha cInt(A)$. Hence $\alpha cInt(A)$=$\cup B_i$.

$(\boldsymbol{ii})$ By(i), $\alpha cInt(A)$=$\cup B_i$ where B_i is αc-open sets contained in A. Hence by Theorem(2.10), we have $\alpha cInt(A)$ is an αc-open set in X.

$(\boldsymbol{iii})$ Let A be αc-open set. Then By(i), $A \subseteq \alpha cInt(A)$. Conversely, Let A=$\alpha cInt(A)$. By(ii), A is αc- open set.

$(\boldsymbol{iv})$ By(ii), $\alpha cInt(A)$ is αc-open set in X and By(iii), $\alpha cInt(A)$= $\alpha cInt\big(\alpha cInt(A)\big)$.

$(\boldsymbol{v})$ Since $\emptyset$ and X are αc-open sets, from(iii), $\alpha cInt(\emptyset)$=$\emptyset$ and $\alpha cInt(X)$=X.

$(\boldsymbol{vi})$ From(i), $\alpha cInt(A)$=$\cup B_i$ where B_i is αc-open set contained in A. Hence $\cup B_i \subset A$ and by(iii), $\alpha cInt(A)$ is an αc - open set implies $\alpha cInt(A) \subset A$.

$(\boldsymbol{vii})$ Let $x \in \alpha cInt(A)$. Then there exists an αc-open set U such that $x \in U \subset A. A \subset B$ implies $x \in U \subset B$. Thus $x \in \alpha cInt(B)$. Hence $\alpha cInt(A) \subset \alpha cInt(B)$.

$(\boldsymbol{viii})$ Since $A \subset A \cup B$ and $B \subset A \cup B$, by(vii), $\alpha cInt(A) \subset \alpha cInt(A \cup B)$ and $\alpha cInt(B) \subset \alpha cInt(A \cup B)$. Hence $\alpha cInt(A) \cup \alpha cInt(B) \subset \alpha cInt(A \cup B)$.

$(\boldsymbol{ix})$ Since $(A \cap B) \subseteq A$ and $(A \cap B) \subseteq B$, by (vii) $\alpha cInt(A \cap B) \subset \alpha cInt(A) \cap \alpha cInt(B)$.

$(\boldsymbol{x})$ Let $x \in \alpha cInt(A)$, then there exists an αc-open set U such that $x \in U \subset A$. Since every αc-open set is α- open set, we have U is α-open set. It follows that $x \in \alpha Int(A)$. Hence $\alpha cInt(A) \subset \alpha Int(A)$.

3.3 αc-Limit Points:

The concept of limit points is essential to explore more properties of a given set. In this section we introduce αc-limit point of a set induced by αc-open set.

Definition 3.3.1: Let A be a subset of a topological space (X, τ). A point $x \in X$ is said to be ***αc-limitpoint of A*** if for each αc-open set U containing x, $U \cap (A \backslash \{x\}) \neq \emptyset$. The set of all αc-limit points of A is called αc-Derived set of A denoted by $(\alpha cD(A))$.

Theorem 3.3.1: Let A be a subset of X. If for each closed set F of X containing x such that $F \cap (A \backslash \{x\}) \neq \emptyset$, then the point $x \in X$ is an αc-limit point of A.

Proof: Let U be any αc-open set containing x. By the definition (2.4), for each $x \in U$, there exists a closed set F such that $x \in F \subset U$. By hypothesis $F \cap (A \backslash \{x\}) \neq \emptyset$.

Hence $U \cap (A \backslash \{x\}) \neq \emptyset$. Therefore x is an αc-limit point of A.

The following Theorem gives the properties of αc-Derived sets in the space X.

Theorem 3.3.2: Let A and B be subsets of a topological space X. Then the following properties hold:
- (i) $\alpha cD(\emptyset)=\emptyset$.
- (ii) If $x \in \alpha cD(A)$, then $x \in \alpha cD(A \backslash \{x\})$.
- (iii) If $A \subset B$, then $\alpha cD(A) \subset \alpha cD(B)$.
- (iv) $\alpha cD(A) \cup \alpha cD(B) \subset \alpha cD(A \cup B)$.
- (v) $\alpha cD(A \cap B) \subset \alpha cD(A) \cap \alpha cD(B)$.
- (vi) $\alpha cD(\alpha cD(A)) \backslash A \subset \alpha cD(A)$.
- (vii) $\alpha cD(A \cup \alpha cD(A)) \subset A \cup \alpha cD(A)$.
- $(viii)$ $\alpha cD(A) \subset \alpha D(A)$.

Proof:
$(\boldsymbol{i})$ Suppose not, let $x \in \alpha cD(\emptyset)$, then for each αc-open set U containing x, we have $U \cap (\emptyset \backslash \{x\}) \neq \emptyset$. Then $U \cap \emptyset \neq \emptyset$, which is a contradiction.

$(\boldsymbol{ii})$ Let $x \in \alpha cD(A)$, then for each αc-open set U containing x,we have $U \cap (A \backslash \{x\}) \neq \emptyset$.

Since $A \backslash \{x\} = (A \backslash \{x\}) \backslash \{x\}$, $U \cap ((A \backslash \{x\}) \backslash \{x\}) \neq \emptyset$. Hence $x \in \alpha cD(A \backslash \{x\})$.

$(\boldsymbol{iii})$ Let $x \in \alpha cD(A)$, then for each αc-open set U containing x, we have $U \cap (A \backslash \{x\}) \neq \emptyset$.

If $A \subset B$, then $U \cap (A \backslash \{x\}) \subset U \cap (B \backslash \{x\})$. Therefore $U \cap (B \backslash \{x\}) \neq \emptyset$, which implies $x \in \alpha cD(B)$. Hence $\alpha cD(A) \subset \alpha cD(B)$.

$(\boldsymbol{iv})$ As $A \subset A \cup B$, from (iii), $\alpha cD(A) \subset \alpha cD(A \cup B)$. As $B \subset A \cup B$, from(iii), $\alpha cD(B) \subset \alpha cD(A \cup B)$. Hence $\alpha cD(A) \cup \alpha cD(B) \subset \alpha cD(A \cup B)$.

$(\boldsymbol{v})$ Since $A \cap B \subset A$ and $A \cap B \subset B$, by(iii), Hence $\alpha cD(A \cap B) \subset \alpha cD(A) \cap \alpha cD(B)$.

$(\boldsymbol{vi})$ Let $x \in \alpha cD(\alpha cD(A)) \backslash A$ .Then $x \in \alpha cD(\alpha cD(A))$ and $x \notin A$.
Then for each αc-open set U containing x, we have $U \cap (\alpha cD(A) \backslash \{x\}) \neq \emptyset$. There exists $y \in X$ such that $y \in U \cap (\alpha cD(A) \backslash \{x\})$ implies $y \in U$ and $y \in (\alpha cD(A) \backslash \{x\})$.

So y is a αc-limit point of A and $y \in U$. Hence there exists $z \in X$ such that $z \in U \cap (A \backslash \{y\})$ then $z \neq x$ since $x \notin A$ and $z \in A$. Hence $U \cap (A \backslash \{x\}) \neq \emptyset$ implies $x \in \alpha cD(A)$. Thus $\alpha cD(\alpha cD(A)) \backslash A \subset \alpha cD(A)$.

$(\boldsymbol{vii})$ Let $x \in \alpha cD(A \cup \alpha cD(A))$. If $x \in A$ then the result is obvious. If $x \notin A$, then $x \in \alpha cD(A \cup \alpha cD(A))\backslash A$. Then for each αc-open set U containing x, we have $U \cap (A \cup \alpha cD(A)\backslash\{x\}) \neq \emptyset$. Hence $(U \cap A\backslash\{x\}) \cup (U \cap \alpha cD(A)\backslash \{x\}) \neq \emptyset$ implies $U \cap A\backslash\{x\} \neq \emptyset$ or $U \cap (\alpha cD(A)\backslash\{x\}) \neq \emptyset$.Thus $x \in \alpha cD(A)$ (or) $x \in \alpha cD(\alpha cD(A))$. Since $x \notin A$, latter implies $x \in \alpha cD(\alpha cD(A))\backslash A$.

From(vi), since $\alpha cD(\alpha cD(A))\backslash A \subset \alpha cD(A)$, we have $x \in \alpha cD(A)$. Hence in both cases we have $x \in \alpha cD(A)$.

Thus $\alpha cD(A \cup \alpha cD(A)) \subset A \cup \alpha cD(A)$.

$(\boldsymbol{viii})$Let $x \in \alpha cD(A)$, then for each αc-open set U containing x, we have $U \cap (A\backslash\{x\}) \neq \emptyset$. Since every αc-open set is α-open, U is α-open set. Thus $x \in \alpha D(A)$. Hence $\alpha cD(A) \subset \alpha D(A)$.

3.4 αc-Closure:

In this section we define, αc-closure of a set with respect to αc- limit points.

Definition 3.4.1: For any subset A of a topological space X, the ***αc-closure of A*** denoted by$(\alpha cCl(A))$ is defined as the intersection of all αc-closed sets containing A.

Definition 3.4.2: A point $x \in X$ is said to be in αc-closure of A if for each αc-open set U containing x such that $U \cap A \neq \emptyset$.

The following Theorem gives the characterization of αc-closed sets.

Theorem 3.4.1: A subset A of a topological space X is αc-closed set if and only if it contains all of its αc- limit points.

Proof: Let A be an αc-closed set. Suppose A does not contain all of its αc- limit points.

Let x be the αc- limit point of A such that $x \notin A$. Then $x \in X\backslash A$, $X\backslash A$ is αc-open.

This implies $(X\backslash A) \cap (A\backslash\{x\}) \neq \emptyset$. i.e, $(X\backslash A) \cap A \neq \emptyset$ as $x \notin A$, which is a contradiction.

Conversely, Let A contains all of its αc- limit points. Therefore for each $x \in X\backslash A$, x is not an αc- limit point of A.

This implies that there exists an αc-open set U containing x such that $U \cap (A\backslash\{x\}) = \emptyset$. $x \notin A$ implies $U \cap A = \emptyset$.

This implies $x \in U \subset X\backslash A$. By Theorem (2.11), we have $X\backslash A$ is αc-open set. Hence A *is* αc-closed.

Theorem 3.4.2: Let A be a subset of a topological space X. Then $\alpha cCl(A) = A \cup \alpha cD(A)$.

Proof: First let us show that $A \cup \alpha cD(A) \subset \alpha cCl(A)$.

We know that $A \subset \alpha cCl(A)$. By the definition of the αc-closure of A, $A \subset \cap B_i$, where B_i- is αc-closed set containing A. Since $A \subset \cap B_i$, by Theorem 3.3.2(iii), $\alpha cD(A) \subset \alpha cD(\cap B_i)$. From Theorem3.3.2$(v)$, $\alpha cD(A) \subset \cap\, \alpha cD(B_i) = \cap B_i$, since each B_i *is* αc-closed containing A. Thus $\alpha cD(A) \subset \alpha cCl(A)$. Hence $A \cup \alpha cD(A) \subset \alpha cCl(A)$

On the other hand Suppose $x \in \alpha cCl(A)$. Since $\alpha cCl(A)$ is the smallest αc-closed set containing x, it is sufficient to show that $A \cup \alpha cD(A)$ is αc-closed set.

Let $x \in X\backslash(A \cup \alpha cD(A))$, then $x \notin A \cup \alpha cD(A)$. This implies that $x \notin A$ and $x \notin \alpha cD(A)$. $G_x \cap (A\backslash\{x\}) = \phi$. $x \notin A$ implies that $G_x \cap A = \phi$. Then

$$G_x \subset X\backslash A \tag{3.4.1}$$

Again, since G_x is an αc-open set of each of its points and $G_x \subset X\backslash A$, no points of G_x is an αc-limit point of A implies $G_x \nsubseteq \alpha cD(A)$. Hence

$$G_x \subseteq X\backslash \alpha cD(A) \tag{3.4.2}$$

From (3.4.1) and (3.4.2), we have $G_x \subseteq (X\backslash A) \cap (X\backslash \alpha cD(A))$.

For all $x \in X\backslash(A \cup \alpha cD(A))$, there exists an αc-open set G_x containing x such that $x \in G_x \subseteq X\backslash(A \cup \alpha cD(A))$.

This implies that $X\backslash(A \cup \alpha cD(A))$ is αc-open. Hence $A \cup \alpha cD(A)$ is αc-closed. Since $A \subseteq A \cup \alpha cD(A)$, we have $\alpha cCl(A) \subseteq A \cup \alpha cD(A)$. Hence $\alpha cCl(A) = A \cup \alpha cD(A)$.

Corollary 3.4.2: Let A be a subset of a topological space X. A point $x \in X$ is in the αc-closure of A if and only if $A \cap U \neq \emptyset$ for every αc-open set U containing x.

Proof: By definition (3.4.2), implies $A \cap U \neq \emptyset$ for every αc-open set U containing x.

Conversely, Suppose $x \notin \alpha cCl(A)$. Then by Theorem (3.4.2), $x \notin A \cup \alpha cD(A)$ implies $x \notin A$ and $x \notin \alpha cD(A)$. Thus there exists an αc-open set U containing x such that $U \cap (A \backslash \{x\}) = U \cap A = \emptyset$, which is a contradiction.

Theorem 3.4.3: Let A be any subset of a space X. If $A \cap F \neq \emptyset$ for every closed set F of X containing x, then the point x is in the αc-closure of A.

Proof: Assume that U be any αc-open set containing x, by Definition (2.4), there exists a closed set F such that $x \in F \subset A$. By hypothesis $A \cap F \neq \emptyset$ implies $A \cap U \neq \emptyset$ for every αc-open set$U$ containing x.

By Corollary (3.4.2), $x \in \alpha cCl(A)$.

The following Theorem gives the properties of αc-Closure of sets.

Theorem 3.4.4: For subsets A, B of a space X, the following statements are true.

- (i) αc-closure of A is the intersection of all αc-closed sets containing A.
- (ii) $A \subset \alpha cCl(A)$.
- (iii) $\alpha cCl(A)$ is an αc-closed set in X.
- (iv) A is αc-closed if and only if $A = \alpha cCl(A)$.
- (v) $\alpha cCl(\alpha cCl(A)) = \alpha cCl(A)$.
- (vi) $\alpha cCl(\emptyset) = \emptyset$ and $\alpha cCl(X) = X$.
- (vii) If $A \subset B$, then $\alpha cCl(A) \subset \alpha cCl(B)$.
- $(viii)$ If $\alpha cCl(A) \cap \alpha cCl(B) = \emptyset$, then $A \cap B = \emptyset$.
- (ix) $\alpha cCl(A) \cup \alpha cCl(B) \subset \alpha cCl(A \cup B)$.
- (x) $\alpha cCl(A \cap B) \subset \alpha cCl(A) \cap \alpha cCl(B)$.

Proof:
$(\boldsymbol{i})$ and $(\boldsymbol{ii})$ are obvious.

$(\boldsymbol{iii})$ By the definition of $\alpha cCl(A)$, $\alpha cCl(A) = \cap B_i$ where B_i is the αc-closed set containing A. By Theorem (2.12), $\cap B_i$ is αc- closed. Hence $\alpha cCl(A)$ is αc-closed in X.

$(\boldsymbol{iv})$ Let A be αc-closed set. Since $A \subset A$ and A is αc-closed set, we have from $\alpha cCl(A) = \cap F$ with $A \subset F$ and F is αc-closed set that $\alpha cCl(A) = A$.

Conversely, Let $A = \alpha cCl(A)$, By(iii) we have A is αc-closed set in X.

$(\boldsymbol{v})$ From(iii), $\alpha cCl(A)$ is αc-closed set in X. From(iv), we have, $\alpha cCl(\alpha cCl(A)) = \alpha cCl(A)$.

$(\boldsymbol{vi})$ Since $\emptyset$ and X are αc-closed sets, from(iv), we have $\alpha cCl(\emptyset) = \emptyset$ and $\alpha cCl(X) = X$.

$(\boldsymbol{vii})$ Let $x \in \alpha cCl(A)$. By Corollary (3.4.2), $A \cap U \neq \emptyset$ for each αc-open set U containing x. If $A \subset$ B, then $B \cap U \neq \emptyset$. Hence $x \in \alpha cCl(B)$. Thus $\alpha cCl(A) \subset \alpha cCl(B)$.

$(\boldsymbol{viii})$ Supppose $A \cap B \neq \emptyset$, then $x \in A \cap B$ implies $x \in \alpha cCl(A \cap B)$ .Then for all αc- open sets U containing x, $(A \cap B) \cap U \neq \emptyset$ implies $(A \cap U) \cap (B \cap U) \neq \emptyset$. Consequently $A \cap U \neq \emptyset$ and $B \cap U \neq \emptyset$. By Corollary(3.4.2), $x \in \alpha cCl(A)$ and $x \in \alpha cCl(B)$. Thus $x \in \alpha cCl(A) \cap \alpha cCl(B)$, which is a contradiction.

$(\boldsymbol{ix})$ Since $A \subset A \cup B$ and $B \subset A \cup B$, by(vii), $\alpha cCl(A) \cup \alpha cCl(B) \subset \alpha cCl(A \cup B)$.

$(\boldsymbol{x})$ Since $(A \cap B) \subseteq A$ and $(A \cap B) \subseteq B$, by(vii), $\alpha cCl(A \cap B) \subset \alpha cCl(A)$ and $\alpha cCl(A \cap B) \subset \alpha cCl(B)$.
Thus $\alpha cCl(A \cap B) \subset \alpha cCl(A) \cap \alpha cCl(B)$.

Proposition 3.4.5: For any subset A of a topological space X, the following statements are true:

- (i) $X \backslash \alpha cCl(A) = \alpha cInt(X \backslash A)$.
- (ii) $X \backslash \alpha cInt(A) = \alpha cCl(X \backslash A)$.
- (iii) $\alpha cCl(A) = X \backslash \alpha cInt(X \backslash A)$.
- (iv) $\alpha cInt(A) = X \backslash \alpha cCl(X \backslash A)$.

Proof:
$(\boldsymbol{i})$ Let $x \in X\backslash\alpha cCl(A)$.Then by Corollary(3.4.2) $x \notin \alpha cCl(A) \Leftrightarrow$ There exists an αc-open set U containing x such that $A \cap U = \emptyset \Leftrightarrow x \in U \subset X\backslash A \Leftrightarrow x \in \alpha cInt(X\backslash A)$.

$(\boldsymbol{ii})$ From(i), $\alpha cInt(A) = X\backslash\alpha cCl(X\backslash A)$. This implies that $X\backslash\alpha cInt(A)$= $\alpha cCl(X\backslash A)$.

$(\boldsymbol{iii})$ and $(\boldsymbol{iv})$ follows from (i) and (ii).

3.5. Filter Space:

In this chapter we introduce several definitions on convergent and accumulation of a filter base.

Definition 3.5.1: [4] A filter is a non-empty collection $\mathfrak{F}$ of subsets of a topological space X such that
i. $\emptyset \notin \mathfrak{F}$
ii. If $A \in \mathfrak{F}$ and $B \supseteq A$, then $B \in \mathfrak{F}$.
iii. If $A \in \mathfrak{F}$ and $B \in \mathfrak{F}$,then $A \cap B \in \mathfrak{F}$.

The following definitions are introduced.

Definition 3.5.2: A subset A of a topological space X is called $\boldsymbol{\theta c}$ ***-open set*** (denoted by $\boldsymbol{\theta} cO(X)$) if for each $x \in A \in \theta O(X)$, there exists a closed set F, such that $x \in F \subset A$.

Definition 3.5.3: Let $\mathfrak{F}$ be a filter base in a topological space (X, τ). We say $\mathfrak{F}$,
(i)*αc-converges* to a point $x \in X$ if for every αc-open set V containing x, there exists an $F \in \mathfrak{F}$ such that $F \subset V$.
(ii) *θc-converges* to a point $x \in X$ if for every θc-open set V containing x, there exists an $F \in \mathfrak{F}$ such that $F \subset V$.
(iii) *αc-accumulates* to a point $x \in X$ if $F \cap V \neq \emptyset$ for every αc-open set V containing x and every $F \in \mathfrak{F}$.
(iv) *θc-accumulates* to a point $x \in X$ if $F \cap V \neq \emptyset$, for every θc-open set V containing x and every $F \in \mathfrak{F}$.

The following Theorem gives the properties of αc-convergent and αc-accumalation of filter base in (X, τ).

Theorem 3.5.1: Let $\mathfrak{F}$ be a filter base in a topological space (X, τ). The following assertion hold.
(i) If $\mathfrak{F}$ αc-converges to a point x, then $\mathfrak{F}$ θc-converges to the point x.
(ii) If $\mathfrak{F}$ αc-accumulates to a point x, then $\mathfrak{F}$ θc-accumulates to the point x.

Proof:
$(\boldsymbol{i})$ Let $\mathfrak{F}$ αc-converge to a point $x \in X$, and V be any θc-open set containing x. By definition (3.5.2) and Theorem (2.14), V is αc-open set. Since $\mathfrak{F}$ αc-converges to x, there exist an $F \in \mathfrak{F}$ such that $F \subset V$. This shows that $\mathfrak{F}$ θc-converges to x.

$(\boldsymbol{ii})$ Let $\mathfrak{F}$ αc-accumulate to a point $x \in X$, and V be any θc-open set containing x. By definition (3.5.2) and Theorem(2.14), implies V is αc-open set. Since $\mathfrak{F}$ αc-accumulates to x, $F \cap V \neq \emptyset$ for every $F \in \mathfrak{F}$. This shows that $\mathfrak{F}$ θc-accumulates **to** a point x.

Theorem 3.5.2: Let $\mathfrak{F}$ be a filter base in a topological space (X, τ) and E be any closed set containing x. Then the following statements hold.
(i)If there exist an $F \in \mathfrak{F}$, such that $F \subset E$, then $\mathfrak{F}$ αc-converges to $x \in X$.
(ii) If for each $F \in \mathfrak{F}$, such that $F \cap E \neq \emptyset$, then $\mathfrak{F}$ θc-accumulates to $x \in X$.

Proof:
$(\boldsymbol{i})$ Let V be any αc-open set V containing x. Then for each $x \in V$, there exist a closed set E such that $x \in E \subset V$. By hypothesis, there exists an $F \in \mathfrak{F}$, such that $F \subset E \subset V$. Hence $\mathfrak{F}$ αc-converges to x.

$(\boldsymbol{ii})$ Let V be any θc-open set containing x. Then for each $x \in V$, there exist a closed set E such that $x \in E \subset V$. By hypothesis, for every $F \in \mathfrak{F}$, $F \cap E \neq \emptyset$. Then $F \cap V \neq \emptyset$. Hence $\mathfrak{F}$ θc-accumulates to $x \in X$.

3.6 αc-Compactness:

We introduce two types of compactness namely αc-compactness and θc-compactness.

Definition 3.6.1: A topological space (X,τ) is $\boldsymbol{\alpha c}$***-compact*** if for every αc-open cover $\{V_\alpha : \alpha \in \Delta\}$ of X, there exist a finite subset Δ_0 of Δ such that X=∪ $\{V_\alpha : \alpha \in \Delta_0\}$.

Definition 3.6.2: A topological space (X,τ) is ***θc-compact*** if for every θc-open cover $\{V_\alpha : \alpha \in \Delta\}$ of X, there exist a finite subset Δ_0 of Δ such that $X = \cup \{V_\alpha : \alpha \in \Delta_0\}$.

The following Theorem gives the properties of αc-compactness.

Theorem 3.6.1: If every closed cover of a space X has finite subcover, then X is αc-compact.

Proof: Let $\{ V_\alpha : \alpha \in \Delta \}$ be any αc-open cover of X and $x \in X$, then for each $x \in V_\alpha(x)$, $\alpha \in \Delta$ there exist a closed set $F_\alpha(x)$ such that $x \in F_\alpha(x) \subset V_\alpha(x)$. So the family $\{ F_\alpha(x) : x \in X \}$ is a cover of X by closed sets. By hypothesis, this family has a finite sub-cover such that $X = \cup \{ F_\alpha(x_i) : (i=1,2,\ldots.n) \} \subset \cup \{ V_\alpha(x_i) : (i=1,2,\ldots.n) \}$.

Therefore $X = \cup \{ V_\alpha(x_i) : (i=1,2,\ldots.n) \}$. Hence X is αc-compact.

Theorem 3.6.2: Let (X, τ) be αc-compact. The following properties hold:
(i)If the space X is Locally indiscrete, then X is compact.
(ii) If the space X is Regular, then X is compact.

Proof:
$(\boldsymbol{i})$ Let $\{ V_\alpha : \alpha \in \Delta \}$ be any open cover of X. Since every open set is α-open, this implies that $\{ V_\alpha : \alpha \in \Delta\}$ is a α-open cover of X. Since the space X be locally indiscrete, Every open subset of X is closed. This implies that $\{ V_\alpha : \alpha \in \Delta\}$ is a αc-open cover of X. By hypothesis, X is αc-compact. So there exists a finite subset Δ_0 of Δ such that $X = \cup \{ V_\alpha : \alpha \in \Delta_0\}$. Hence X is compact.

$(\boldsymbol{ii})$ Let $\{ V_\alpha : \alpha \in \Delta \}$ be any open cover of X. Since the space X is Regular, by Theorem (2.13), Every open set is αc-open. This implies that every $\{ V_\alpha : \alpha \in \Delta\}$ is a αc-open cover of X. By hypothesis, X is αc-compact.So there exists a finite subset Δ_0 of Δ such that X=$\cup \{ V_\alpha : \alpha \in \Delta_0\}$. Hence X is compact.

Theorem 3.6.3: If a topological space (X, τ) be αc-compact, then it is is θc-compact.

Proof: Let $\{V_\alpha : \alpha \in \Delta \}$ be any θc open cover of X. By definition(3.5.2) and Theorem(2.14) $\{V_\alpha : \alpha \in \Delta \}$ is a αc-open cover of X. Since X is αc-compact, there exists a finite subset Δ_0 of Δ in X such that $X = \cup \{ V_\alpha : \alpha \in \Delta_0\}$.

Hence X is θc- compact.

Theorem 3.6.4: Let (X, τ) be a topological space, then α-compactness implies αc-compactness.

Proof: Let $\{V_\alpha : \alpha \in \Delta \}$ be any αc- open cover of X. Since every αc-open set is α-open set, $\{V_\alpha : \alpha \in \Delta \}$ is a α- open cover of X. Since X is α-compact, there exists a finite subset Δ_0 of Δ in$X$ such that $X = \cup \{ V_\alpha : \alpha \in \Delta_0\}$. Hence X is αc-compact.

Theorem 3.6.5: Every αc-compact space that is T_1-space must be α-compact.

Proof: Let X be αc-compact and T_1-space. Let $\{V_\alpha : \alpha \in \Delta \}$ be any α- open cover of X. Then for every $x \in X$, there exists $\alpha(x) \in \Delta$ such that $x \in V_\alpha(x)$. Since X is T_1-space every singleton set is closed. Then$\{ V_\alpha(x)\}$ is closed. Therefore for each $x \in V_\alpha(x) \subset V_\alpha(x)$.Thus $V_\alpha(x)$ is αc-open cover of X.

Since X is αc-compact, there exists a finite subset Δ_0 of Δ in X such that $X = \cup \{ V_\alpha : \alpha \in \Delta_0\}$. Hence X is α- compact.

3.7 αc- CONTINUOUS FUNCTIONS

In this chapter we introduce the αc-Continuous functions.

Definition 3.7.1: A function $f: X \to Y$ is called αc-continuous at a point $x \in X$ if for each open set V of Y containing $f(x)$, there exists an αc-open set U of X contaning x such that $f(U) \subseteq V$. If f is αc-continuos at every point x of X, then it is called αc-continuous.

Theorem 3.7.1: A function $f : X \to Y$ is αc-continuous if and only if the inverse image of every open set in Y is αc-open in X.

Proof: Let f be αc-continuous and V be open set in Y. Let $x \in f^{-1}(V)$. This implies $f(x) \in V$. Hence by definition, there exists an αc-open set U_x in X containing x such that $f(U_x) \subseteq V$. Therefore $f^{-1}(V) = \cup (U_x)$. Since by Theorem (2.10), we have f^{-1}(V) is αc-open in X.

Conversely, let us assume that $f^{-1}(V)$ is αc-open in X for every open set V in Y.

Let V be open in Y. By assumption, $f^{-1}(V)$ is αc-open in X. Let $U = f^{-1}(V)$, then $f(U) = f(f^{-1}(V)) \subseteq V$. Hence f is αc-continuous.

The following Theorem gives the characterization of αc-continuois function.

Theorem 3.7.2: A function $f : X \to Y$ is αc-continuous if and only if f is α-continuous and for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a closed set F of X containig x such that $f(F) \subseteq V$.

Proof: Let $f : X \to Y$ is αc-continuous, then it is α-continuous. Let $x \in X$ and V be any open set of Y containning $f(x)$. By hypothesis, there exists an αc-open set U of X containing x such that $f(U) \subseteq V$. Since U is αc-open set, then for each $x \in U$, there exists a closed set F of X such that $x \in F \subseteq U$. Therefore, we have $f(F) \subseteq V$.

Conversely, let V be any open set of Y. We have to show that $f^{-1}(V)$ is αc-open set in X. Since f is α-continuous, then $f^{-1}(V)$ is α-open set in X. Let $x \in f^{-1}(V)$. Then $f(x) \in V$. By hypothesis, there exists a closed set F of X containig x such that $f(F) \subseteq V$, which implies that $x \in F \subseteq f^{-1}(V)$. Therefore, $f^{-1}(V)$ is αc-open set in X. Thus, f is αc-continuous.

Theorem 3.7.3: Let $f : X \to Y$ be an αc-continuous and $Y \subseteq Z$. If Y is an open subset of a topological space Z, then $f : X \to Z$ is αc-continuous.

Proof: Let V be an open set in Z. Then $V \cap Y$ is open in Y. Since f is αc-continuous, by Theorem (3.7.1), $f^{-1}(V \cap Y)$ is αc-open set in X. But $f(x) \in Y$ for each $x \in X$, and thus $f^{-1}(V) = f^{-1}(V \cap Y)$ is an αc-open subset of X. Therefore, by Theorem (3.7.1), $f : X \to Z$ is αc-continuous.

Theorem 3.7.4: Let $f, g : X \to Y$ be functions and Y is Hausdorff. If f is αc-continuous, and g is clopen continuous, then the set $E = \{x \in X : f(x) = g(x)\}$ is αc-closed in X.

Proof: Let $x \notin E$. Then $f(x) \neq g(x)$. Since Y is Hausdorff, there exist open sets V_1 and V_2 of Y such that $f(x) \subseteq V_1$, $g(x) \subseteq V_2$, and $V_1 \cap V_2 = \emptyset$. Since f is αc-continuous, there exists an αc-open set U_1 of X containing x such that $f(U_1) \subseteq V_1$. Since g is clopen continuous, there exists a clopen set U_2 of X containing x such that $f(U_2) \subseteq V_2$. Put $U = U_1 \cap U_2$ is an αc-open set of X containing x, By definition (3.4.2), $U \cap E = \emptyset$. Therefore, we obtain $x \notin \alpha cCl(E)$. This shows that E is αc-closed in X.

Theorem 3.7.5: Let $f : X \to Y$ and $g : Y \to Z$ be two functions in which f is αc-continuous and g is continuous. Then the composition function $g \circ f : X \to Z$ is αc-continuous.

Proof: Let W be any open subset of Z. Since g is continuous $g^{-1}(W)$ is open subset of Y. Since f is αc-continuous, then by Theorem(3.7.1), $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is αc-open subset set in X. Therefore, by Theorem(3.7.1), $g \circ f$ is αc-continuous.

Theorem 3.7.6: Let $f : X_1 \to Y$ and $g : X_2 \to Y$ be two αc-continuous. If Y is Hausdroff, then the set $E = \{(x_1, x_2) \in X_1 \times X_2 : f(x_1) = g(x_2)\}$ is αc-closed in the product space $X_1 \times X_2$.

Proof: Let $(x_1, x_2) \notin E$. Then $f(x_1) \neq g(x_2)$. Since Y is Hausdorff, there exist open sets V_1 and V_2 of Y such that $f(x_1) \subseteq V_1$, $g(x_2) \subseteq V_2$, and $V_1 \cap V_2 = \emptyset$. Since f and g are αc-continuous, there exists an αc-open set U_1 and U_2 of X_1 and X_2 containing x_1 and x_2 such that $f(U_1) \subseteq V_1$ and $g(U_2) \subseteq V_2$, respectively. Put $U = U_1 \times U_2$, then $(x_1, x_2) \in U$ and by Theorem(2.15), U is αc-open set in $X_1 \times X_2$ with $U \cap E = \emptyset$.

Therefore, we obtain $(x_1, x_2) \notin \alpha cCl(E)$. Hence E is αc-closed in the product space $X_1 \times X_2$.

REFERENCES:

1. Alias Khalaf B., Zanyar Ameen A., sc-Open sets and sc-Continuity in topological spaces, Journal of Advanced Research in Pure Mathematics, Vol. 2, No. 3, (2010), 87-101.
2. Andrijevic D., Some properties of the topology of α-sets, Math. Vesnik, (1984), 1-10.
3. Hariwan Ibrahim Z., Bc-open sets in topological spaces, Advances in Pure Mathematics, Vol. 3, No. 1, (2013), 34-40.
4. Hyland J.M.E., Filter spaces and continuous Funtionals, University of Cambridge, England Received (1977).
5. James Munkers R., Topology, Pearson Education, Inc., Prentice Hall (2013).
6. Levine N., Semi-Open Sets and Semi-Continuity in Topological Spaces, American Mathematical Monthly, Vol.70, No. 1, (1963), 36-41.

7. Maki H., Devi R and Balachandran K., Generalized α-closed sets in topology, Bull. Fukuoka Univ. Ed., Part-III, 42 (1993), 13-21.
8. Mashhour A.S., Hasanein I.A and El-Deeb S.N., α-continuous and α-open Mappings, Acta Mathe. Hungarica, 41, (1983), 213-218.
9. Njastad O., On Some Classes of Nearly Open Sets, Pacific Journal of Mathematics, Vol.15, No. 3, (1965), 961- 970.
10. Stella Irene Mary J. and Sindhu M., On Some properties of αc - open sets in Topological spaces, International Journal of Mathematical Archive, Vol.6(11), (2015), 135-141
11. VelickoN. V., H-Closed Topological Spaces, American Mathematical society, Vol. 78, No. 2, (1968), 103-118.

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