International Journal of Mathematical Archive-7(1), 2016, 29-36
Available online through www.ijma.info ISSN 2229 – 5046

(1, 2)*-GENERALIZED CLOSED SETS IN FUZZY BITOPOLOGICAL SPACES

P. SARAVANAPERUMAL*[1], S. MURUGESAN[2]

[1]Department of Mathematics,
Sri Vidya College of Engineering and Technology, Virudhunagar-626 005, India.

[2]Department of Mathematics, Sri. S. R. Naidu Memorial College, Sattur-626 203, India.

(Received On: 19-12-15; Revised & Accepted On: 28-01-16)

ABSTRCT

In this paper, a new class of fuzzy sets called (1, 2)-fuzzy generalized closed set and (1, 2)*-fuzzy $\ddot{g}$ -closed set in fuzzy bitopological space are introduced and study their relations with various generalized (1, 2)*-fuzzy closed sets*

***Key words**: Fuzzy bitopological space, (1, 2)*-fg- closed, (1, 2)*-f $\ddot{g}$ -closed set.*

1. INTRODUCTION

The concept of fuzzy sets was introduced by L.A. Zadeh in his classical paper [21]. After the discovery of the fuzzy subsets, much attention has been paid to generalize the basic concepts of classical topology in fuzzy setting and thus modern theory of fuzzy topology is developed. The notion of fuzzy subsets naturally plays a very significant role in image processing and in the study of fuzzy topology which was introduced by C.L. Chang [5] in 1968. In 1989, [9] Kandil introduced the concept of fuzzy bitopological spaces and since then many concepts in bitopological space have been extended of fuzzy bitopological space. In 1970, [10] Levine introduced generalized closed sets in general topology as a generalization of closed sets. Recently Ravi et al [14], Ravi and Thivagar [13] and Duszynski *et. al* [6] introduced (1,2)*-g-closed sets, (1,2)*-sg-closed sets and (1,2)*- $\hat{g}$ -closed sets in bitopological space respectively. In this paper, we introduce (1,2)*-fuzzy generalized set and (1,2)*-fuzzy $\ddot{g}$ -closed set and study their relations with various generalized (1,2)*-fuzzy closed sets.

2. PRELIMINARIES

In this section, we list out the definitions and the results which are needed in sequel. Fuzzy sub sets in (X, τ_1, τ_2) will be denoted by λ, μ, γ .

Definition 2.1: A fuzzy subset set λ of (X, τ) is called:
(i) Fuzzy semiopen (briefly, fs-open) if $\lambda \leq Cl(Int(\lambda))$ and a fuzzy semiclosed (briefly, fs-closed) if $Int(Cl(\lambda)) \leq \lambda$ [1];
(ii) Fuzzy preopen (briefly, fp-open) if $A \leq Int(Cl(\lambda))$ and a fuzzy preclosed (briefly, fp-closed) if $Cl(Int(\lambda)) \leq \lambda$ [4];
(iii) Fuzzy α-open (briefly, fα-open) if $\lambda \leq IntCl(Int(\lambda))$ and a fuzzy α-closed (briefly, fα-closed) if $ClInt(Cl(\lambda) \leq \lambda$ [4];
(iv) Fuzzy semi-preopen (briefly, fsp-open) if $\lambda \leq ClInt(Cl(\lambda))$ and a fuzzy semi-preclosed (briefly, fsp-closed) if $IntCl(Int(\lambda)) \leq \lambda$ [20].

By FSPO (X, τ), we denote the family of all fuzzy semi-preopen sets of fts X. The semiclosure (resp. α-closure, semi-preclosure) of a fuzzy set A of (X, τ) is the intersection of all fs-closed (resp. fα-closed, fsp-closed) sets that contain A and is denoted by sCl(λ) (resp. αCl(λ) and spCl(λ)).

Lemma 2.2: Let λ be a fuzzy set in a fuzzy topological space (X, τ). Then
(i) $\alpha Cl(\lambda) = \lambda \vee ClIntCl(\lambda)$.
(ii) $SCl(\lambda) = \lambda \vee IntCl(\lambda)$.
(iii) $PCl(\lambda) \geq \lambda \vee ClInt(\lambda)$.
(iv) $SPCl(\lambda) \geq \lambda \vee IntClInt(\lambda)$.

Corresponding Author: P. Saravanaperumal[1]*

Definition 2.3: A fuzzy set λ in a fuzzy topological space (X, τ) is called:

(i) [2] Fuzzy generalized closed set if $Cl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open. We briefly denote it as fg-closed set.
(ii) [11] Fuzzy semi-generalized closed set if $sCl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy semiopen. We briefly denote it as fsg-closed set.
(iii) [9] Fuzzy generalized almost strongly semi-closed set if $\alpha Cl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy α -open. We briefly denote it as fαg-closed set.
(iv) [19] Fuzzy semi-pre-generalized closed set if $spCl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy semi -pre-open. We briefly denote it as fspg-closed set.
(v) [3] Fuzzy generalized strongly closed set if $\alpha Cl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open. We briefly denote it as fgα-closed set.
(vi) [7] Fuzzy $\ddot{g}$ -closed set (briefly f $\ddot{g}$ -closed set) if $cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is fsg-open.
(vii) [8] Fuzzy αgs-closed set if $\alpha\ cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy semi -open. We briefly denote it as fαs-closed set.
(viii) [18] Fuzzy generalized semi-closed set if $sCl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open. We briefly denote it as fgs-closed set.
(ix) [18] Fuzzy generalized semi-pre-closed set if $spCl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open. We briefly denote it as fgsp-closed set.

Definition 2.4: [16] Let S be a fuzzy subset of (X,τ_1,τ_2). Then S is said to be $\tau_{1,2}$-fuzzy open if $S = \lambda \cup \mu$ where $\lambda \in \tau_1$ and $\mu \in \tau_2$. The complement of $\tau_{1,2}$-fuzzy open set is called $\tau_{1,2}$-fuzzy-closed.

Definition 2.5:[16] Let S be a fuzzy subset of a Fuzzy Bitopological space X. Then

(i) the $\tau_{1,2}$-closure of S, denoted by $\tau_{1,2}$-cl(S), is defined as $\cap \{F : S \leq F$ and F is $\tau_{1,2}$-closed$\}$.
(ii) the $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S), is defined as $\cup \{F : F \leq S$ and F is $\tau_{1,2}$-open$\}$.

Definition 2.6: A fuzzy subset λ of a bts (X, τ_1, τ_2) is called

(i) (1,2)*-sg-closed set[13] if (1,2)*-scl$(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is (1,2)*-fuzzy semi-open in X. The complement of (1,2)*-sg-closed set is called (1,2)*-sg-open set;
(ii) (1,2)*-gs-closed set[13] if (1,2)*-scl $(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is $\tau_{1,2}$-open in X. The complement of (1,2)*-gs-closed set is called (1,2)*-gs-open set;
(iii) (1,2)*-α g-closed set [15] if (1,2)*-α cl$(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is $\tau_{1,2}$-open in X. The complement of (1,2)*-α g-closed set is called (1,2)*-α g-open set;
(iv) (1,2)*-ĝ-closed set or ω-closed set[6] if $\tau_{1,2}$-cl$(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is (1,2)*-s-open in X. The complement of (1,2)*-ĝ-closed (resp. ω-closed) set is called (1,2)*-ĝ-open (resp. ω-open)set;
(v) (1,2)*-ψ -closed set[12] if (1,2)*-scl$(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is (1,2)*-fsg-open in X. The complement of (1,2)*-fψ -closed set is called (1,2)*-fψ -open set;
(vi) (1,2)*- $\ddot{g}_{\alpha}$-closed set[12] if (1,2)*-α cl$(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is (1,2)*-sg- open in X. The complement of (1,2)*- $\ddot{g}_{\alpha}$-closed set is called (1,2)*- $\ddot{g}_{\alpha}$-open set;
(vii) (1,2)*-gsp-closed set[17] if (1,2)*- spcl$(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is $\tau_{1,2}$-open in X. The complement of (1,2)*-gsp-closed set is called (1,2)*-gsp-open set;
(viii) (1,2)*-g-closed set[14] if τ_{12} -cl$(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is τ_{12} -open in X. The complement of (1,2)*- g-closed set is called (1,2)*- g-open set;

3. (1, 2)*-FUZZY GENERALIZED CLOSED SET

We introduce the following definitions.

Definition 3.1: A fuzzy subset λ of a fbts (X, τ_1, τ_2) is called

(i) (1,2)*-fsg-closed set if (1,2)*-scl$(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is (1,2)*-fuzzy semi-open in X. The complement of (1,2)*-fsg-closed set is called (1,2)*-fsg-open set;
(ii) (1,2)*-fgs-closed set if (1,2)*-scl $(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is $\tau_{1,2}$-open in X. The complement of (1,2)*-fgs-closed set is called (1,2)*-fgs-open set;
(iii) (1,2)*-fα g-closed set if (1,2)*-α cl$(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is $\tau_{1,2}$-open in X. The complement of (1,2)*-f α g-closed set is called (1,2)*-fα g-open set;
(iv) (1,2)*-fĝ-closed set or fω-closed set if $\tau_{1,2}$-cl$(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is (1,2)*-fs-open in X. The complement of (1,2)*-fĝ-closed (resp. ω-closed) set is called (1,2)*-fĝ-open (resp. ω-open)set;
(v) (1,2)*-fψ -closed set if (1,2)*-scl$(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is (1,2)* -fsg-open in X. The complement of (1,2)*-fψ -closed set is called (1,2)*-fψ -open set;

(vi) (1,2)*-f $\ddot{g}_{\alpha}$-closed set if (1,2)*-α cl(λ) ≤ μ whenever λ ≤ μ and μ is (1,2)*-fsg-open in X. The complement of (1,2)*-f $\ddot{g}_{\alpha}$-closed set is called (1,2)*-f $\ddot{g}_{\alpha}$-open set;

(vii) (1,2)*-fgsp-closed set if (1,2)*- spcl(λ) ≤ μ whenever λ ≤ μ and μ is $\tau_{1,2}$-open in X. The complement of (1,2)*-fgsp-closed set is called (1,2)*-fgsp-open set;

(viii)(1,2)*-fαgs-closed set if (1,2)*-αcl(λ) ≤ μ whenever λ ≤ μ and μ is (1,2)*-fs-open in X. The complement of (1,2)*-fα gs-closed set is called (1,2)*-fα gs-open set;

Definition 3.2: A fuzzy subset λ of a fbts X is called (1,2)*-fg closed set if $\tau_{1,2}-$cl(λ) ≤ μ whenever λ ≤ μ and μ is $\tau_{1,2}-$open subset in X . The complement of (1,2)*- fg- closed set is called (1,2)*- fg-open set.

Definition 3.3: A fuzzy subset λ of a fbts X is called (1,2)*-f $\ddot{g}$ -closed set if $\tau_{1,2}$-cl(λ) ≤ μ whenever λ ≤ μ and μ is (1,2)*-fsg-open in X. The complement of (1,2)*-f $\ddot{g}$ -closed set is called (1,2)*-f $\ddot{g}$ -open set.

Proposition 3.4: Every (1,2)*-f α -closed set is (1,2)*-f $\ddot{g}$ α-closed.

Proof : If λ is a (1,2)*-f α closed and μ is any (1,2)*-fsg-open set Containing λ , we have μ ≥ $\tau_{1,2}$-cl(λ) ≥ αcl(λ). Hence λ is (1,2)*-f $\ddot{g}$ α-closed in X.

The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5: Consider the fbts (X, τ_1, τ_2) where X= $\{a, b\}$

$\tau_1 = \left\{0,1, \mu = \frac{0.3}{a} + \frac{0.4}{b}\right\}; \tau_2 \{0,1\}$

$\tau_{1,2}-$open set = $\left\{0,1, \mu = \frac{0.3}{a} + \frac{0.4}{b}\right\}$ and $\tau_{1,2}-$closed set = $\left\{0,1, \mu' = \frac{0.7}{a} + \frac{0.6}{b}\right\}$

Let λ $= \frac{0.4}{a} + \frac{0.5}{b}$ be any fuzzy subset of X. Since ClIntCl(λ) = $\mu' \not\leq$ λ . Therefore λ is not (1,2)*-fα-closed but it is not (1,2)*-f $\ddot{g}$ α-closed, for αCl(λ) = μ'≤ 1which is (1,2)*-fsg- open set containing λ.

Proposition 3.6: Every (1,2)*-f $\ddot{g}$ α-closed is (1,2)*-fαgs-closed

Proof If λ is a (1,2)*-f $\ddot{g}$ α-closed subset of X and μ is any (1,2)*-fs-open set containing λ, since every (1,2)*-fs-open set is (1,2)*-fsg-open, we have (1,2)*-αcl(λ) ≤ μ. Hence λ is (1,2)*-fαgs-closed in X.

The converse of Proposition 3.6 need not be true as seen from the following example.

Example 3.7: Consider the fbts (X, τ_1, τ_2) where X= $\{a, b\}$

$\tau_1 = \left\{0,1, \mu = \frac{0.4}{a} + \frac{0.6}{b}\right\}; \tau_2 \{0,1\}$

$\tau_{1,2}-$open set = $\left\{0,1, \mu = \frac{0.4}{a} + \frac{0.6}{b}\right\}$

and $\tau_{1,2}$ closed set = $\left\{0,1, \mu' = \frac{0.6}{a} + \frac{0.4}{b}\right\}$

$\tau_{1,2}$ -closed set containing $\lambda = \frac{0.4}{a} + \frac{0.5}{b}$ is 1. Therefore $\tau_{1,2} - cl(\lambda) = 1$

$\tau_{1,2}-$open set containing λ are 1 and μ. Therefore $\tau_{1,2} - cl(\lambda) = 1 \not\leq \mu$

Therefore λ is not (1,2)*-f $\ddot{g}$ -closed but it is (1,2)*-fαgs-closed, for αcl(λ) = 1≤ 1,which is (1,2)*-fs-open.

Proposition 3.8: Every (1,2)*-fαgs-closed set is (1,2)*-fαg-closed.

Proof: If λ is (1,2)*-fαgs-closed subset of X and μ is any $\tau_{1,2}$ - open set containing λ, since every $\tau_{1,2}$ - open set is (1,2)*- fs-open, we have αcl(λ)≤ μ . Hence λ is (1,2)*-fαg-closed in X.

The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.9: Consider the fbts (X, τ_1, τ_2) where X= $\{a, b\}$

$$\tau_1 = \left\{0,1, \mu = \frac{0.3}{a} + \frac{0.6}{b}\right\}; \tau_2 = \{0,1\}$$

$\tau_{1,2}$ − open set = $\left\{0,1, \mu = \frac{0.3}{a} + \frac{0.6}{b}\right\}$ and $\tau_{1,2}$ − closed set = $\left\{0,1, \mu' = \frac{0.7}{a} + \frac{0.4}{b}\right\}$

$\tau_{1,2}$ -closed set containing $\lambda = \frac{0.4}{a} + \frac{0.5}{b}$ is 1 . Therefore $\tau_{1,2} - cl(\lambda) = 1$

$\tau_{1,2}$ − open set containing λ is 1 . Therefore $\tau_{1,2} - cl(\lambda) = 1 \leq 1$, hence λ is(1,2)*-fαg-closed but it is not (1,2)*-fαgs-closed.

Proposition 3.10: Every (1,2)*-fuzzy closed set is (1,2)*-f $\ddot{g}$ -closed.

Proof: If λ is a (1,2)*-fuzzy closed subset set of X and μ is any fsg-open set such that μ≥ λ $= \tau_{1,2}$ - cl(λ). Hence λ is (1,2)*- f $\ddot{g}$ - closed.

The converse of Proposition 3.10 need not be true as seen from the following example.

Example 3.11: Consider the fbts (X, τ_1, τ_2) where X= $\{a, b, c\}$

$$\tau_1 = \left\{0,1, \lambda = \frac{0.7}{a} + \frac{0.3}{b} + \frac{0.8}{c}\right\} \text{ and } \tau_2 = \left\{1,0, \mu = \frac{0.3}{a} + \frac{0.5}{b} + \frac{0.7}{c}\right\},$$

$\tau_{1,2}$ − open sets are $\left\{0,1, \lambda, \mu, \frac{0.7}{a} + \frac{0.5}{b} + \frac{0.8}{c}\right\}$

$\tau_{1,2}$ -closed set are $\left\{0,1, \lambda' = \frac{0.3}{2} + \frac{0.7}{b} + \frac{0.2}{c}, \mu = \frac{0.7}{a} + \frac{0.5}{b} + \frac{0.3}{c}, \frac{0.3}{a} + \frac{0.5}{b} + \frac{0.2}{c}\right\}$.

Let $\gamma = \frac{.8}{a} + \frac{.9}{b} + \frac{.8}{c}$ be fuzzy set in X. since $\tau_{1,2} cl(\gamma) = 1 \leq 1$, 1 is $\tau_{1,2}$ -open in X. Thus γ is (1,2)*-f $\ddot{g}$ - closed but it is not $\tau_{1,2}$ -fuzzy closed.

Proposition 3.12: Every (1,2)*-f $\ddot{g}$ - closed set is(1,2)*-f ψ -closed.

Proof: If λ is a (1,2)*-f $\ddot{g}$ - closed subset of X and μ is any fs -open set such that μ ≥ λ , since every fs- open set is fsg-open, we have μ ≥ $\tau_{1,2}$ − cl(λ). Hence λ is(1.2)*-f ψ -closed in X .

The converse of Proposition 3.12 need not be true as seen from the following example.

Example 3.13: Consider the fbts (X, τ_1, τ_2) where X= $\{a, b\}$

$\tau_1 = \left\{0,1, \lambda = \frac{0.4}{a} + \frac{0.7}{b}\right\}; \tau_2 = \{0,1\}$. $\tau_{1,2}$ − open set = $\{0,1, \lambda\}$.

and $\tau_{1,2}$ − closed set = $\left\{0,1, \lambda' = \frac{0.6}{a} + \frac{0.3}{b}\right\}$

Let $\mu = \left\{\frac{0.4}{a} + \frac{0.3}{b}\right\}$ be a fuzzy subset of X. Clearly μ is (1,2)*-f $\ddot{g}$ -closed but not (1,2)*-f ψ - closed.

Proposition 3.14: Every (1,2)*-f $\ddot{g}$ -closed set is (1,2)*-f $\ddot{g}$ α closed.

Proof: If λ is a (1,2)*-f $\ddot{g}$ -closed subset of X and μ is any fsg-open set such that λ ≤ μ , then μ ≥ cl(λ) ≥ α cl(λ). Hence λ is (1,2)*- f $\ddot{g}$ α closed.

The converse of Proposition 3.14 need not be true as seen from the following example.

Example 3.15: Consider the fbts (X, τ_1, τ_2) where X= $\{a, b\}$

$\tau_1 = \left\{0,1, \mu = \frac{0.4}{a} + \frac{0.7}{b}\right\}; \tau_2 = \{0,1\}$. $\tau_{1,2}$ − Open set = $\{0,1, \mu\}$. $\tau_{1,2}$ − Closed set = $\{0,1, \mu'\}$. Let λ = $\frac{0.4}{a} + \frac{0.3}{b}$ a fuzzy subset of X. $\tau_{1,2}$ -open set containing λ are 1 and μ. $\tau_{1,2}$ -closed set containing λ are 1 and μ'.

Therefore $\tau_{1,2} - cl(\lambda) = \mu' \not\leq \mu$. Hence λ is not (1,2)*-f $\ddot{g}$ -closed but its (1,2)*-f $\ddot{g}$ α-closed.

Proposition 3.16: Every (1, 2)*-f ω -closed set is (1,2)*- fg-closed.

Proof: If λ is a (1,2)*- f ω -closed subset of X and μ is any (1,2)*- fuzzy open set containing λ, then μ ≥ $\tau_{1,2}$ − cl(λ). Hence λ is (1,2)*- fg-closed in X.

The converse of Proposition 3.16 need not be true as seen from the following example.

Example 3.17: Consider the fbts (X, τ_1, τ_2) where X= $\{a, b\}$

$\tau_1 = \left\{0,1, \mu = \frac{0.5}{a} + \frac{0.5}{b}\right\}; \tau_2 \{0,1\}$

$\tau_{1,2}$ -open set = $\left\{0,1, \mu = \frac{0.5}{a} + \frac{0.5}{b}\right\}$ and $\tau_{1,2}$ closed set = $\left\{0,1, \mu' = \frac{0.5}{a} + \frac{0.5}{b}\right\}$

$\tau_{1,2}$ -closed set containing $\lambda = \frac{0.4}{a} + \frac{0.4}{b}$ are 1& μ'. Therefore $\tau_{1,2} - cl(\lambda) = \mu'$

$\tau_{1,2}$ -open set containing λ are 1 and μ. Therefore $\tau_{1,2} - cl(\lambda) = \mu' \leq \mu \,\&1$

Therefore λ is (1, 2)*- fg-closed but not (1, 2)*-f ω -closed.

Proposition 3.18: Every (1,2)*- f ψ -closed set is (1,2)*- fsg closed.

Proof: If λ is a (1,2)*- f ψ -closed subset of X and μ is any (1,2)*-fs- open set containing λ, we have (1,2)*- scl(λ) ≤ μ. Hence λ is (1,2)*- fsg-closed.

The converse of Proposition 3.18 need not be true as seen from the following example.

Example 3.19: Consider the fbts (X, τ_1, τ_2) where X= $\{a, b\}$

$\tau_1 = \left\{0,1, \lambda = \frac{0.4}{a} + \frac{0.7}{b}\right\}; \tau_2 = \{0,1\}$. $\tau_{1,2}$ − open set = $\{0,1, \lambda\}$. $\tau_{1,2}$ − closed set = $\{0,1, \lambda'\}$. Let μ = $\frac{0.4}{a} + \frac{0.3}{b}$ a fuzzy subset of X. $\tau_{1,2}$ -open set containing μ are 1 and λ. $\tau_{1,2}$ -closed set containing μ are 1 and λ'.

Therefore $\tau_{1,2} - cl(\mu) = \lambda' \not\leq \lambda$. Hence μ is not (1,2)*-fψ-closed but its (1,2)*-fsg-closed , for Intλ'≤ μ ≤ λ' hence μ is semi closed. Therefore Scl (μ) = $\mu \leq 1$& λ.

Proposition 3.20: Every (1,2)*- fg-closed set is (1,2)*- fgs-closed.

Proof : If λ is a (1,2)*- fg-closed subset of X and μ is any $\tau_{1,2}$-open set containing λ, since every $\tau_{1,2}$-open set is (1,2)*-fsg-open, we have μ ≥ $\tau_{1,2}$-cl(λ) ≥ (1,2)*-scl(λ). Hence λ is (1,2)*- fgs-closed in X.

The converse of Proposition 3.20 need not be true as seen from the following example.

Example 3.21: Consider the fbts (X, τ_1, τ_2) where X= $\{a, b\}$

$\tau_1 = \left\{0,1, \mu = \frac{0.4}{a} + \frac{0.6}{b}\right\}; \tau_2 \{0,1\}$

$\tau_{1,2}$-open set = $\left\{0,1, \mu = \frac{0.4}{a} + \frac{0.6}{b}\right\}$ and $\tau_{1,2}$ closed set = $\left\{0,1, \mu' = \frac{0.6}{a} + \frac{0.4}{b}\right\}$

$\tau_{1,2}$-closed set containing $\lambda = \frac{0.4}{a} + \frac{0.5}{b}$ is 1 . Therefore $\tau_{1,2} - cl(\lambda) = 1$

$\tau_{1,2}$-open set containing λ are 1 and μ. Therefore $\tau_{1,2} - cl(\lambda) = 1 \not\leq \mu$

Therefore λ is not (1,2)*- fg-closed. Also int μ'≤ λ≤ μ', hence λ is semi closed. Therefore scl (λ) = $\lambda \leq 1$ & μ.

Hence λ is (1,2)*- fgs-closed.

Proposition 3.22: Every (1,2)*- fg-closed set is (1,2)*-fαg-closed.

Proof: If λ is a (1,2)*- fg-closed subset of X and μ is any $\tau_{1,2}$-open set containing λ. we have $\mu \geq \tau_{1,2}cl(\lambda) \geq (1,2)^* - \alpha cl(\lambda)$ .Hence λ is (1.2)*-fαg-closed in X.

The converse of Proposition 3.22 need not be true as seen from the following example.

Example 3.23: Consider the fbts (X, τ_1, τ_2) where X= $\{a, b\}$

$\tau_1 = \left\{0,1, \lambda = \frac{0.4}{a} + \frac{0.7}{b}\right\}; \tau_2 = \{0,1\}$. $\tau_{1,2}$ – open set = $\{0,1, \lambda\}$. $\tau_{1,2}$ – closed set = $\{0,1, \lambda'\}$.

Let μ = $\frac{0.4}{a} + \frac{0.3}{b}$ a fuzzy subset of X. $\tau_{1,2}$-open set containing μ are 1 and λ. $\tau_{1,2}$-closed set containing μ are 1 and λ'.

Therefore $\tau_{1,2} - cl(\mu) = \lambda' \not\leq \lambda$. Hence μ is not (1,2)*-fg-closed but its (1,2)*-fαg-closed for, $\alpha cl(\lambda) = \lambda \leq 1 \,\&\, \mu$.

Proposition 3.24: Every (1,2)*- fuzzy ω -closed set is (1,2)*- fuzzy sg closed.

Proof: If λ is a (1,2)*- fuzzy ω-closed subset of X and μ is any (1,2)*- fuzzy semi open set containing λ, we have $\mu \geq \tau_{1,2}cl(\lambda) \geq (1,2)^* - scl(\lambda)$. Hence λ is (1,2)*- fuzzy sg-closed in X.

The converse of Proposition 3.24 need not be true as seen from the following example.

Example 3.25: Consider the fbts (X, τ_1, τ_2) where X= $\{a, b\}$

$\tau_1 = \left\{0,1, \mu = \frac{0.4}{a} + \frac{0.6}{b}\right\}; \tau_2 \{0,1\}$

$\tau_{1,2}$-open set = $\left\{0,1, \mu = \frac{0.4}{a} + \frac{0.6}{b}\right\}$ and $\tau_{1,2}$ closed set = $\left\{0,1, \mu' = \frac{0.6}{a} + \frac{0.4}{b}\right\}$

$\tau_{1,2}$-closed set containing $\lambda = \frac{0.4}{a} + \frac{0.4}{b}$ is 1& μ . Therefore $\tau_{1,2} - cl(\lambda) = \mu' \not\leq \mu$

Hence λ is not a (1,2)*-fω-closed. Also Int μ'≤ λ≤ μ', hence λ is semi closed. Therefore scl (λ) = λ≤ 1 & μ .Hence λ is (1,2)*- fsg-closed.

Proposition 3.26: Every (1,2)*-fuzzy sg-closed set is (1,2)*-fuzzy gs –closed.

Proof: If λ is (1,2)*-fuzzy sg-closed subset of X and μ is any $\tau_{1,2}$-open set containing λ, since every $\tau_{1,2}$-open set is (1,2)*-fs-open, we have $scl(\lambda) \leq \mu$. Hence λ is (1,2)*-fuzzy-gs-closed.

The converse of proposition 3.26 need not be true as seen from the following example.

Example 3.27: Consider the fbts (X, τ_1, τ_2) where X= $\{a, b\}$

$$\tau_1 = \left\{0,1, \mu = \frac{0.3}{a} + \frac{0.6}{b}\right\}; \tau_2 \{0,1\}$$

$\tau_{1,2}$ -open set = $\left\{0,1, \mu = \frac{0.3}{a} + \frac{0.6}{b}\right\}$ and $\tau_{1,2}$ closed set = $\left\{0,1, \mu' = \frac{0.7}{a} + \frac{0.4}{b}\right\}$

Let $\lambda = \frac{0.3}{a} + \frac{0.4}{b}$ be any fuzzy subset of X. Therefore Int μ'≤ λ≤ μ', hence λ is semi closed.
Therefore scl (λ) = λ ≤ 1 & μ .Hence λ is (1,2)*- fgs-closed, but it is not (1,2)*-fuzzy sg-closed.

Proposition 3.28: Every (1,2)*-fuzzy ω- closed set is (1,2)*-fαgs closed.

Proof: If λ is (1,2)*-fuzzy ω-closed and μ is any (1,2)*-fuzzy semi open set containing λ, we have αcl(λ) ≤ $\tau_{1,2}$-cl(λ) ≤ μ. Hence λ is (1,2)*-fαgs-closed.

The converse of the proposition 3.28 is not true as seen from the following example.

Example 3.29: Consider the fbts { X,τ_1,τ_2} where X = {a,b}.

$$\tau_1 = \left\{0,1, \mu = \frac{0.4}{a} + \frac{0.7}{b}\right\}; \tau_2 \{0,1\}$$

$\tau_{1,2}$ -open set = $\left\{0,1, \mu = \frac{0.4}{a} + \frac{0.7}{b}\right\}$ and $\tau_{1,2}$ closed set = $\left\{0,1, \mu' = \frac{0.6}{a} + \frac{0.3}{b}\right\}$

Let $\lambda = \frac{0.4}{a} + \frac{0.3}{b}$ be any fuzzy subset of X. Therefore $\tau_{1,2}$ -closed set containing $\lambda = \frac{0.4}{a} + \frac{0.3}{b}$ is 1& μ'.

Therefore $\tau_{1,2} - cl(\lambda) = \mu' \not\leq \mu$. Hence λ is not a (1,2)*-fω-closed but it is (1,2)*-fαgs –closed, for αcl(λ) = λ ≤ 1& μ .

From the above propositions (3.4, 3.6, 3.8, 3.10, 3.12, 3.14, 3.16, 3.18, 3.20, 3.22, 3.24, 3.26, 3.28), Examples (3.5, 3.7, 3.9, 3.11, 3.13, 3.15, 3.17, 3.19, 3.21, 3.23, 3.25, 3.27,3.29) we obtain the following diagram, where A → B (resp. A $\longleftrightarrow$ B) represents A implies B but not conversely (resp. A and B are independent of each other).

$$\begin{tikzcd}
1 \arrow[r] & 2 \arrow[r] & 3 \arrow[r] & 4 \\
5 \arrow[u] \arrow[r] \arrow[d] & 6 \arrow[u] \arrow[r] \arrow[d] & 7 \arrow[u] \arrow[r] \arrow[d] & 8 \arrow[u] \arrow[d] \\
9 \arrow[r] & 10 \arrow[r] & 11 \arrow[r] & 12
\end{tikzcd}$$

Where

(1) is (1,2)*-f α -closed	(7) is (1,2)*-f ω -closed
(2) is (1,2)*- f $\ddot{g}$ α-closed	(8) is (1,2)*-f g-closed
(3) is (1,2)*- f αgs -closed	(9) is (1,2)*-fs-closed
(4) is (1,2)*- f α g-closed	(10) is (1,2)*-f ψ -closed
(5) is (1,2)*- fuzzy closed	(11) is (1,2)*-fsg-closed
(6) is (1,2)*-f $\ddot{g}$ -closed	(12) is (1,2)*-fgs-closed

ACKNOWLEDGEMENT

The authors would like to thank the reviewers for their valuable comments and helpful suggestions for improvement of the original manuscript.

REFERENCES

1. K. K. Azad, On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl. 82(1) (1981), 14--32.
2. G. Balasubramanian and P. Sundaram, On some generalizations of fuzzy discontinuous functions, Fuzzy Sets and Systems 86(1)(1997), 93--100.
3. O. Bedre Ozbakir, On generalized fuzzy strongly semi closed sets in fuzzy topological spaces, Int. J. Math. Math. Sci. 30(11)(2002), 651--657.
4. A. S. Bin Shahna, On fuzzy strong semicontinuity and fuzzy precontinuity, Fuzzy Sets /
5. C.L.Chang, Fuzzy topological spaces, J, Math. Anal. Appl. 24 (1968) 182-190.
6. Duszynski, Z., Joseph Israel, M. and Ravi, O.: A new generalization of closedsetsinbitopology, International Mathematical Forum (To appear).
7. M. Jeyaraman,S. Vijayalakshmi and O. Ravi, New Notion Of Closed Sets In Fuzzy Topology, IJMA- 4(8),(2013) 67-73.
8. M. Jeyaraman, J. Rajalakshmi and O. Ravi.: Another Generalization Of Closed Sets In Fuzzy Topological Spaces IJMA- 4(8)(2013),187-192.
9. Kandil, Biproximities and fuzzy bitopologicalspaces, Simon Stevin.63 (1989) 45-66.
10. Levine, N.: Generalized closed sets in topology, Rend. Circ. Math. Palermo, 19(2) (1970), 89-96.
11. H. Maki et al., Generalized closed sets in fuzzy topological space, I, Meetings on Topological Spaces Theory and its Applications (1998), 23--36.
12. Ravi, O. and Ganesan, S.: (1,2)*- $\ddot{g}$ -closed sets in bitopology (submitted).
13. Ravi, O. and Thivagar, M. L.: A bitopological (1,2)*-semi-generalized continuous maps, Bull.Malaysian Math. Sci. Soc., (2)(29)(1)(2006), 76-88.
14. Ravi, O., Thivagar, M. L. and Jinjinli. Remarks on extensions of (1,2)*-g-closed maps, Archimedes J. Math., 1(2)(2011), 177-187.
15. Ravi, O. Thivagar, M. L. and Nagarajan, A.: (1,2)*-αg-closed sets and (1,2)*-gα-closed sets(submitted).
16. Ravi, O. and Lellis Thivagar, M.: On stronger forms of (1,2)*-quotient mappings in bitopological spaces, Internat. J. Math. Game Theory and Algebra., 14(6)(2004), 481-492.
17. Ravi, O. and Thivagar, M. L.: Remarks on λ-irresolute functions via (1,2)*-sets, Advances inApp. Math. Analysis, 5(1) (2010), 1-15.
18. K. Saraf and M. Khana, Fuzzy generalized semi preclosed sets, J. Tripura Math. Soc., 3(2001), 59-68.
19. R. K. Saraf, G. Navalagi and Meena Khanna, On fuzzy semi-pre-generalized closed sets, Bull. Mal. Mat. Sci. Soc. 28(1)(2005), 19--30.
20. Veerakumar, M. K. R. S.: $\hat{g}$ -closed sets in topological spaces, Bull. Allah. Math. Soc., 18 (2003), 99-112.
21. L. A. Zadeh, Fuzzy sets Information and Control 8 (1965), 338-353.

Source of support: Nil, Conflict of interest: None Declared