

**(1, 2)\*-GENERALIZED CLOSED SETS IN FUZZY BITOPOLOGICAL SPACES**

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**ABSTRACT**

*In this paper, a new class of fuzzy sets called (1, 2)\*-fuzzy generalized closed set and (1, 2)\*-fuzzy  $\ddot{g}$ -closed set in fuzzy bitopological space are introduced and study their relations with various generalized (1, 2)\*-fuzzy closed sets*

*Key words: Fuzzy bitopological space, (1, 2)\*-fg-closed, (1, 2)\*- $\ddot{g}$ -closed set.*

**1. INTRODUCTION**

The concept of fuzzy sets was introduced by L.A. Zadeh in his classical paper [21]. After the discovery of the fuzzy subsets, much attention has been paid to generalize the basic concepts of classical topology in fuzzy setting and thus modern theory of fuzzy topology is developed. The notion of fuzzy subsets naturally plays a very significant role in image processing and in the study of fuzzy topology which was introduced by C.L. Chang [5] in 1968. In 1989, [9] Kandil introduced the concept of fuzzy bitopological spaces and since then many concepts in bitopological space have been extended of fuzzy bitopological space. In 1970, [10] Levine introduced generalized closed sets in general topology as a generalization of closed sets. Recently Ravi et al [14], Ravi and Thivagar [13] and Duszynski *et. al* [6] introduced (1,2)\*-g-closed sets, (1,2)\*-sg-closed sets and (1,2)\*- $\hat{g}$ -closed sets in bitopological space respectively. In this paper, we introduce (1,2)\*-fuzzy generalized set and (1,2)\*-fuzzy  $\ddot{g}$ -closed set and study their relations with various generalized (1,2)\*-fuzzy closed sets.

**2. PRELIMINARIES**

In this section, we list out the definitions and the results which are needed in sequel. Fuzzy sub sets in  $(X, \tau_1, \tau_2)$  will be denoted by  $\lambda, \mu, \gamma$ .

**Definition 2.1:** A fuzzy subset set  $\lambda$  of  $(X, \tau)$  is called:

- (i) Fuzzy semiopen (briefly, fs-open) if  $\lambda \leq \text{Cl}(\text{Int}(\lambda))$  and a fuzzy semiclosed (briefly, fs-closed) if  $\text{Int}(\text{Cl}(\lambda)) \leq \lambda$  [1];
- (ii) Fuzzy preopen (briefly, fp-open) if  $A \leq \text{Int}(\text{Cl}(\lambda))$  and a fuzzy preclosed (briefly, fp-closed) if  $\text{Cl}(\text{Int}(\lambda)) \leq \lambda$  [4];
- (iii) Fuzzy  $\alpha$ -open (briefly,  $\alpha$ -open) if  $\lambda \leq \text{IntCl}(\text{Int}(\lambda))$  and a fuzzy  $\alpha$ -closed (briefly,  $\alpha$ -closed) if  $\text{ClInt}(\text{Cl}(\lambda)) \leq \lambda$  [4];
- (iv) Fuzzy semi-preopen (briefly, fsp-open) if  $\lambda \leq \text{ClInt}(\text{Cl}(\lambda))$  and a fuzzy semi-preclosed (briefly, fsp-closed) if  $\text{IntCl}(\text{Int}(\lambda)) \leq \lambda$  [20].

By FSPO  $(X, \tau)$ , we denote the family of all fuzzy semi-preopen sets of fts X. The semiclosure (resp.  $\alpha$ -closure, semi-preclosure) of a fuzzy set A of  $(X, \tau)$  is the intersection of all fs-closed (resp.  $\alpha$ -closed, fsp-closed) sets that contain A and is denoted by  $s\text{Cl}(\lambda)$  (resp.  $\alpha\text{Cl}(\lambda)$  and  $sp\text{Cl}(\lambda)$ ).

**Lemma 2.2:** Let  $\lambda$  be a fuzzy set in a fuzzy topological space  $(X, \tau)$ . Then

- (i)  $\alpha\text{Cl}(\lambda) = \lambda \vee \text{ClIntCl}(\lambda)$ .
- (ii)  $s\text{Cl}(\lambda) = \lambda \vee \text{IntCl}(\lambda)$ .
- (iii)  $\text{PCl}(\lambda) \geq \lambda \vee \text{ClInt}(\lambda)$ .
- (iv)  $\text{SPCl}(\lambda) \geq \lambda \vee \text{IntClInt}(\lambda)$ .

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**Definition 2.3:** A fuzzy set  $\lambda$  in a fuzzy topological space  $(X, \tau)$  is called:

- (i) [2] Fuzzy generalized closed set if  $Cl\lambda \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is fuzzy open. We briefly denote it as fg-closed set.
- (ii) [11] Fuzzy semi-generalized closed set if  $sCl\lambda \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is fuzzy semiopen. We briefly denote it as fsg-closed set.
- (iii) [9] Fuzzy generalized almost strongly semi-closed set if  $\alpha Cl\lambda \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is fuzzy  $\alpha$ -open. We briefly denote it as  $\alpha$ fg-closed set.
- (iv) [19] Fuzzy semi-pre-generalized closed set if  $spCl\lambda \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is fuzzy semi-pre-open. We briefly denote it as fspg-closed set.
- (v) [3] Fuzzy generalized strongly closed set if  $\alpha Cl\lambda \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is fuzzy open. We briefly denote it as  $\alpha$ fg-closed set.
- (vi) [7] Fuzzy  $\ddot{g}$ -closed set (briefly  $f\ddot{g}$ -closed set) if  $cl(\lambda) \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is fsg-open.
- (vii) [8] Fuzzy  $\alpha$ gs-closed set if  $\alpha cl(\lambda) \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is fuzzy semi-open. We briefly denote it as  $\alpha$ gs-closed set.
- (viii) [18] Fuzzy generalized semi-closed set if  $sCl\lambda \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is fuzzy open. We briefly denote it as fgs-closed set.
- (ix) [18] Fuzzy generalized semi-pre-closed set if  $spCl\lambda \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is fuzzy open. We briefly denote it as fgsp-closed set.

**Definition 2.4:** [16] Let  $S$  be a fuzzy subset of  $(X, \tau_1, \tau_2)$ . Then  $S$  is said to be  $\tau_{1,2}$ -fuzzy open if  $S = \lambda \cup \mu$  where  $\lambda \in \tau_1$  and  $\mu \in \tau_2$ . The complement of  $\tau_{1,2}$ -fuzzy open set is called  $\tau_{1,2}$ -fuzzy-closed.

**Definition 2.5:** [16] Let  $S$  be a fuzzy subset of a Fuzzy Bitopological space  $X$ . Then

- (i) the  $\tau_{1,2}$ -closure of  $S$ , denoted by  $\tau_{1,2}\text{-cl}(S)$ , is defined as  $\bigcap \{F : S \leq F \text{ and } F \text{ is } \tau_{1,2}\text{-closed}\}$ .
- (ii) the  $\tau_{1,2}$ -interior of  $S$ , denoted by  $\tau_{1,2}\text{-int}(S)$ , is defined as  $\bigcup \{F : F \leq S \text{ and } F \text{ is } \tau_{1,2}\text{-open}\}$ .

**Definition 2.6:** A fuzzy subset  $\lambda$  of a bts  $(X, \tau_1, \tau_2)$  is called

- (i) (1,2)\*-sg-closed set [13] if (1,2)\*-scl( $\lambda$ )  $\subset$   $\mu$  whenever  $\lambda \subset \mu$  and  $\mu$  is (1,2)\*-fuzzy semi-open in  $X$ . The complement of (1,2)\*-sg-closed set is called (1,2)\*-sg-open set;
- (ii) (1,2)\*-gs-closed set [13] if (1,2)\*-scl( $\lambda$ )  $\subset$   $\mu$  whenever  $\lambda \subset \mu$  and  $\mu$  is  $\tau_{1,2}$ -open in  $X$ . The complement of (1,2)\*-gs-closed set is called (1,2)\*-gs-open set;
- (iii) (1,2)\*- $\alpha$ g-closed set [15] if (1,2)\*- $\alpha$ cl( $\lambda$ )  $\subset$   $\mu$  whenever  $\lambda \subset \mu$  and  $\mu$  is  $\tau_{1,2}$ -open in  $X$ . The complement of (1,2)\*- $\alpha$ g-closed set is called (1,2)\*- $\alpha$ g-open set;
- (iv) (1,2)\*- $\hat{g}$ -closed set or  $\omega$ -closed set [6] if  $\tau_{1,2}\text{-cl}(\lambda) \subset \mu$  whenever  $\lambda \subset \mu$  and  $\mu$  is (1,2)\*-s-open in  $X$ . The complement of (1,2)\*- $\hat{g}$ -closed (resp.  $\omega$ -closed) set is called (1,2)\*- $\hat{g}$ -open (resp.  $\omega$ -open) set;
- (v) (1,2)\*- $\psi$ -closed set [12] if (1,2)\*-scl( $\lambda$ )  $\subset$   $\mu$  whenever  $\lambda \subset \mu$  and  $\mu$  is (1,2)\*-fsg-open in  $X$ . The complement of (1,2)\*- $\psi$ -closed set is called (1,2)\*- $\psi$ -open set;
- (vi) (1,2)\*- $\ddot{g}_\alpha$ -closed set [12] if (1,2)\*- $\alpha$ cl( $\lambda$ )  $\subset$   $\mu$  whenever  $\lambda \subset \mu$  and  $\mu$  is (1,2)\*-sg-open in  $X$ . The complement of (1,2)\*- $\ddot{g}_\alpha$ -closed set is called (1,2)\*- $\ddot{g}_\alpha$ -open set;
- (vii) (1,2)\*-gsp-closed set [17] if (1,2)\*-spcl( $\lambda$ )  $\subset$   $\mu$  whenever  $\lambda \subset \mu$  and  $\mu$  is  $\tau_{1,2}$ -open in  $X$ . The complement of (1,2)\*-gsp-closed set is called (1,2)\*-gsp-open set;
- (viii) (1,2)\*-g-closed set [14] if  $\tau_{1,2}\text{-cl}(\lambda) \subset \mu$  whenever  $\lambda \subset \mu$  and  $\mu$  is  $\tau_{1,2}$ -open in  $X$ . The complement of (1,2)\*-g-closed set is called (1,2)\*-g-open set;

### 3. (1, 2)\*-FUZZY GENERALIZED CLOSED SET

We introduce the following definitions.

**Definition 3.1:** A fuzzy subset  $\lambda$  of a fbts  $(X, \tau_1, \tau_2)$  is called

- (i) (1,2)\*-fsg-closed set if (1,2)\*-scl( $\lambda$ )  $\leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is (1,2)\*-fuzzy semi-open in  $X$ . The complement of (1,2)\*-fsg-closed set is called (1,2)\*-fsg-open set;
- (ii) (1,2)\*-fgs-closed set if (1,2)\*-scl( $\lambda$ )  $\leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is  $\tau_{1,2}$ -open in  $X$ . The complement of (1,2)\*-fgs-closed set is called (1,2)\*-fgs-open set;
- (iii) (1,2)\*- $f\alpha$ g-closed set if (1,2)\*- $\alpha$ cl( $\lambda$ )  $\leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is  $\tau_{1,2}$ -open in  $X$ . The complement of (1,2)\*- $f\alpha$ g-closed set is called (1,2)\*- $f\alpha$ g-open set;
- (iv) (1,2)\*- $f\hat{g}$ -closed set or  $f\omega$ -closed set if  $\tau_{1,2}\text{-cl}(\lambda) \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is (1,2)\*-fs-open in  $X$ . The complement of (1,2)\*- $f\hat{g}$ -closed (resp.  $f\omega$ -closed) set is called (1,2)\*- $f\hat{g}$ -open (resp.  $f\omega$ -open) set;
- (v) (1,2)\*- $f\psi$ -closed set if (1,2)\*-scl( $\lambda$ )  $\leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is (1,2)\*-fsg-open in  $X$ . The complement of (1,2)\*- $f\psi$ -closed set is called (1,2)\*- $f\psi$ -open set;

- (vi) (1,2)\*-f  $\ddot{g}$   $\alpha$ -closed set if (1,2)\*-  $\alpha \text{ cl}(\lambda) \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is (1,2)\*-fsg-open in X. The complement of (1,2)\*-f  $\ddot{g}$   $\alpha$ -closed set is called (1,2)\*-f  $\ddot{g}$   $\alpha$ -open set;
- (vii) (1,2)\*-fsgsp-closed set if (1,2)\*-  $\text{spcl}(\lambda) \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is  $\tau_{1,2}$ -open in X. The complement of (1,2)\*-fsgsp-closed set is called (1,2)\*-fsgsp-open set;
- (viii) (1,2)\*-fags-closed set if (1,2)\*- $\alpha \text{cl}(\lambda) \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is (1,2)\*-fs-open in X. The complement of (1,2)\*-f  $\alpha$  gs-closed set is called (1,2)\*-f  $\alpha$  gs-open set;

**Definition 3.2:** A fuzzy subset  $\lambda$  of a fbts X is called (1,2)\*-fg closed set if  $\tau_{1,2} - \text{cl}(\lambda) \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is  $\tau_{1,2} -$  open subset in X . The complement of (1,2)\*- fg- closed set is called (1,2)\*- fg-open set.

**Definition 3.3:** A fuzzy subset  $\lambda$  of a fbts X is called (1,2)\*-f  $\ddot{g}$  -closed set if  $\tau_{1,2}\text{-cl}(\lambda) \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is (1,2)\*-fsg-open in X. The complement of (1,2)\*-f  $\ddot{g}$  -closed set is called (1,2)\*-f  $\ddot{g}$  -open set.

**Proposition 3.4:** Every (1,2)\*-f  $\alpha$  -closed set is (1,2)\*-f  $\ddot{g}$   $\alpha$ -closed.

**Proof :** If  $\lambda$  is a (1,2)\*-f  $\alpha$  closed and  $\mu$  is any (1,2)\*-fsg-open set Containing  $\lambda$  , we have  $\mu \geq \tau_{1,2}\text{-cl}(\lambda) \geq \alpha \text{cl}(\lambda)$ . Hence  $\lambda$  is (1,2)\*-f  $\ddot{g}$   $\alpha$ -closed in X.

The converse of Proposition 3.4 need not be true as seen from the following example.

**Example 3.5:** Consider the fbts  $(X, \tau_1, \tau_2)$  where  $X = \{a, b\}$

$$\tau_1 = \left\{ 0, 1, \mu = \frac{0.3}{a} + \frac{0.4}{b} \right\}; \tau_2 \{0, 1\}$$

$$\tau_{1,2} - \text{open set} = \left\{ 0, 1, \mu = \frac{0.3}{a} + \frac{0.4}{b} \right\} \text{ and } \tau_{1,2} - \text{closed set} = \left\{ 0, 1, \mu' = \frac{0.7}{a} + \frac{0.6}{b} \right\}$$

Let  $\lambda = \frac{0.4}{a} + \frac{0.5}{b}$  be any fuzzy subset of X. Since  $\text{CIIntCl}(\lambda) = \mu' \not\leq \lambda$  . Therefore  $\lambda$  is not (1,2)\*-f $\alpha$ -closed but it is not (1,2)\*-f  $\ddot{g}$   $\alpha$ -closed, for  $\alpha \text{Cl}(\lambda) = \mu' \leq 1$  which is (1,2)\*-fsg- open set containing  $\lambda$ .

**Proposition 3.6:** Every (1,2)\*-f  $\ddot{g}$   $\alpha$ -closed is (1,2)\*-fags-closed

**Proof** If  $\lambda$  is a (1,2)\*-f  $\ddot{g}$   $\alpha$ -closed subset of X and  $\mu$  is any (1,2)\*-fs-open set containing  $\lambda$ , since every (1,2)\*-fs-open set is (1,2)\*-fsg-open, we have (1,2)\*- $\alpha \text{cl}(\lambda) \leq \mu$ . Hence  $\lambda$  is (1,2)\*-fags-closed in X.

The converse of Proposition 3.6 need not be true as seen from the following example.

**Example 3.7:** Consider the fbts  $(X, \tau_1, \tau_2)$  where  $X = \{a, b\}$

$$\tau_1 = \left\{ 0, 1, \mu = \frac{0.4}{a} + \frac{0.6}{b} \right\}; \tau_2 \{0, 1\}$$

$$\tau_{1,2} - \text{open set} = \left\{ 0, 1, \mu = \frac{0.4}{a} + \frac{0.6}{b} \right\}$$

$$\text{and } \tau_{1,2} \text{ closed set} = \left\{ 0, 1, \mu' = \frac{0.6}{a} + \frac{0.4}{b} \right\}$$

$$\tau_{1,2} \text{-closed set containing } \lambda = \frac{0.4}{a} + \frac{0.5}{b} \text{ is } 1. \text{ Therefore } \tau_{1,2} - \text{cl}(\lambda) = 1$$

$$\tau_{1,2} - \text{open set containing } \lambda \text{ are } 1 \text{ and } \mu. \text{ Therefore } \tau_{1,2} - \text{cl}(\lambda) = 1 \not\leq \mu$$

Therefore  $\lambda$  is not (1,2)\*-f  $\ddot{g}$  -closed but it is (1,2)\*-fags-closed, for  $\alpha \text{cl}(\lambda) = 1 \leq 1$ , which is (1,2)\*-fs-open.

**Proposition 3.8:** Every (1,2)\*-fags-closed set is (1,2)\*-fag-closed.

**Proof:** If  $\lambda$  is (1,2)\*-fags-closed subset of  $X$  and  $\mu$  is any  $\tau_{1,2}$  - open set containing  $\lambda$ , since every  $\tau_{1,2}$  - open set is (1,2)\*- fs-open, we have  $\text{acl}(\lambda) \leq \mu$ . Hence  $\lambda$  is (1,2)\*-fag-closed in  $X$ .

The converse of Proposition 3.8 need not be true as seen from the following example.

**Example 3.9:** Consider the fbts  $(X, \tau_1, \tau_2)$  where  $X = \{a, b\}$

$$\tau_1 = \left\{ 0, 1, \mu = \frac{0.3}{a} + \frac{0.6}{b} \right\}; \tau_2 = \{0, 1\}$$

$$\tau_{1,2} \text{ - open set} = \left\{ 0, 1, \mu = \frac{0.3}{a} + \frac{0.6}{b} \right\} \text{ and } \tau_{1,2} \text{ - closed set} = \left\{ 0, 1, \mu' = \frac{0.7}{a} + \frac{0.4}{b} \right\}$$

$$\tau_{1,2} \text{-closed set containing } \lambda = \frac{0.4}{a} + \frac{0.5}{b} \text{ is } 1. \text{ Therefore } \tau_{1,2} \text{-cl}(\lambda) = 1$$

$\tau_{1,2}$  - open set containing  $\lambda$  is 1. Therefore  $\tau_{1,2} \text{-cl}(\lambda) = 1 \leq 1$ , hence  $\lambda$  is (1,2)\*-fag-closed but it is not (1,2)\*-fags-closed.

**Proposition 3.10:** Every (1,2)\*-fuzzy closed set is (1,2)\*-f  $\ddot{g}$  -closed.

**Proof:** If  $\lambda$  is a (1,2)\*-fuzzy closed subset set of  $X$  and  $\mu$  is any fsg-open set such that  $\mu \geq \lambda = \tau_{1,2} \text{-cl}(\lambda)$ . Hence  $\lambda$  is (1,2)\*- f  $\ddot{g}$  - closed.

The converse of Proposition 3.10 need not be true as seen from the following example.

**Example 3.11:** Consider the fbts  $(X, \tau_1, \tau_2)$  where  $X = \{a, b, c\}$

$$\tau_1 = \left\{ 0, 1, \lambda = \frac{0.7}{a} + \frac{0.3}{b} + \frac{0.8}{c} \right\} \text{ and } \tau_2 = \left\{ 1, 0, \mu = \frac{0.3}{a} + \frac{0.5}{b} + \frac{0.7}{c} \right\},$$

$$\tau_{1,2} \text{ - open sets are } \left\{ 0, 1, \lambda, \mu, \frac{0.7}{a} + \frac{0.5}{b} + \frac{0.8}{c} \right\}$$

$$\tau_{1,2} \text{-closed set are } \left\{ 0, 1, \lambda' = \frac{0.3}{2} + \frac{0.7}{b} + \frac{0.2}{c}, \mu = \frac{0.7}{a} + \frac{0.5}{b} + \frac{0.3}{c}, \frac{0.3}{a} + \frac{0.5}{b} + \frac{0.2}{c} \right\}.$$

Let  $\gamma = \frac{.8}{a} + \frac{.9}{b} + \frac{.8}{c}$  be fuzzy set in  $X$ . since  $\tau_{1,2} \text{cl}(\gamma) = 1 \leq 1$ , 1 is  $\tau_{1,2}$ -open in  $X$ . Thus  $\gamma$  is (1,2)\*-f  $\ddot{g}$  - closed but it is not  $\tau_{1,2}$ -fuzzy closed.

**Proposition 3.12:** Every (1,2)\*-f  $\ddot{g}$  - closed set is (1,2)\*-f  $\psi$  -closed.

**Proof:** If  $\lambda$  is a (1,2)\*-f  $\ddot{g}$  - closed subset of  $X$  and  $\mu$  is any fs -open set such that  $\mu \geq \lambda$ , since every fs- open set is fsg-open, we have  $\mu \geq \tau_{1,2} \text{-cl}(\lambda)$ . Hence  $\lambda$  is (1,2)\*-f  $\psi$  -closed in  $X$ .

The converse of Proposition 3.12 need not be true as seen from the following example.

**Example 3.13:** Consider the fbts  $(X, \tau_1, \tau_2)$  where  $X = \{a, b\}$

$$\tau_1 = \left\{ 0, 1, \lambda = \frac{0.4}{a} + \frac{0.7}{b} \right\}; \tau_2 = \{0, 1\}. \tau_{1,2} \text{ - open set} = \{0, 1, \lambda\}.$$

$$\text{and } \tau_{1,2} \text{ - closed set} = \left\{ 0, 1, \lambda' = \frac{0.6}{a} + \frac{0.3}{b} \right\}$$

Let  $\mu = \left\{ \frac{0.4}{a} + \frac{0.3}{b} \right\}$  be a fuzzy subset of  $X$ . Clearly  $\mu$  is (1,2)\*-f  $\ddot{g}$  -closed but not (1,2)\*-f  $\psi$  - closed.

**Proposition 3.14:** Every  $(1,2)^* \text{-} \ddot{g}$ -closed set is  $(1,2)^* \text{-} \ddot{g} \alpha$  closed.

**Proof:** If  $\lambda$  is a  $(1,2)^* \text{-} \ddot{g}$ -closed subset of  $X$  and  $\mu$  is any fsg-open set such that  $\lambda \leq \mu$ , then  $\mu \geq cl(\lambda) \geq \alpha cl(\lambda)$ . Hence  $\lambda$  is  $(1,2)^* \text{-} \ddot{g} \alpha$  closed.

The converse of Proposition 3.14 need not be true as seen from the following example.

**Example 3.15:** Consider the fbts  $(X, \tau_1, \tau_2)$  where  $X = \{a, b\}$

$$\tau_1 = \left\{ 0, 1, \mu = \frac{0.4}{a} + \frac{0.7}{b} \right\}; \tau_2 = \{0, 1\}. \tau_{1,2} \text{- Open set} = \{0, 1, \mu\}. \tau_{1,2} \text{- Closed set} = \{0, 1, \mu'\}. \text{ Let } \lambda = \frac{0.4}{a} + \frac{0.3}{b}$$

a fuzzy subset of  $X$ .  $\tau_{1,2}$ -open set containing  $\lambda$  are 1 and  $\mu$ .  $\tau_{1,2}$ -closed set containing  $\lambda$  are 1 and  $\mu'$ .

Therefore  $\tau_{1,2} \text{-} cl(\lambda) = \mu' \not\leq \mu$ . Hence  $\lambda$  is not  $(1,2)^* \text{-} \ddot{g}$ -closed but its  $(1,2)^* \text{-} \ddot{g} \alpha$ -closed.

**Proposition 3.16:** Every  $(1, 2)^* \text{-} \omega$ -closed set is  $(1,2)^* \text{-} fg$ -closed.

**Proof:** If  $\lambda$  is a  $(1,2)^* \text{-} \omega$ -closed subset of  $X$  and  $\mu$  is any  $(1,2)^* \text{-}$  fuzzy open set containing  $\lambda$ , then  $\mu \geq \tau_{1,2} \text{-} cl(\lambda)$ . Hence  $\lambda$  is  $(1,2)^* \text{-} fg$ -closed in  $X$ .

The converse of Proposition 3.16 need not be true as seen from the following example.

**Example 3.17:** Consider the fbts  $(X, \tau_1, \tau_2)$  where  $X = \{a, b\}$

$$\tau_1 = \left\{ 0, 1, \mu = \frac{0.5}{a} + \frac{0.5}{b} \right\}; \tau_2 = \{0, 1\}$$

$$\tau_{1,2} \text{-open set} = \left\{ 0, 1, \mu = \frac{0.5}{a} + \frac{0.5}{b} \right\} \text{ and } \tau_{1,2} \text{ closed set} = \left\{ 0, 1, \mu' = \frac{0.5}{a} + \frac{0.5}{b} \right\}$$

$$\tau_{1,2} \text{-closed set containing } \lambda = \frac{0.4}{a} + \frac{0.4}{b} \text{ are } 1 \text{ \& } \mu'. \text{ Therefore } \tau_{1,2} \text{-} cl(\lambda) = \mu'$$

$$\tau_{1,2} \text{-open set containing } \lambda \text{ are } 1 \text{ and } \mu. \text{ Therefore } \tau_{1,2} \text{-} cl(\lambda) = \mu' \leq \mu \text{ \& } 1$$

Therefore  $\lambda$  is  $(1, 2)^* \text{-} fg$ -closed but not  $(1, 2)^* \text{-} \omega$ -closed.

**Proposition 3.18:** Every  $(1,2)^* \text{-} \psi$ -closed set is  $(1,2)^* \text{-} fsg$  closed.

**Proof:** If  $\lambda$  is a  $(1,2)^* \text{-} \psi$ -closed subset of  $X$  and  $\mu$  is any  $(1,2)^* \text{-} fs$ -open set containing  $\lambda$ , we have  $(1,2)^* \text{-} scl(\lambda) \leq \mu$ . Hence  $\lambda$  is  $(1,2)^* \text{-} fsg$ -closed.

The converse of Proposition 3.18 need not be true as seen from the following example.

**Example 3.19:** Consider the fbts  $(X, \tau_1, \tau_2)$  where  $X = \{a, b\}$

$$\tau_1 = \left\{ 0, 1, \lambda = \frac{0.4}{a} + \frac{0.7}{b} \right\}; \tau_2 = \{0, 1\}. \tau_{1,2} \text{- open set} = \{0, 1, \lambda\}. \tau_{1,2} \text{- closed set} = \{0, 1, \lambda'\}. \text{ Let } \mu = \frac{0.4}{a} + \frac{0.3}{b}$$

a fuzzy subset of  $X$ .  $\tau_{1,2}$ -open set containing  $\mu$  are 1 and  $\lambda$ .  $\tau_{1,2}$ -closed set containing  $\mu$  are 1 and  $\lambda'$ .

Therefore  $\tau_{1,2} \text{-} cl(\mu) = \lambda' \not\leq \mu$ . Hence  $\mu$  is not  $(1,2)^* \text{-} \psi$ -closed but its  $(1,2)^* \text{-} fsg$ -closed, for  $Int \lambda' \leq \mu \leq \lambda'$  hence  $\mu$  is semi closed. Therefore  $Scl(\mu) = \mu \leq 1 \text{ \& } \lambda$ .

**Proposition 3.20:** Every  $(1,2)^* \text{-} fg$ -closed set is  $(1,2)^* \text{-} fgs$ -closed.

**Proof :** If  $\lambda$  is a  $(1,2)^* \text{-} fg$ -closed subset of  $X$  and  $\mu$  is any  $\tau_{1,2}$ -open set containing  $\lambda$ , since every  $\tau_{1,2}$ -open set is  $(1,2)^* \text{-} fsg$ -open, we have  $\mu \geq \tau_{1,2} \text{-} cl(\lambda) \geq (1,2)^* \text{-} scl(\lambda)$ . Hence  $\lambda$  is  $(1,2)^* \text{-} fgs$ -closed in  $X$ .

The converse of Proposition 3.20 need not be true as seen from the following example.

**Example 3.21:** Consider the fbts  $(X, \tau_1, \tau_2)$  where  $X = \{a, b\}$

$$\tau_1 = \left\{ 0, 1, \mu = \frac{0.4}{a} + \frac{0.6}{b} \right\}; \tau_2 = \{0, 1\}$$

$$\tau_{1,2} \text{-open set} = \left\{ 0, 1, \mu = \frac{0.4}{a} + \frac{0.6}{b} \right\} \text{ and } \tau_{1,2} \text{ closed set} = \left\{ 0, 1, \mu' = \frac{0.6}{a} + \frac{0.4}{b} \right\}$$

$$\tau_{1,2} \text{-closed set containing } \lambda = \frac{0.4}{a} + \frac{0.5}{b} \text{ is } 1. \text{ Therefore } \tau_{1,2} \text{-} cl(\lambda) = 1$$

$$\tau_{1,2} \text{-open set containing } \lambda \text{ are } 1 \text{ and } \mu. \text{ Therefore } \tau_{1,2} \text{-} cl(\lambda) = 1 \not\leq \mu$$

Therefore  $\lambda$  is not (1,2)\*-fg-closed. Also  $\text{int } \mu' \leq \lambda \leq \mu'$ , hence  $\lambda$  is semi closed. Therefore  $scl(\lambda) = \lambda \leq 1 \& \mu$ .

Hence  $\lambda$  is (1,2)\*-fsg-closed.

**Proposition 3.22:** Every (1,2)\*-fg-closed set is (1,2)\*-fag-closed.

**Proof:** If  $\lambda$  is a (1,2)\*-fg-closed subset of  $X$  and  $\mu$  is any  $\tau_{1,2}$ -open set containing  $\lambda$ . we have  $\mu \geq \tau_{1,2} cl(\lambda) \geq (1,2)^* \text{-} \alpha cl(\lambda)$ . Hence  $\lambda$  is (1,2)\*-fag-closed in  $X$ .

The converse of Proposition 3.22 need not be true as seen from the following example.

**Example 3.23:** Consider the fbts  $(X, \tau_1, \tau_2)$  where  $X = \{a, b\}$

$$\tau_1 = \left\{ 0, 1, \lambda = \frac{0.4}{a} + \frac{0.7}{b} \right\}; \tau_2 = \{0, 1\}. \tau_{1,2} \text{-open set} = \{0, 1, \lambda\}. \tau_{1,2} \text{-closed set} = \{0, 1, \lambda'\}.$$

$$\text{Let } \mu = \frac{0.4}{a} + \frac{0.3}{b} \text{ a fuzzy subset of } X. \tau_{1,2} \text{-open set containing } \mu \text{ are } 1 \text{ and } \lambda. \tau_{1,2} \text{-closed set containing } \mu \text{ are } 1 \text{ and } \lambda'.$$

$$\text{Therefore } \tau_{1,2} \text{-} cl(\mu) = \lambda' \not\leq \lambda. \text{ Hence } \mu \text{ is not (1,2)*-fg-closed but its (1,2)*-fag-closed for, } \alpha cl(\lambda) = \lambda \leq 1 \& \mu.$$

**Proposition 3.24:** Every (1,2)\*-fuzzy  $\omega$ -closed set is (1,2)\*-fuzzy sg-closed.

**Proof:** If  $\lambda$  is a (1,2)\*-fuzzy  $\omega$ -closed subset of  $X$  and  $\mu$  is any (1,2)\*-fuzzy semi open set containing  $\lambda$ , we have  $\mu \geq \tau_{1,2} cl(\lambda) \geq (1,2)^* \text{-} scl(\lambda)$ . Hence  $\lambda$  is (1,2)\*-fuzzy sg-closed in  $X$ .

The converse of Proposition 3.24 need not be true as seen from the following example.

**Example 3.25:** Consider the fbts  $(X, \tau_1, \tau_2)$  where  $X = \{a, b\}$

$$\tau_1 = \left\{ 0, 1, \mu = \frac{0.4}{a} + \frac{0.6}{b} \right\}; \tau_2 = \{0, 1\}$$

$$\tau_{1,2} \text{-open set} = \left\{ 0, 1, \mu = \frac{0.4}{a} + \frac{0.6}{b} \right\} \text{ and } \tau_{1,2} \text{ closed set} = \left\{ 0, 1, \mu' = \frac{0.6}{a} + \frac{0.4}{b} \right\}$$

$$\tau_{1,2} \text{-closed set containing } \lambda = \frac{0.4}{a} + \frac{0.4}{b} \text{ is } 1 \& \mu. \text{ Therefore } \tau_{1,2} \text{-} cl(\lambda) = \mu' \not\leq \mu$$

Hence  $\lambda$  is not a (1,2)\*-f $\omega$ -closed. Also  $\text{Int } \mu' \leq \lambda \leq \mu'$ , hence  $\lambda$  is semi closed. Therefore  $scl(\lambda) = \lambda \leq 1 \& \mu$ . Hence  $\lambda$  is (1,2)\*-fsg-closed.

**Proposition 3.26:** Every (1,2)\*-fuzzy sg-closed set is (1,2)\*-fuzzy gs-closed.

**Proof:** If  $\lambda$  is (1,2)\*-fuzzy sg-closed subset of  $X$  and  $\mu$  is any  $\tau_{1,2}$ -open set containing  $\lambda$ , since every  $\tau_{1,2}$ -open set is (1,2)\*-fs-open, we have  $scl(\lambda) \leq \mu$ . Hence  $\lambda$  is (1,2)\*-fuzzy-gs-closed.

The converse of proposition 3.26 need not be true as seen from the following example.

**Example 3.27:** Consider the fbts  $(X, \tau_1, \tau_2)$  where  $X = \{a, b\}$

$$\tau_1 = \left\{ 0, 1, \mu = \frac{0.3}{a} + \frac{0.6}{b} \right\}; \tau_2 = \{0, 1\}$$

$$\tau_{1,2}\text{-open set} = \left\{ 0, 1, \mu = \frac{0.3}{a} + \frac{0.6}{b} \right\} \text{ and } \tau_{1,2}\text{ closed set} = \left\{ 0, 1, \mu' = \frac{0.7}{a} + \frac{0.4}{b} \right\}$$

Let  $\lambda = \frac{0.3}{a} + \frac{0.4}{b}$  be any fuzzy subset of X. Therefore  $\text{Int } \mu' \leq \lambda \leq \mu'$ , hence  $\lambda$  is semi closed.

Therefore  $\text{scl}(\lambda) = \lambda \leq 1 \& \mu$ . Hence  $\lambda$  is (1,2)\*- fgs-closed, but it is not (1,2)\*-fuzzy sg-closed.

**Proposition 3.28:** Every (1,2)\*-fuzzy  $\omega$ - closed set is (1,2)\*-fags closed.

**Proof:** If  $\lambda$  is (1,2)\*-fuzzy  $\omega$ -closed and  $\mu$  is any (1,2)\*-fuzzy semi open set containing  $\lambda$ , we have  $\alpha\text{cl}(\lambda) \leq \tau_{1,2}\text{-cl}(\lambda) \leq \mu$ . Hence  $\lambda$  is (1,2)\*-fags-closed.

The converse of the proposition 3.28 is not true as seen from the following example.

**Example 3.29:** Consider the fbts  $\{X, \tau_1, \tau_2\}$  where  $X = \{a, b\}$ .

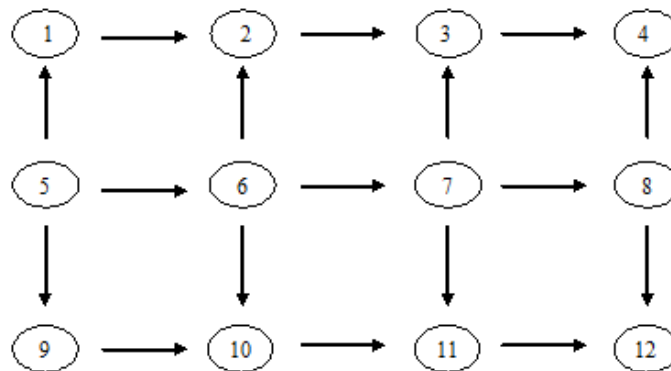
$$\tau_1 = \left\{ 0, 1, \mu = \frac{0.4}{a} + \frac{0.7}{b} \right\}; \tau_2 = \{0, 1\}$$

$$\tau_{1,2}\text{-open set} = \left\{ 0, 1, \mu = \frac{0.4}{a} + \frac{0.7}{b} \right\} \text{ and } \tau_{1,2}\text{ closed set} = \left\{ 0, 1, \mu' = \frac{0.6}{a} + \frac{0.3}{b} \right\}$$

Let  $\lambda = \frac{0.4}{a} + \frac{0.3}{b}$  be any fuzzy subset of X. Therefore  $\tau_{1,2}$ -closed set containing  $\lambda = \frac{0.4}{a} + \frac{0.3}{b}$  is  $1 \& \mu'$ .

Therefore  $\tau_{1,2}\text{-cl}(\lambda) = \mu' \not\leq \mu$ . Hence  $\lambda$  is not a (1,2)\*-f $\omega$ -closed but it is (1,2)\*-fags –closed, for  $\alpha\text{cl}(\lambda) = \lambda \leq 1 \& \mu$ .

From the above propositions (3.4, 3.6, 3.8, 3.10, 3.12, 3.14, 3.16, 3.18, 3.20, 3.22, 3.24, 3.26, 3.28), Examples (3.5, 3.7, 3.9, 3.11, 3.13, 3.15, 3.17, 3.19, 3.21, 3.23, 3.25, 3.27, 3.29) we obtain the following diagram, where  $A \rightarrow B$  (resp.  $A \leftarrow B$ ) represents A implies B but not conversely (resp. A and B are independent of each other).



Where

- |  |                                  |
|--|----------------------------------|
| (1) is (1,2)*-f $\alpha$ -closed             | (7) is (1,2)*-f $\omega$ -closed |
| (2) is (1,2)*- f $\ddot{g}$ $\alpha$ -closed | (8) is (1,2)*-f g-closed         |
| (3) is (1,2)*- f $\alpha$ gs-closed          | (9) is (1,2)*-fs-closed          |
| (4) is (1,2)*- f $\alpha$ g-closed           | (10) is (1,2)*-f $\psi$ -closed  |
| (5) is (1,2)*- fuzzy closed                  | (11) is (1,2)*-fsg-closed        |
| (6) is (1,2)*-f $\ddot{g}$ -closed           | (12) is (1,2)*-fgs-closed        |

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