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ON SOME PROPERTIES OF (1, 2)*-$g\alpha b\hat{g}$-CLOSED SETS IN BITOPOLOGICAL SPACES

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ABSTRACT

In this paper a new class of closed sets called (1, 2)-$g\alpha b\hat{g}$-closed sets in bitopological spaces is introduced. Several properties of this class and its inclusion relationship with other known classes of closed sets are analyzed. This class contains the class of all (1, 2)*-closed sets and is contained in the class of all (1, 2)*-αg- closed sets and (1, 2)*-$g\alpha$-closed sets. Also several new classes of spaces induced by the class of (1, 2)*-$g\alpha b\hat{g}$-closed sets are defined and their properties are investigated.*

Keywords: *(1,2)*-α-closed sets, (1,2)*-b-closed sets, (1,2)*-$\hat{g}$-closed sets, (1,2)*-$b\hat{g}$-closed sets, (1,2)*-$\alpha b\hat{g}$-closed sets, (1,2)*-$g\alpha b\hat{g}$-closed sets, (1,2)*-$g\alpha b\hat{g}$ continuous and (1,2)*-$T_{g\alpha b\,\hat{g}}$-space.*

AMS subject classification: *54A05, 54C05, 54E55, 54F65, 54H05.*

1. INTRODUCTION

Njastad [17] introduced and investigated the concept of alpha open sets (briefly α-open sets). Maki et.al [16] defined generalized alpha and alpha generalized closed sets (briefly $g\alpha$ and αg closed sets) in 1993 and 1994 respectively in topological spaces as an extension of alpha and generalized closed sets. Andrijevic [2] in 1996 exhibited a new class called b-open sets in a topological space. This class is contained in the class of semi pre-opens sets and contains all semi-open and pre-open sets. Norman Levine introduced the concept of generalized closed sets [13] (briefly g-closed set) in topological spaces in 1963. As an extension of g-closed sets, Veera Kumar [21] defined $\hat{g}$-closed sets in topological spaces in 2003. Subasree and Maria Singam [20] defined a new class namely $b\hat{g}$-closed sets in topological spaces which is a subclass of gb-closed sets and contains b-closed set. Followed by this, Mary and Nagajothi [18] [19] defined and characterized the class $\alpha b\hat{g}$-closed sets which is a subclass of $b\alpha\hat{g}$-closed sets and contains α-closed sets.

In 1963 Kelly [16] introduced the concept of Bitopological Spaces. A set X equipped with two topologies τ_1 and τ_2 is called Bitoplogical spaces and it is denoted by(X,τ_1,τ_2). The concept and various class of closed sets defined in topological spaces (X,τ) have been extended to bitopological spaces(X,τ_1,τ_2). Fukutake [8], [9] defined generalized closed sets and semi open sets in bitopological space in 1986 and 1989 respectively. In 1990, Jelic [11] introduced the concept of alpha open sets in bitopological space. El-Tantawy and Abu-Donia [7] extends the class of α-closed set to alpha generalized closed sets in bitopological spaces. Recently the authors introduced (1,2)*-$\alpha b\hat{g}$-closed sets and analyzed their properties[20].

In this paper another new class of closed sets namely (1,2)*-g$\alpha b\hat{g}$-closed sets is introduced in bitopological spaces that satisfies the inclusion relation given below:

$\{(1,2)^*\text{-}closed\ sets\} \subset \{(1,2)^*\text{-}g\alpha b\hat{g}\text{-}closed\ sets\} \subset \{(1,2)^*\text{-}\alpha g\text{-}closed\ sets\}$

Based on the definition of (1,2)*-$g\alpha b\hat{g}$-$closed\ sets$, a new space namely (1,2)*-$T_{g\alpha b\,\hat{g}}$ -space is defined and further several theorems on its relationship with other known bitopological spaces are proved.

2. PRELIMINARIES:

Throughout this paper, (X,τ_1,τ_2) denote a bitopological space with the topologies τ_1 and τ_2.

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Definition 2.1.1: [10] A $\boldsymbol{topology}$ on a set X is a collection τ of subsets of X having the following properties:

1) ϕ and X are in τ.
2) The union of the elements of any sub collection of τ is in τ.
3) The intersection of the elements of any finite sub collection of τ is in τ.

A set X for which a topology τ has been specified is called a $\boldsymbol{topological\ space.}$

Definition 2.1.2: [12] A set X with two topologies τ_1 and τ_2 is said to be a $\boldsymbol{bitopological\ space}$ and it is denoted by $(\boldsymbol{X}, \boldsymbol{\tau_1}, \boldsymbol{\tau_2})$.

Definition 2.1.3: [6] A subset A of a bitopological space (X, τ_1, τ_2) is called $\boldsymbol{\tau_{1,2}}$-$\boldsymbol{open\ set}$ if $A \in \tau_1 \cup \tau_2$. The complement of a $\tau_{1,2}$-open set is $\boldsymbol{\tau_{1,2}}$-$\boldsymbol{closed\ set.}$

Definition 2.1.4: [6] Let A be a subset of a bitopological space (X, τ_1, τ_2), then

1. The $\boldsymbol{\tau_{1,2}}$-$\boldsymbol{interior\ of\ A}$ in (X, τ_1, τ_2), denoted by $\tau_{1,2}$-$int(A)$, is defined as $\cup\{F/F \subset A\ and\ F\ is\ \tau_{1,2}$ -$open\ set\}$.
2. The $\boldsymbol{\tau_{1,2}}$-$\boldsymbol{closure\ of\ A}$ in (X, τ_1, τ_2), denoted by $\tau_{1,2}$-$cl(A)$, is defined as $\cap\{G/A \subset G\ and\ G\ is\ \tau_{1,2}$-$closed\ set\}$.

Definition 2.1.5: [6] A subset A of a bitopological space (X, τ_1, τ_2) is called

1. a (1,2)*-$semi\ open\ set$ if $A \subseteq \tau_{1,2}$-$cl(\tau_{1,2}$-$int(A))$ and a (1,2)*-$semi\ closed\ set$ if $\tau_{1,2}$-$int(A)(\tau_{1,2}$-$cl(A)) \subseteq A$.
2. a (1,2)*-$pre\ open\ set$ if $A \subseteq \tau_{1,2}$-$int(\tau_{1,2}$-$cl(A))$ and a (1,2)*-$pre\ closed\ set$ if $\tau_{1,2}$-$cl(A)(\tau_{1,2}$-$int(A)) \subseteq A$.
3. a (1,2)*-α- $open\ set$ if $A \subseteq \tau_{1,2}$-$int(\tau_{1,2}$-$cl(\tau_{1,2}$-$int(A)))$ and a (1,2)*-α- $closed\ set$ if $\tau_{1,2}$-$cl(\tau_{1,2}$-$int(\tau_{1,2}$-$cl(A))) \subseteq A$.
4. a (1,2)*-b- $open\ set$ if $A \subseteq \tau_{1,2}$-$cl(\tau_{1,2}$-$int(A)) \cup \tau_{1,2}$-$int(\tau_{1,2}$-$cl(A))$ and a (1,2)*-b- $closed\ set$ if $\tau_{1,2}$-$int(\tau_{1,2}$-$cl(A)) \cap \tau_{1,2}$-$cl(\tau_{1,2}$-$int(A)) \subseteq A$.
5. a (1,2)*-$semi\ pre\ open\ set$ if $A \subseteq\ \tau_{1,2}$-$cl(\tau_{1,2}$-$int(\tau_{1,2}$-$cl(A)))$ and a (1,2)*-$semi\ pre\ closed\ set$ if $\tau_{1,2}$-$int(\tau_{1,2}$-$cl(\tau_{1,2}$-$int(A))) \subseteq A$.

Definition 2.1.6: [6]

1. The intersection of all (1,2)*-$semi\ closed\ \ sets$ containing A is called (1,2)*-$semi\ closure\ of\ A$ and it is denoted by $\tau_{1,2}$-$scl(A)$.
2. The intersection of all (1,2)*-α-$closed\ sets$ containing A is called (1,2)*-α-$closure\ of\ A$ and it is denoted by $\tau_{1,2}$-$\alpha cl(A)$.
3. The intersection of all (1,2)*-b-$closed\ sets$ containing A is called (1,2)*-b-$closure\ of\ A$ and it is denoted by $\tau_{1,2}$-$bcl(A)$.
4. The intersection of all (1,2)*-$pre\ closed\ \ sets$ containing A is called (1,2)*-$pre\ closure\ of\ A$ and it is denoted by $\tau_{1,2}$-$pcl(A)$.
5. The intersection of all (1,2)*-$semi\ pre\ closed\ sets$ containing A is called (1,2)*-$semi\ pre\ closure\ of\ A$ and it is denoted by $\tau_{1,2}$-$spcl(A)$.

Definition 2.1.7: [6] The family of all (1,2)*-open sets, (1,2)*-α-open sets, (1,2)*-b-open sets, (1,2)*-$semi$ open sets in X are denoted by (1,2)*-$O(X)$, (1,2)*-$\alpha O(X)$, (1,2)*-$BO(X)$ and (1,2)*-$SO(X)$ respectively.

Definition 2.1.8: [6] A subset A of a bitopological space (X, τ_1, τ_2) or X is called

1. a (1,2)*-$generalized\ closed\ set$ (briefly (1,2)*-g-$closed\ set$) if $\tau_{1,2}$-$cl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$O(X)$.
2. a (1,2)*-$generalized\ semi\ closed\ set$ (briefly (1,2)*-gs-$closed\ set$) if $\tau_{1,2}$-$scl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$O(X)$.
3. a (1,2)*- $semi\ generalized\ closed\ set$ (briefly (1,2)*-sg-$closed\ set$) if $\tau_{1,2}$-$scl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$SO(X)$.
4. a (1,2)*-$\alpha\ generalized\ closed\ set$ (briefly (1,2)*-αg-$closed\ set$) if $\tau_{1,2}$-$\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$O(X)$.
5. a (1,2)*- $generalized\ \alpha\ closed\ set$ (briefly (1,2)*-$g\alpha$-$closed\ set$) if $\tau_{1,2}$-$\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$\alpha O(X)$.
6. a (1,2)*-$generalized\ pre\ closed\ set$ (briefly (1,2)*-gp-$closed\ set$) if $\tau_{1,2}$-$pcl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$O(X)$.
7. a (1,2)*-$generalized\ semi\ pre\ closed\ set$ (briefly (1,2)*-gsp-$closed\ set$) if $\tau_{1,2}$-$spcl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$O(X)$.
8. a (1,2)*-$strongly\ generalized\ closed\ set$ (briefly (1,2)*-$strongly$-g-$closed\ set$) if $\tau_{1,2}cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and $U \in$ (1,2)*-$O(X)$.

Definition 2.1.9: [6] A subset A of a bitopological space (X, τ_1, τ_2) or X is called

1. a (1,2)*-$\hat{g}$-$closed\ set$ if $\tau_{1,2}$-$cl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$SO(X)$ and the complement of (1,2)*-$\hat{g}$-$closed\ set$ is called a (1,2)*-$\hat{g}$-$open\ set$.
2. a (1,2)*-gb-$closed\ set$ if $\tau_{1,2}$-$bcl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$O(X)$ and the complement of (1,2)*-gb-$closed\ set$ is called a (1,2)*-gb-$open\ set$.
3. a (1,2)*-$b\hat{g}$-$closed\ set$ if $\tau_{1,2}$-$bcl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$\hat{G}O(X)$ and the complement of (1,2)*-$b\hat{g}$-$closed\ set$ is called a (1,2)*-$b\hat{g}$-$open\ set$.
4. a (1,2)*-$\alpha\hat{g}$-$closed\ set$ if $\tau_{1,2}$-$\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$\hat{G}O(X)$ and the complement of (1,2)*-$\alpha\hat{g}$-$closed\ set$ is called a (1,2)*-$\alpha\hat{g}$-$open\ set$.
5. a (1,2)*-$b\alpha\hat{g}$-$closed\ set$ if $\tau_{1,2}$-$bcl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$\alpha\hat{G}O(X)$ and the complement of (1,2)*-$b\alpha\hat{g}$-$closed\ set$ is called a (1,2)*-$b\alpha\hat{g}$-$open\ set$.

Definition 2.1.10: The family of all (1,2) *-g-open sets, (1,2) *-$\hat{g}$- open sets, (1,2) *-$\alpha\hat{g}$-open sets and (1,2) *-$b\hat{g}$-open sets in X are denoted by (1,2) *-$GO(X)$, (1,2) *-$\hat{G}O(X)$, (1,2) *-$\alpha\hat{G}O(X)$ and (1,2) *-$B\hat{G}O(X)$ respectively.

Definition 2.1.11: A function $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is called

1. a (1,2)*-$continuous\ function$ if $f^{-1}(V)$ is (1,2)*-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).
2. a (1,2)*-g-$continuous\ function$ if $f^{-1}(V)$ is (1,2)*-g-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).
3. a (1,2)*-$\alpha\ generalized\ continuous\ function$ (briefly (1,2)*-αg-continuous) if $f^{-1}(V)$ is (1,2)*-αg-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).
4. a (1,2)*- $generalized\ \alpha\ continuous\ function$ (briefly (1,2)*-$g\alpha$-continuous) if $f^{-1}(V)$ is (1,2)*-$g\alpha$-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).
5. a (1,2)*- $generalized\ semi\ continuous\ function$ (briefly (1,2)*-gs-continuous) if $f^{-1}(V)$ is (1,2)*-gs-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).
6. a (1,2)*- $semi\ generalized\ continuous\ function$ (briefly (1,2)*-sg-continuous) if $f^{-1}(V)$ is (1,2)*-sg-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).
7. a (1,2)*- $generalized\ semi$-$pre\ continuous\ function$ (briefly (1,2)*-gsp-continuous) if $f^{-1}(V)$ is (1,2)*-gsp-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).
8. a (1,2)*- $generalized\ pre\ continuous\ function$ (briefly (1,2)*-gp-continuous) if $f^{-1}(V)$ is (1,2)*-gp-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).
9. a (1,2)*-gb-$continuous\ function$ if $f^{-1}(V)$ is (1,2)*-gb-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).

Definition 2.1.12:[6] A bitopological space (X, τ_1, τ_2) is called

1. a (1,2)*-$T_{1/2}$-$space$ if every (1,2)*-g-closed set in it is (1,2)*-closed set.
2. a (1,2)*-T_b-$space$ if every (1,2)*-gs-closed set in it is (1,2)*-closed set.
3. a (1,2)*- ${}_{\alpha}T_b$-$space$ if every (1,2)*-αg-closed set in it is (1,2)*-closed set.
4. a (1,2)*-$T^c_{b\alpha\hat{g}}$-$space$ if every (1,2)*-$b\alpha\hat{g}$-closed set in it is (1,2)*-closed set.

3. (1, 2)*-$g\alpha b\hat{g}$-CLOSED SETS

In this section we introduce a new class of closed sets called (1, 2)*-$g\alpha b\hat{g}$-closed sets ((1,2)*-generalized $\alpha b\hat{g}$-closed sets) which lie between the class of (1,2)*-closed sets and the class of (1,2)*-αg-closed sets.

Definition 3.1: A subset A of a bitopological space (X,τ_1,τ_2) is said to be **(1,2)*- $g\alpha b\hat{g}$-$closed\ set$** if $\tau_{1,2}$-cl(A) ⊆ U, whenever A ⊆ U and U ∈ (1,2)*- αb$\hat{g}$-open set in (X,τ_1,τ_2). The family of all (1,2)*- $g\alpha b\hat{g}$-$open\ sets$ in X is denoted by (1,2)*-$g\alpha b\hat{g}O(X)$.

3.1 Relationship of (1,2)*- $g\alpha b\hat{g}$ closed sets with other classes of (1,2)*-closed sets:

Theorem 3.1.1:

(i) Every (1,2)*-closed set is (1,2)*-$g\alpha b\hat{g}$-closed set. But the converse need not be true.
(ii) If a (1,2)*- $g\alpha b\hat{g}$-closed set is (1,2)*- $\alpha b\hat{g}$-closed set, then it is (1,2)*-closed set.

Proof:

(i) Let A be an (1,2)*-closed set and U ∈ (1,2)*-$\alpha b\hat{g}$-open set such that A ⊆ U. Since A is (1,2)*-closed set, we have $\tau_{1,2}$-$cl(A) = A \subseteq U$. Therefore, $\tau_{1,2}$-$cl(A) \subseteq U$, whenever $A \subseteq U$ and U ∈ (1,2)*-$\alpha b\hat{g}$-open set. Hence A is (1,2)*-$g\alpha b\hat{g}$-closed set.The converse of the above theorem need not be true. This is proved in the following example:

Example 3.1.1: Let X = {a, b, c} with τ_1= {ϕ , X, {a}} and τ_2= {ϕ , X, {a, b}}.

Clearly A= {a, c} is (1,2)*-$g\alpha b\hat{g}$-closed set, but not (1,2)*-closed set.

(ii) Let A be (1,2)*-$\alpha b\hat{g}$-open set and (1,2)*-$g\alpha b\hat{g}$-closed set. Since $A \subseteq A$, and by our hypothesis we have $\tau_{1,2}$-$cl(A) \subseteq A$. It is obvious that $A \subseteq \tau_{1,2}$-$cl(A)$. Hence A is a (1,2)*-closed set.

Theorem 3.1.2: Let A be a (1,2)*-$g\alpha$b$\hat{g}$ closed set in a bitopological space (X,τ_1,τ_2). Then A is a) (1,2)*-g-closed set, b) (1,2)*-αg-closed set, c) (1,2)*-gs-closed set, d) (1,2)*- gp-closed set, e) (1,2)*- gsp-closed set, f) (1,2)*- gb-closed set, g) (1,2)*-$g\alpha$-closed set.

Proof: Let A be a (1,2)*-$g\alpha b\hat{g}$-closed set. Then by definition $\tau_{1,2}$-$cl(A) \subseteq$ U whenever $A \subseteq U$ and U be a (1,2)*-$\alpha b\hat{g}$-closed set.

a) Let U be an (1,2)*-open set such that $A \subseteq U$. Since every (1,2)*-open set is (1,2)*- $\alpha b\hat{g}$-open set [20], U is a (1,2)*-$\alpha b\hat{g}$-open set, and hence $\tau_{1,2}$-$cl(A) \subseteq U$. Thus A is (1,2)*-g-closed set.
b) Let U be an (1,2)*-open set such that $A \subseteq U$. Since every (1,2)*-open set is (1,2)*- $\alpha b\hat{g}$-open set [20], U is a (1,2)*-$\alpha b\hat{g}$-open set. Always $\tau_{1,2}$-$\alpha cl(A \subseteq \tau_{1,2}$-$cl(A)$. Hence by hypothesis $\tau_{1,2}$-$\alpha cl(A) \subseteq \tau_{1,2}$-$cl(A \subseteq U$. Thus A is (1,2)*-αg-closed set.
c) Let U be an (1,2)*-open set such that $A \subseteq U$. Since every (1,2)*-open set is (1,2)*- $\alpha b\hat{g}$-open set [20], U is a (1,2)*-$\alpha b\hat{g}$-open set. Always $\tau_{1,2}$-$scl(A \subseteq \tau_{1,2}$-$cl(A)$.Hence by hypothesis $\tau_{1,2}$-$scl(A) \subseteq \tau_{1,2}$-$cl(A) \subseteq U$. Thus A is (1,2)*-gs-closed set.
d) Let U be an (1,2)*-open set such that $A \subseteq U$. Since every (1,2)*-open set is (1,2)*- $\alpha b\hat{g}$-open set [20], U is a (1,2)*-$\alpha b\hat{g}$-open set. It is well known that, $\tau_{1,2}$-$pcl(A) \subseteq \tau_{1,2}$-$cl(A)$.Hence by hypothesis $\tau_{1,2}$-$pcl(A) \subseteq \tau_{1,2}$-$cl(A) \subseteq U$. Thus A is (1,2)*-gp-closed set.
e) Let U be an (1,2)*-open set such that $A \subseteq U$. Since every (1,2)*-open set is (1,2)*- $\alpha b\hat{g}$-open set [20], U is a (1,2)*-$\alpha b\hat{g}$-open set. It is well known that, $\tau_{1,2}$-$spcl(A) \subseteq \tau_{1,2}$-$cl(A)$.Hence by hypothesis $\tau_{1,2}$-$spcl(A) \subseteq \tau_{1,2}$-$cl(A) \subseteq U$. Thus A is (1,2)*-gsp-closed set.
f) Let U be an (1,2)*-open set such that $A \subseteq U$. Since every (1,2)*-open set is (1,2)*- $\alpha b\hat{g}$-open set [20], U is a (1,2)*-$\alpha b\hat{g}$-open set. It is well known that, $\tau_{1,2}$-$bcl(A) \subseteq \tau_{1,2}$-$cl(A)$.Hence by hypothesis $\tau_{1,2}$-$bcl(A) \subseteq \tau_{1,2}$-$cl(A) \subseteq U$. Thus A is (1,2)*-gb-closed set.
g) Let U be an (1,2)*-α-open set such that $A \subseteq U$. Since every (1,2)*-α-open set is (1,2)*- $\alpha b\hat{g}$-open set [20], U is a (1,2)*-$\alpha b\hat{g}$-open set. Always , $\tau_{1,2}$-$\alpha cl(A) \subseteq \tau_{1,2}$-$cl(A)$.Hence by hypothesis $\tau_{1,2}$-$\alpha cl(A) \subseteq \tau_{1,2}$-$cl(A) \subseteq U$. Thus A is (1,2)*-$g\alpha$-closed set.

Remark 3.1.1: Note that the converse of the above Theorem need not be true. The following examples prove this statement:

Example 3.1.2: Let X = {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a, b}}. Clearly A= {a, c} is (1,2)*-g-closed set, but not (1,2)*-$g\alpha b\hat{g}$-closed set.

Example 3.1.3: Let X = {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a, b}}. Clearly A= {a, c} is (1,2)*-αg-closed set, but not (1,2)*-$g\alpha b\hat{g}$-closed set.

Example 3.1.4: Let X = {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a, b}}. Clearly A= {b} is (1,2)*-$g\alpha$-closed set, but not (1,2)*-$g\alpha b\hat{g}$-closed set.

Example 3.1.5: Let X = {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a, b}}. Clearly A= {b} is (1,2)*-gs-closed set, but not (1,2)*-$g\alpha b\hat{g}$-closed set.

Example 3.1.6: Let X = {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a, b}}. Clearly A= {a, c} is (1,2)*-gp-closed set, but not (1,2)*-$g\alpha b\hat{g}$-closed set.

Example 3.1.7: Let X = {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a}, {b}, {a, b}}. Clearly A= {a} is (1,2)*-gsp-closed set, but not (1,2)*-$g\alpha b\hat{g}$-closed set.

Example 3.1.8: Let X = {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a}, {b}, {a, b}}. Clearly A= {a} is (1,2)*-gb-closed set, but not (1,2)*-$g\alpha b\hat{g}$-closed set.

Remark 3.1.2: From Theorem 3.1.1 and Theorem 3.1.2 it is observe that the following inclusion relations holds:
{(1,2)*-closed sets} $\subset$ {(1,2)*-$g\alpha b\hat{g}$-closed sets} $\subset$ {(1,2)*-αg-closed sets}

Remark 3.1.3: The following examples reveal that the class of (1,2)*-$g\alpha b\hat{g}$-closed set are ***independent*** from the class of (1,2)*-α-closed sets, class of (1,2)*-$\alpha b\hat{g}$-closed sets and class of (1,2)*-semi closed sets.

Example 3.1.9:

(i) Let X= {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a, b}}. Clearly A= {b} is (1,2)*-α-closed set, but not (1,2)*-$g\alpha b\hat{g}$-closed set.

(ii) Let X= {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {b,c}}. Clearly A= {a, b} is (1,2)*-$g\alpha b\hat{g}$-closed set, but not (1,2)*-α-closed set.

Example 3.1.10:

(i) Let X = {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {b,c}}. Clearly A= {a, b} is (1,2)*-$g\alpha b\hat{g}$-closed set, but not (1,2)*-$\alpha b\hat{g}$-closed set.

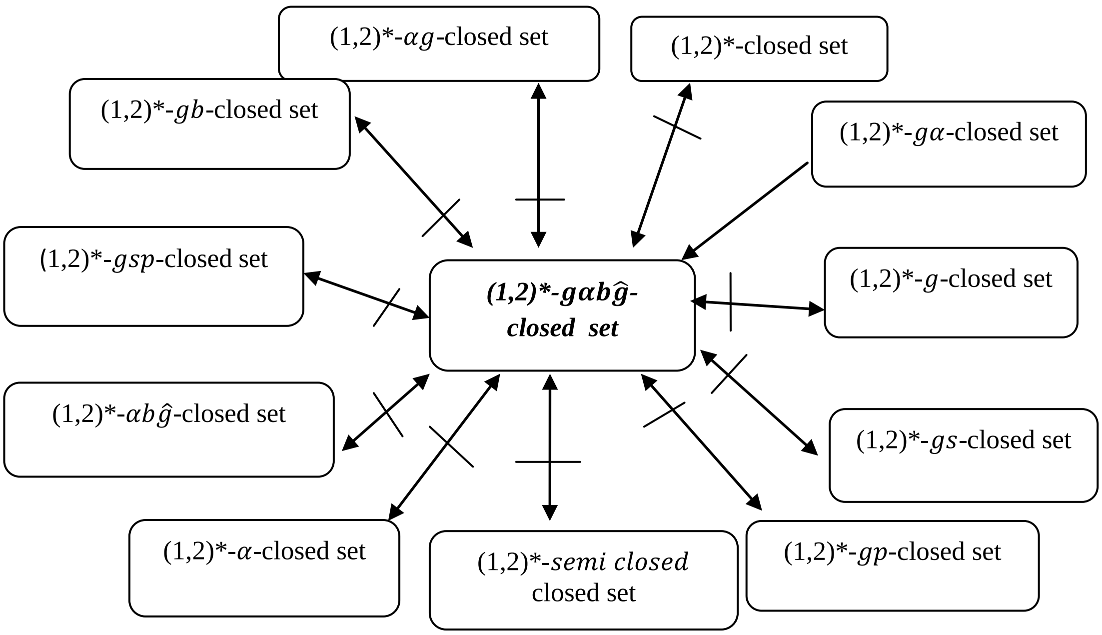
(ii) Let X = {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a, b}}. Clearly A= {b} is (1,2)*-$\alpha b\hat{g}$-closed set, but not (1,2)*-$g\alpha b\hat{g}$-closed set.

Example 3.1.11:

(i) Let X = {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a, b}}. Clearly A= {b} is (1,2)*-semi closed set, but not (1,2)*-$g\alpha b\hat{g}$-closed set.

(ii) Let X = {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {b, c}}. Clearly A= {a, c} is (1,2)*-$g\alpha b\hat{g}$-closed set, but not (1,2)*-α-closed set.

Relationships of (1,2)*-$g\alpha b\hat{g}$-closed sets with other closed sets are represented by the following diagram:



In the above diagram, A⟶B denotes A implies B, A $\longleftrightarrow$ B denotes A implies B but B does not imply A, A $\longleftrightarrow$ B denotes B implies A but A does not imply B, A $\longleftrightarrow$ B denotes A and B are independent.

3.2 PROPERTIES OF (1,2)*-$g\alpha b\hat{g}$-CLOSED SETS

Theorem 3.2.1: If A and B are (1,2)*-$g\alpha b\hat{g}$-closed sets in (X,τ_1,τ_2) then $A\cup B$ is (1,2)*-$g\alpha b\hat{g}$-closed set.

Proof: Let A and B be (1,2)*-$g\alpha b\hat{g}$-closed sets in (X,τ_1,τ_2) and U be any (1,2)*-$\alpha b\hat{g}$-open set containing $A\cup B$. Since $A\subseteq U$ and $B\subseteq U$, we have $\tau_{1,2}$-$cl(A)\subseteq U$ and $\tau_{1,2}$-$cl(B)\subseteq U$. Also $\tau_{1,2}$-$cl(A\cup B)=\tau_{1,2}$-$cl(A)\cup\tau_{1,2}$-$cl(B)\subseteq U$. Hence $A\cup B$ is (1,2)*- $g\alpha b\hat{g}$-closed set.

Theorem 3.2.2: If a set A is (1,2)*- $g\alpha b\hat{g}$-closed set then $\tau_{1,2}$-$cl(A)\backslash A$ contains no non empty (1,2)*-closed set in (X,τ_1,τ_2).

Proof: Suppose F is a (1,2)*-closed subset of $\tau_{1,2}$-$cl(A)\backslash A$. Then F$\subseteq\tau_{1,2}$-$cl(A)$ and $A\subseteq F^c$. Since A is (1,2)*- $g\alpha b\hat{g}$-closed set and F^c is (1,2)*-open set, since every (1,2)*-open set is (1,2)*- $\alpha b\hat{g}$-open set, F^c is (1,2)*- $\alpha b\hat{g}$-open set such that $A\subseteq F^c$, we have $\tau_{1,2}$-$cl(A)\subseteq F^c$. Hence F $\subseteq\tau_{1,2}$-$(cl(A))^c$. We have F$\subseteq\tau_{1,2}$-$cl(A)\cup\tau_{1,2}$-$(cl(A))^c$ and hence F is empty. Therefore $\tau_{1,2}$-$cl(A)\backslash A$ contains no non empty (1,2)*-closed set in (X,τ_1,τ_2).

Remark 3.2.1: The converse of the above Theorem need not be true. This is proved in the following example:

Example 3.2.1: Let X = {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a, b}}. If A= {a, c}, $\tau_{1,2}$-$cl(A) - A$ = X – {a, c} = {b} does not contain non empty closed set then A is not (1,2)*- $g\alpha b\hat{g}$-closed set.

Theorem 3.2.3: A set A is (1,2)*- $g\alpha b\hat{g}$-closed set if and only if $\tau_{1,2}$-$cl(A)\backslash A$ contains no non empty (1,2)*-$\alpha b\hat{g}$-closed set in (X, τ_1, τ_2).

Proof: Suppose that A is (1,2)*- $g\alpha b\hat{g}$-closed set. Let F be a (1,2)*-$\alpha b\hat{g}$-closed subset of $\tau_{1,2}$-$cl(A)\backslash A$. Thus, $A \subseteq F^c$. Since A is (1,2)*- $g\alpha b\hat{g}$-closed set, we have $\tau_{1,2}$-$cl(A) \subseteq F^c$.Therefore $F \subseteq \tau_{1,2}$-$(cl(A))^c$. Consequently $F \subseteq \tau_{1,2}$-$cl(A) \cap \tau_{1,2}$-$(cl(A))^c = \phi$. Thus F is empty. Hence $\tau_{1,2}$-$cl(A)\backslash A$ contains no non empty (1,2)*-$\alpha b\hat{g}$-closed set.

Conversely, Suppose that $\tau_{1,2}$-$cl(A)\backslash A$ contains no non empty (1,2)*-$\alpha b\hat{g}$-closed set. Let $A \subseteq G$ and G be (1,2)*-$\alpha b\hat{g}$-open set. If $\tau_{1,2}$-$cl(A)$is not a subset of a G, then $\tau_{1,2}$-$cl(A) \cap G^c$, then $\tau_{1,2}$-$cl(A) \cap G^c$ is a non empty subset of a $\tau_{1,2}$-$cl(A)\backslash A$. Since $\tau_{1,2}$-$cl(A)$ is a closed set and G^c is a (1,2)*-$\alpha b\hat{g}$-closed set, $\tau_{1,2}$-$cl(A) \cap G^c$ is a non empty (1,2)*-$\alpha b\hat{g}$-closed subset of $\tau_{1,2}$-$cl(A)\backslash A$ which is a contradiction. Therefore $\tau_{1,2}$-$cl(A) \subseteq G$ and hence A is (1,2)*- $g\alpha b\hat{g}$-closed set.

Corollary 3.2.1: If A is (1,2)*- $g\alpha b\hat{g}$-closed set in (X, τ_1, τ_2), and $A \subseteq B \subseteq \tau_{1,2}$-$cl(A)$, then B is (1,2)*- $g\alpha b\hat{g}$-closed set in (X, τ_1, τ_2).

Proof: Since $B \subseteq \tau_{1,2}$-$cl(A)$, we have $\tau_{1,2}$-$cl(B) \subseteq \tau_{1,2}$-$cl(A)$. Then $\tau_{1,2}$-$cl(B)\backslash B \subseteq \tau_{1,2}$-$cl(A)\backslash A$. By theorem 3.2.3, $\tau_{1,2}$-$cl(A)\backslash A$ contains no non empty (1,2)*-$\alpha b\hat{g}$-closed subset of (X, τ_1, τ_2) and hence $\tau_{1,2}$-$cl(B)\backslash B$ contains no non empty (1,2)*-$\alpha b\hat{g}$-closed subset of (X, τ_1, τ_2). Hence again by Theorem 3.2.3, B is (1,2)*- $g\alpha b\hat{g}$-closed set in (X, τ_1, τ_2).

Theorem 3.2.4: Suppose Y is a subspace of (X, τ_1, τ_2) and $A \subseteq Y$ is a (1,2)*- $g\alpha b\hat{g}$-closed set in (X, τ_1, τ_2) then A is (1,2)*- $g\alpha b\hat{g}$-closed relative to Y.

Proof: Let $A \subseteq Y \cap G$ where $G \in$ (1,2)*-$\alpha b\hat{g}$-open set in (X, τ_1, τ_2). Then $A \subseteq G$ and since A is a (1,2)*- $g\alpha b\hat{g}$-closed set, we have $\tau_{1,2}$-$cl(A) \subseteq G$. This implies that $Y \cap \tau_{1,2}$-$cl(A) \subseteq Y \cap G$ whenever $A \subseteq Y \cap G$ where $G \in$ (1,2)*-$\alpha b\hat{g}$-open set in (X, τ_1, τ_2). Thus A is (1,2)*- $g\alpha b\hat{g}$-closed relative to Y.

Theorem 3.2.5: In a bitopological space (X, τ_1, τ_2), let F denotes the class of (1,2)*-closed sets in X, then (1,2)*-$\alpha b\hat{g}O(X) = F$ if and only if every subset of X is (1,2)*- $g\alpha b\hat{g}$-closed set in (X, τ_1, τ_2).

Proof: Suppose (1,2)*-$\alpha b\hat{g}O(X) = F$. Let A be a subset of X such that $A \subseteq G$ where $G \in$ (1,2)*-$\alpha B\hat{G}$O(X) then $\tau_{1,2}$-$cl(G) = G$. Also $\tau_{1,2}$-$cl(A) \subseteq \tau_{1,2}$-$cl(G) = G$. Hence A is (1,2)*- $g\alpha b\hat{g}$-closed set in (X, τ_1, τ_2).
Conversely,

Suppose every subset of X is (1,2)*- $g\alpha b\hat{g}$-closed set in X.

Let $G \in$ (1,2)*-$\alpha B\hat{G}$O(X). Since $G \subseteq G$ and G is (1,2)*- $g\alpha b\hat{g}$-closed set in X, we have $\tau_{1,2}$-$cl(G) \subseteq G$. Thus $\tau_{1,2}$-$cl(G) = G$. Therefore (1,2)*-$\alpha B\hat{G}$O(X) $\subseteq F$.

If $S \in F$, then S^c is (1,2)*-open set and hence it is (1,2)*- $\alpha b\hat{g}$-open set. Therefore $S^c \in$ (1,2)*-$\alpha B\hat{G}$O(X) $\subseteq F$ and hence $S \in F^c$. This implies that S is (1,2)*-open set and hence it is (1,2)*-$\alpha b\hat{g}$-open set. Therefore $F \subseteq$ (1,2)*-$\alpha B\hat{G}$O(X). Thus (1,2)*-$\alpha B\hat{G}$O(X)=F.

3.3 CHARACTERIZATION OF (1,2)*-$g\alpha b\hat{g}$-OPEN SETS

Theorem 3.3.1: A set A is (1,2)*-$g\alpha b\hat{g}$-open set if and only if $F \subseteq \tau_{1,2}$-$int(A)$, where F is (1,2)*-$\alpha b\hat{g}$-closed set and $F \subseteq A$.

Proof: Suppose A is (1,2)*-$g\alpha b\hat{g}$-open set, $F \subseteq A$ and F is (1,2)*-$\alpha b\hat{g}$-closed set. Then F^c is (1,2)*-$\alpha b\hat{g}$-open set and $A^c \subseteq F^c$. Since A^c is (1,2)*-$g\alpha b\hat{g}$-closed set, we have $\tau_{1,2}$-$cl(A^c) \subseteq F^c$. Hence $F \subseteq \tau_{1,2}$-$int(A)$.

Conversely, Suppose $F \subseteq \tau_{1,2}$-$int(A)$, where F is (1,2)*-$\alpha b\hat{g}$-closed set and $F \subseteq A$. Let $A^c \subseteq G$ where $G = F^c$ is (1,2)*-$\alpha b\hat{g}$-open set. Then $G^c \subseteq \tau_{1,2}$-$int(A)$. This implies that, $\tau_{1,2}$-$cl(A^c) \subseteq G$. Thus A^c is (1,2)*-$g\alpha b\hat{g}$-closed set. Hence A is (1,2)*-$g\alpha b\hat{g}$-open set.

Theorem 3.3.2: In a bitopological space (X, τ_1, τ_2), if $A \subseteq B \subseteq X$ where A is (1,2)*-$g\alpha b\hat{g}$-open set relative to B and B is (1,2)*-$g\alpha b\hat{g}$-open set in (X, τ_1, τ_2), then A is (1,2)*-$g\alpha b\hat{g}$-open set in (X, τ_1, τ_2).

Proof: Let F be a (1,2)*-$\alpha b\hat{g}$-closed set in X and suppose that $F \subseteq A$. Then $F = F \cap B$ is (1,2)*-$\alpha b\hat{g}$-closed set in B. Since A is (1,2)*-$g\alpha b\hat{g}$-open set relative to B, we have $F \subseteq \tau_{1,2}\text{-}int_B(A)$. Since $\tau_{1,2}\text{-}int_B(A)$ is an (1,2)*-open set relative to B, we have $F \subseteq G \cap B \subseteq A$ for some (1,2)*-open set G in X. Since B is (1,2)*-$g\alpha b\hat{g}$-open set in X, we have $F \subseteq \tau_{1,2}\text{-}int(B) \subseteq B$. Therefore $F \subseteq \tau_{1,2}\text{-}int(B) \cap G \subseteq B \cap G \subseteq A$. It follows that, $F \subseteq \tau_{1,2}\text{-}int(A)$. Hence A is (1,2)*-$g\alpha b\hat{g}$-open set in (X, τ_1, τ_2).

Theorem 3.3.3: If $\tau_{1,2}\text{-}int(A) \subseteq B \subseteq A$ and if A is (1,2)*-$g\alpha b\hat{g}$-open set in X, then B is (1,2)*-$g\alpha b\hat{g}$-open set in (X, τ_1, τ_2).

Proof: Suppose that $\tau_{1,2}\text{-}int(A) \subseteq B \subseteq A$ and if A is (1,2)*-$g\alpha b\hat{g}$-open set in X then $A^c \subseteq B^c \subseteq \tau_{1,2}\text{-}cl(A^c)$. Since A^c is (1,2)*-$g\alpha b\hat{g}$-closed set in (X, τ_1, τ_2), by corollary 3.2.1 B is (1,2)*-$g\alpha b\hat{g}$-open set in (X, τ_1, τ_2).

Theorem 3.3.4: A set A is (1,2)*- $g\alpha b\hat{g}$-closed set in (X, τ_1, τ_2) if and only if $\tau_{1,2}\text{-}cl(A)\backslash A$ is (1,2)*-$g\alpha b\hat{g}$-open set in (X, τ_1, τ_2).

Proof: Suppose that A is (1,2)*- $g\alpha b\hat{g}$-closed set. Let F $\subseteq \tau_{1,2}\text{-}cl(A)\backslash A$ where F is (1,2)*-$\alpha b\hat{g}$-closed. By theorem 3.2.3, F is empty. Therefore $F \subseteq \tau_{1,2}\text{-}int(\tau_{1,2}\text{-}cl(A)\backslash A)$. By theorem 3.3.1, we have $\tau_{1,2}\text{-}cl(A)\backslash A$ is (1,2)*-$g\alpha b\hat{g}$-open set in (X, τ_1, τ_2).

Conversely, Suppose that $\tau_{1,2}\text{-}cl(A)\backslash A$ is (1,2)*-$g\alpha b\hat{g}$-open set in (X, τ_1, τ_2). Let $A \subseteq G$ where G be (1,2)*-$\alpha b\hat{g}$-open set. Then $\tau_{1,2}\text{-}cl(A) \cap G^c \subseteq \tau_{1,2}\text{-}cl(A) \cap A^c = \tau_{1,2}\text{-}cl(A)\backslash A$. Since $\tau_{1,2}\text{-}cl(A) \cap G^c$ is (1,2)*-$\alpha b\hat{g}$-closed set and by hypothesis, we have $\tau_{1,2}\text{-}cl(A) \cap G^c \subseteq \tau_{1,2}\text{-}int(\tau_{1,2}\text{-}cl(A)\backslash A) = \phi$. So $\tau_{1,2}\text{-}cl(A) \subseteq G$. Hence A is (1,2)*- $g\alpha b\hat{g}$-closed set in (X, τ_1, τ_2).

3.4 (1,2)*-$g\alpha b\hat{g}$- CONTINUOUS FUNCTION

We introduce the following definition.

Definition 3.4: A function f: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) is called (1,2)*-$g\alpha b\hat{g}$-continuous if f^{-1}(V) is a (1,2)*-$g\alpha b\hat{g}$-closed set of (X, τ_1 ,τ_2) for every closed set V of (Y, σ_1, σ_2).

Theorem 3.4.1: Every continuous map f: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) is (1,2)*-$g\alpha b\hat{g}$-continuous.

Proof: Let V be a (1,2)*- closed set in (Y, σ_1, σ_2) , then f^{-1}(V) is a (1,2)*- closed set in (X, τ_1 ,τ_2). Since every (1,2)*-closed set is (1,2)*-$g\alpha b\hat{g}$-closed set, f^{-1}(V) is a (1,2)*-$g\alpha b\hat{g}$- closed set in (X, τ_1 ,τ_2). Hence f is an (1,2)*- $g\alpha b\hat{g}$- continuous.

Remark 3.4.1: The converse of the above theorem need not true. This is proved in the following example:

Example 3.4.1: Let X={a, b, c}=Y with $\tau_{1,2}$= {ϕ, X, {a}, {b, c}} and $\sigma_{1,2}$= {ϕ, Y, {a}, {a, b}}. Let f: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) be the identity map, then f is (1,2)*-$g\alpha b\hat{g}$-continuous but not (1,2)*- continuous. For the (1,2)*-closed set {c} in (Y, σ_1, σ_2), f^{-1}({c})={c} is (1,2)*-$g\alpha b\hat{g}$-closed set, but not (1,2)*-closed set in (X, τ_1 ,τ_2) .

Theorem 3.4.2: Every (1,2)*-$g\alpha b\hat{g}$-continuous map is a) (1,2)*-g- continuous map, b) (1,2)*-αg- continuous map, c) (1,2)*-gs-continuous map, d) (1,2)*- gp-continuous map, e) (1,2)*-gsp-continuous map, f) (1,2)*-gb-continuous map, g) (1,2)*-$g\alpha$- continuous map.

Proof: Let f: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) be a (1,2)*-$g\alpha b\hat{g}$-continuous map. Let V be a (1,2)*-closed set in (Y, σ_1, σ_2), then f^{-1}(V) is a (1,2)*-$g\alpha b\hat{g}$-closed set in (X, τ_1 ,τ_2). Then by Theorem 3.1.2 a), b), c), d), e), f) and g), f is an (1,2)*-g-continuous, (1,2)*-αg-continuous, (1,2)*-gs-continuous, (1,2)*-gp-continuous, (1,2)*-gsp-continuous, (1,2)*-gb-continuous and (1,2)*-$g\alpha$-continuous respectively.

Remark 3.4.2: The converse of the above Theorem need not be true. This is proved in the following examples:

Example 3.4.2: Let X={a, b, c}=Y with $\tau_{1,2}$= {ϕ, X, {a}, {a, b}} and $\sigma_{1,2}$= {ϕ, Y, {a}, {b}, {a, b}}. Let f: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) be the identity map, then f is (1,2)*-g-continuous but not (1,2)*-$g\alpha b\hat{g}$-continuous. For the (1,2)*-closed set {a, c} in (Y, σ_1, σ_2), f^{-1}({a, c})={a, c} is (1,2)*-g-closed set, but not (1,2)*-$g\alpha b\hat{g}$-closed set in (X, τ_1 ,τ_2).

Example 3.4.3: Let X={a, b, c}=Y with $\tau_{1,2}$= {ϕ, X, {a}, {a, b}} and $\sigma_{1,2}$= {ϕ, Y, {a}, {b}, {a, b}}. Let *f*: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) be the identity map, then f is (1,2)*-αg-continuous but not (1,2)*-$g\alpha b\hat{g}$-continuous. For the (1,2)*-closed set {a, c} in (Y, σ_1, σ_2), f^{-1}({a, c})={a, c} is (1,2)*-αg-closed set, but not (1,2)*-$g\alpha b\hat{g}$-closed set in (X, τ_1 ,τ_2).

Example 3.4.4: Let X={a, b, c}=Y with $\tau_{1,2}$= {ϕ, X, {a}, {a, b}} and $\sigma_{1,2}$= {ϕ, Y, {a}, {b, c}}. Define *f*: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) by *f*(a)={c}, *f*(b)={a}, *f*(c)={b}, and f^{-1}(c)={a}, f^{-1}(a)={b}, f^{-1}(b)={c}, then f is (1,2)*-$g\alpha$-continuous but not (1,2)*- $g\alpha b\hat{g}$-continuous. For the (1,2)*-closed set {a} in (Y, σ_1, σ_2), f^{-1}({a})={b} is (1,2)*-$g\alpha$-closed set, but not (1,2)*-$g\alpha b\hat{g}$-closed set in (X, τ_1 ,τ_2) .

Example 3.4.5: Let X={a, b, c}=Y with $\tau_{1,2}$= {ϕ, X, {a}, {a, b}} and $\sigma_{1,2}$= {ϕ, Y, {a}, {b}, {a, b}}. Let *f*: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) be the identity map, then f is (1,2)*-gs-continuous but not (1,2)*- $g\alpha b\hat{g}$-continuous. For the (1,2)*-closed set {a, c} in (Y, σ_1, σ_2), f^{-1}({a,c})={a,c} is (1,2)*-gs-closed set, but not (1,2)*-$g\alpha b\hat{g}$-closed set in (X, τ_1 ,τ_2) .

Example 3.4.6: Let X={a, b, c}=Y with $\tau_{1,2}$= {ϕ, X, {a}, {a, b}} and $\sigma_{1,2}$= {ϕ, Y, {a}, {b}, {a, b}}. Define *f*: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) by *f*(a)={c}, *f*(b)={b}, *f*(c)={a}, and f^{-1}(c)={a}, f^{-1}(b)={b}, f^{-1}(a)={c}, then f is (1,2)*-gp-continuous but not (1,2)*- $g\alpha b\hat{g}$-continuous. For the (1,2)*-closed set {a, c} in (Y, σ_1, σ_2), f^{-1}({a, c})={a, c} is (1,2)*-gp-closed set, but not (1,2)*-$g\alpha b\hat{g}$-closed set in (X, τ_1 ,τ_2).

Example 3.4.7: Let X={a, b, c}=Y with $\tau_{1,2}$= {ϕ, X, {a}, {a, b}} and $\sigma_{1,2}$= {ϕ, Y, {a}, {b}, {a, b}}. Let *f*: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) be an identity map, then f is (1,2)*-gsp-continuous but not (1,2)*-$g\alpha b\hat{g}$-continuous. For the (1,2)*-closed set {a, c} in (Y, σ_1, σ_2), f^{-1}({a,c})={a,c} is (1,2)*-gsp-closed set, but not (1,2)*-$g\alpha b\hat{g}$-closed set in (X, τ_1 ,τ_2) .

Example 3.4.8: Let X={a, b, c}=Y with $\tau_{1,2}$= {ϕ, X, {a}, {b}, {a, b}} and $\sigma_{1,2}$= {ϕ, Y, {a, c}}. Let *f*: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) be an identity map, then f is (1,2)*-gb-continuous but not (1,2)*-$g\alpha b\hat{g}$-continuous. For the (1,2)*-closed set {c} in (Y, σ_1, σ_2), f^{-1}({c})={c} is (1,2)*-gb-closed set, but not (1,2)*-$g\alpha b\hat{g}$-closed set in (X, τ_1 ,τ_2) .

3.5 APPLICATIONS OF (1,2)*-$g\alpha b\hat{g}$-CLOSED SETS

As an application of (1,2)*-$g\alpha b\hat{g}$-closed sets we introduce a new space namely (1,2)*-$T_{g\alpha b\,\hat{g}}$ -space.

Definition 3.5: A space (X, τ_1 ,τ_2) is called (1,2)*-$T_{g\alpha b\,\hat{g}}$ -space if every (1,2)*-$g\alpha b\hat{g}$-closed sets in it is (1,2)*-closed set.

Theorem 3.5.1: For a space (X, τ_1, τ_2) the following are equivalent:

a) (X, τ_1 ,τ_2) is a (1,2)*-$T_{g\alpha b\,\hat{g}}$ -space.
b) Every singleton of (X, τ_1, τ_2) is either (1,2)*-$\alpha b\hat{g}$-closed set or (1,2)*-open set.

Proof:
$(\boldsymbol{a}) \Rightarrow (\boldsymbol{b})$**:** Let (X, τ_1 ,τ_2) be a (1,2)*-$T_{g\alpha b\,\hat{g}}$ -space. Assume that for some$\in X$, the set $\{x\}$ is not (1,2)*-$\alpha b\hat{g}$-closed set in (X, τ_1 ,τ_2). Then the only (1,2)*-$\alpha b\hat{g}$-open set containing $\{x\}^c$ is the space X itself and so $\{x\}^c$ is (1,2)*-$g\alpha b\hat{g}$-closed set in (X, τ_1 ,τ_2). By our assumption $\{x\}^c$ is (1,2)*-closed set in (X, τ_1 ,τ_2) and hence $\{x\}$ is (1,2)*-open set.

$(\boldsymbol{b}) \Rightarrow (\boldsymbol{a})$**:** Let A be a (1,2)*-$g\alpha b\hat{g}$-closed subset of (X, τ_1 ,τ_2) and let $x \in \tau_{1,2}\text{-}cl(A)$. By assumption $\{x\}$ is either (1,2)*-$\alpha b\hat{g}$-closed set or (1,2)*-open set.

***CASE* 1: Suppose $\{\boldsymbol{x}\}$ is (1,2)*-$\boldsymbol{\alpha b\hat{g}}$-closed set.**

If $\{x\} \notin A$, then $\tau_{1,2}\text{-}cl(A)\backslash A$ contains a non empty (1,2)*-$\alpha b\hat{g}$-closed set $\{x\}$, which is a contradiction. Therefore $\in A$. This implies that $\tau_{1,2}\text{-}cl(A) \subseteq A$. Hence A is (1,2)*-closed set.

***CASE* 2: Suppose $\{\boldsymbol{x}\}$ is (1,2)*-open set.**

Since $x \in \tau_{1,2}\text{-}cl(A)$, $\{x\} \cap A \neq \phi$ and therefore $\tau_{1,2}\text{-}cl(A) \subseteq A$. Hence A is (1,2)*-closed set.

Theorem 3.5.2:

a) Every (1,2)*-$T_{1/2}$-space is a (1,2)*-$T_{g\alpha b\,\hat{g}}$ -space.
b) Every (1,2)*-T_b-space is a (1,2)*-$T_{g\alpha b\,\hat{g}}$ -space.
c) Every (1,2)*- ${}_{\alpha}T_b$-space is a (1,2)*-$T_{g\alpha b\,\hat{g}}$ -space.

Proof:

a) Let (X, τ_1 ,τ_2) be a (1,2)*-$T_{1/2}$-space, and let A be a (1,2)*-$g\alpha b\hat{g}$-closed subset of (X, τ_1 ,τ_2). By theorem 3.1.2(a), A is (1,2)*-g-closed set. Since (X, τ_1 ,τ_2) is a (1,2)*-$T_{1/2}$-space, A is (1,2)*-closed set in (X, τ_1 ,τ_2). Hence (X, τ_1 ,τ_2) is (1,2)*-$T_{g\alpha b\,\hat{g}}$ -space.
b) Let (X, τ_1 ,τ_2) be a (1,2)*-T_b-space, and let A be a (1,2)*-$g\alpha b\hat{g}$-closed subset of (X, τ_1 ,τ_2). By theorem 3.1.2(d), A is (1,2)*-g-closed set. Since (X, τ_1 ,τ_2) is a (1,2)*-T_b-space, A is (1,2)*-closed set in (X, τ_1 ,τ_2). Hence (X, τ_1 ,τ_2) is (1,2)*-$T_{g\alpha b\,\hat{g}}$ -space.
c) Let (X, τ_1 ,τ_2) be a (1,2)*- ${}_{\alpha}T_b$-space, and let A be a (1,2)*-$g\alpha b\hat{g}$-closed subset of (X, τ_1 ,τ_2). By theorem 3.1.2(b), A is (1,2)*-αg-closed set. Since (X, τ_1 ,τ_2) is a (1,2)*- ${}_{\alpha}T_b$-space, A is (1,2)*-closed set in (X, τ_1 ,τ_2). Hence (X, τ_1 ,τ_2) is (1,2)*-$T_{g\alpha b\,\hat{g}}$ -space.

Remark 3.5.1: The converse of the above Theorem need not be true. This is proved in the following examples:

Example 3.5.1: Let X= {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a, b}}. Let (X, τ_1 ,τ_2) be a (1,2)*-$T_{g\alpha b\,\hat{g}}$ -space. Here A = {a, c} is (1,2)*-g-closed set but not (1,2)*-closed set. Hence (X, τ_1 ,τ_2) is not (1,2)*-$T_{1/2}$-space.

Example 3.5.2: Let X= {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a}, {b}, {a, b}}. Let (X, τ_1 ,τ_2) be a (1,2)*-$T_{g\alpha b\,\hat{g}}$ -space. Here A = {a} is (1,2)*-gs-closed set but not (1,2)*-closed set. Hence (X, τ_1 ,τ_2) is not (1,2)*-T_b-space.

Example 3.5.3: Let X= {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a, b}}. Let (X, τ_1 ,τ_2) be a (1,2)*-$T_{g\alpha b\,\hat{g}}$ -space. Here A = {a, c} is (1,2)*-αg-closed set but not (1,2)*-closed set. Hence (X, τ_1 ,τ_2) is not (1,2)*- ${}_{\alpha}T_b$-space.

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