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THE EDGE VERSION OF INVERSE SUM INDEG INDEX OF CONNECTED GRAPH

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ABSTRACT

The inverse sum indeg index is defined as $ISI(G) = \sum_{uv\in E(G)} \frac{1}{\frac{1}{d_u}+\frac{1}{d_v}} = \sum_{uv\in E(G)} \frac{d_u \times d_v}{d_u+d_v}$ *in which degree of vertex v is denoted by* d_v. *Now we define a new version of ISI index as* $ISI_e\ (G) = \sum_{ef\in E(L(G))} \frac{d_e \times d_f}{d_e+d_f}$, *where* d_e *denotes the degree of an edge e in G. The goal of this paper is to the study of the* ISI_e *index of graphs.*

Key words: *sum indeg index, Edge version, circumcronone series, Benzenoid.*

2000 Mathematics subject classification: *05C12, 05C75.*

1. INTRODUCTION

A graph is a collection of points and lines connecting them. The points and lines of a graph are also called vertices and edges respectively. If e is an edge of G, connecting the vertices u and v, then we write $e = uv$ and say "u and v are adjacent". A connected graph is a graph such that there is a path between all pairs of vertices. The distance $d(u, v)$ between two vertices u and v is the length of the shortest path between u and v in G. A simple graph is an unweighted, undirected graph without loops or multiple edges. A single number that can be used to characterize some property of the graph is called a Topological Index for that graph. Obviously, the number of vertices and the number of edges are topological indices. The Wiener index is the first graph invariant reported (distance based) topological index and is defined as a half sum of the distances between all the pairs of vertices in a graph [1]. Also, the edge version of Wiener index which were based on distance between edges introduced by A. Iranmanesh *et al.* in 2008 [2]. These topological indices are formulated as follows:

$$W_v(G) = \sum_{\{u,v\}\subset V(G)} d(u, v) \tag{1}$$

$$W_e(G) = \sum_{\{e,f\}\subset E(G)} d(e, f) \tag{2}$$

in which degree of vertex v and edge e are denoted by d_v *and* d_e. The degree of a vertex v is the number of vertices joining to v. Also, the degree of an edge $e \in E(G)$ is the number of its adjacent vertices in $L(G)$, where the line graph L(G) of a graph G is defined to be the graph whose vertices are the edges of G, with two vertices being adjacent if the corresponding edges share a vertex in G.

$$ISI\ (G) = \sum_{uv\in E(G)} \frac{d_u \times d_v}{d_u+d_v} \tag{3}$$

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Here, we define the new member (edge version of inverse sum indeg index) of this class on the ground of the end-vertex degree d_e and d_f of edges e and f in a line graph of G as follows:

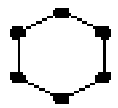
$$ISI_e\ (G) = \sum_{ef \in E(L(G))} \frac{d_e \times d_f}{d_e + d_f} \qquad (4)$$

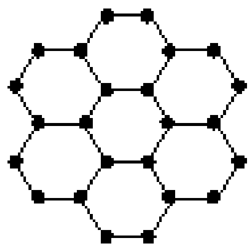
where $d_e$denotes the degree of the edge e in G. The aim of this paper is to study the ISI_e index for some families of graphs.

2. MAIN RESULT

The goal of this section is to study and computing the ISI_e index of the complete graph $K_{n\,,}$ path P_n, cycle C_n and and star graph S_n.

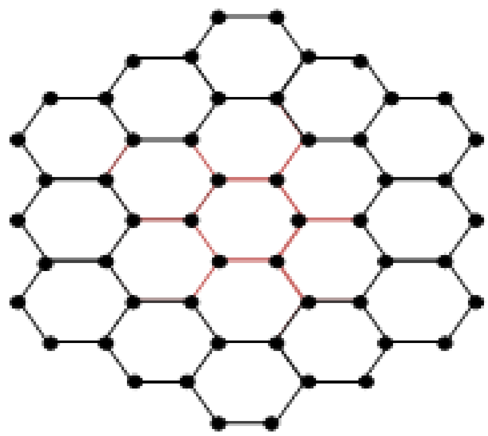
Also we obtain a closed formula of this index for a famous molecular graph that is Circumcoronene Series of Benzenoid H_k. The circumcoronene homologous series of benzenoid is family of molecular graph, which consist several copy of benzene C_6 on circumference. The first terms of this series are $H_1 =$ benzene, $H_2 =$ coronene, $H_3 =$ circumcoronene $H_4 =$ circumcircumcoronene, see Figure 1 and Figure 2, where they are shown.





$H_1 = C_6$
(=Benzene)

$H_2 = Ca(C_6)$
(=Coronene)



$H_3 =$ circumcronone

Figure-1: The first 3 graphs H_1, H_2 *and* H_3 from the circumcronone series.

These molecular graphs are presented in many papers, (see the paper series [18-21]). For every positive integer number k, the general form of circum- coronene series of benzenoid H_k is shown in Figure 1. Also, its line graph is shown in Figure 3. For more information of this family, see the paper series [3, 6, 9-15].

Lemma 1: Let K_n be the complete graph on n vertices. Then$L(K_n)$ will be a $2(n-1)$ regular graph and for every $e \in E(K_n)$ $(e \in V(L(K_n)))$ $d_v = 2(n-1)$ .So $|E(L(K_n)| = \frac{1}{2}|E(K_n)|2(n-1) = \frac{1}{2}n(n-1)^2$. This implies that $ISI_e\ (K_n) = \sum_{ef \in E(L(K_n))} \frac{d_e \times d_f}{d_e + d_f} = |E(L(K_n)| \frac{2(n-1) \times 2(n-1)}{2(n-1)+2(n-1)} = \frac{n(n-1)^3}{2}$. (5)

Lemma 2: Let C_n be the cycle of length n: Then one can see that $L(C_n) = C_n$ and for every $v \in V(c_n)$ and $e \in V\big(L(C_n)\big) d_v = d_e = 2$, so $ISI_e\ (C_n) = |E(L(C_n)|\ \frac{2 \times 2}{2+2} = n$. (6)

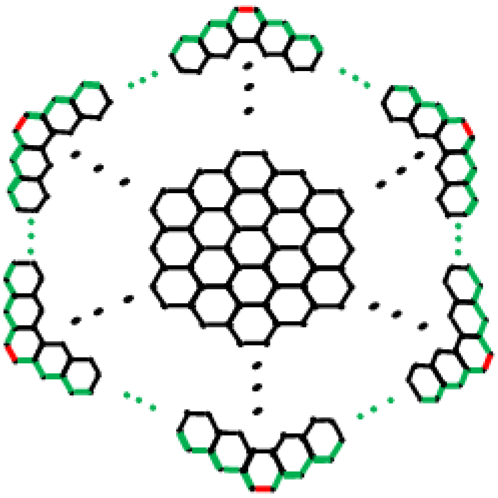


Figure-2: The Circumcoronene Series of Benzenoid $H_k(k \geq 1)$ with dges

Lemma 3: Let P_n be a path of length n.Then $L(P_n) = P_{n-1}$and for all vertices of $L(P_n)$: d_e = 2, (except first and end vertices on path, that are as degree one), thus

$$\mathrm{ISI_e}\ (\mathrm{P_n}) = (\mathrm{n}-3) \times \left(\frac{2 \times 2}{2+2}\right) + 2\ \times \left(\frac{2 \times 1}{2+1}\right)$$
$$= \frac{3\mathrm{n}-5}{3}. \tag{7}$$

Lemma 4: Let S_n be a star graph with $n+1$ vertices. Then $L(S_n)$ will be a$(n-1)$ regular graph (or a complete graph on n vertices) and for every $e \in E(S_n)$ or $(e \in V(L(S_n)))d_v = (n-1)$. So $|E(L(S_n))| = |E(K_n)| = \frac{1}{2}n(n-1)$. This implies that

$$ISI_e\ (S_n) = |E(L(S_n)|\ \frac{(n-1)\times(n-1)}{(n-1)+(n-1)} = \frac{n(n-1)^2}{4} \tag{8}$$

In addition, to achieve our aims we need the following definition.

Definition 1[16]**:** Let G be the molecular graph and d_v is degree of vertex $v \in V(G)$ (Obviously
$1 \leq \delta \leq d_v \leq \Delta \leq n-1$, where
$\delta = min\{d_v/v \in V(G)\}$ and
$\Delta = max\{d_v|v \in V(G)\}$.

Let $G = H_k$, we divide edge set $E(G)$ and vertex set $V(G)$ of graph G to several partitions, as follow: $for\ i \in [2\delta, 2\Delta\,]\,, define\ E_i = \{e = uv \in |\ d_u + d_v = i\}, for\ j \in\ [\delta^2, \Delta^2\,]\,, define$
$E_j = \{e = uv \in |\ d_u \times d_v = j\}, for\ k \in\ [\delta, \Delta\,]\ , define\ V_k = \{v \in V(G)|\ d_v = k\}$.

Theorem 4: Consider the graph $G = H_k\ (k \geq 2)$ is circumcoronene series. Then the Inverse sum indeg index of H_k given by

$$ISI(H_k\,) = \frac{1}{10}(135k^2 - 81k - 54). \tag{9}$$

Proof: Let $G = H_k\ (k \geq 2)$ be the circumcoronene series of benzenoid. Thus, this graph has $6k^2$ vertices and $\frac{3\times6k(k-1)+2\times6k}{2} = 9k^2$ -3k edges. From the structure of H_k .0ne can see that there are two partitions $V_3 = \{v \in V(H_k\,)|\ d_v = 3\}$ and $V_2 = \{v \in V(H_k\,)|\ d_v = 2\}$ with size $6k(k-1)$ and $6k$, respectively. The edge set of graph H_k can be divide in to three partitions $E_4, E_5\ and\ E_6$. In other words, For every $e = uv$ belong to E_6, $d_u = d_v = 3$. Similarly, for every $e = uv$ belong to E_4, $d_u = d_v = 2$ Finally, if $e = uv$ be an edge of E_5 then $d_u = 2\ and\ d_v = 3$. We mark the members of E_4 by red color, the members of E_5 by green color and the members of E_6 by black color, in Figure 2. Thus, we have following computations:

Clearly $|E_4| = 4, |E_5| = 12(k-1)\ and\ |E_6| = 9k^2 - 15k + 6.$ Hence we have

$$ISI(H_k\,) = \sum_{e=uv\in E(H_k\,)} \frac{d_u \times d_v}{d_u + d_v}$$
$$= \sum_{e=uv\in E_9} \frac{d_u \times d_v}{d_u + d_v} + \sum_{e=uv\in E_6} \frac{d_u \times d_v}{d_u + d_v} + \sum_{e=uv\in E_4} \frac{d_u \times d_v}{d_u + d_v}$$

$$= \frac{9}{6}|E_9| + \frac{6}{5}|E_6| + \frac{4}{4}|E_4|$$
$$= \frac{3}{2}[9k^2 - 15k + 6] + \frac{6}{5}[12(k-1)] + 1(6)$$

$$ISI(H_k) = \frac{135k^2 - 81k - 54}{10}$$

Theorem 5: Let G be the graphs from the circumcoronene series of benzenoid H_k $\forall\, k \geq 1$ with $6k^2$ vertices and $9k^2$-3k edges then

$$ISI_e(H_k) = \frac{(1260k^2 - 1170k + 99)}{35} \tag{10}$$

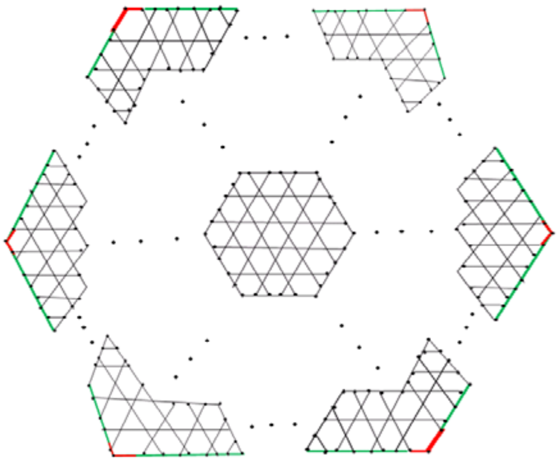


Figure-3: The general representation of line graph of circumcoronene series of Benzenoid H_k $(k \geq 1)$ with edges marking [11]

Proof: Consider the circumcoronene series of benzenoid H_k for all positive integer number k. By theorem 4 the edge version of Inverse sum indeg index of H_k is equivalent with Inverse sum indeg index of its line graph. At first, we partition the vertex set and edge set of H_k as follow:

$V_3 = \{v \in V(H_k) |\, d_v = 3\} \Rightarrow |V_3| = 6k(k-1)$
$V_2 = \{v \in V(H_k) |\, d_v = 2\} \Rightarrow |V_2| = 6k$
$E_4 = \{e = uv \in E(H_k) | d_u = d_v = 2\} \Rightarrow |E_4| = 6$
$E_5 = \{e = uv \in E(H_k) | d_u = 3 \,\&\, d_v = 2\} \Rightarrow |E_5| = 12(k-1)$
$E_6 = \{e = uv \in E(H_k) | d_u = d_v = 3\} \Rightarrow |E_6| = 9k^2 - 15k + 6$

In Figure 1, all edges belong to E_4, E_5 *and* E_6 marked by red, green and black colors, respectively. It is very clear that number of vertices in $L(H_k)$ = number of edges in H_k $= 9k^2 - 15k + 6 + 12k - 12 + 6 = 9k^2 - 3$ and number of edges in $L(H_k)$ = sum of the degrees of the edges of H_k $= \frac{4(9k^2 - 15k + 6) + 3 \times 12(k-1) + 2 \times 6}{2}$ $= 18k^2 - 12k$. Alternatively, we can partition the vertex set and edge set of $L(H_k)$ by using the results of [11] as follow:

$VL_2 = \{e \in E(H_k) |\, d_e = 2\} \Rightarrow |VL_2| = |E_4| = 6$
$VL_3 = \{e \in E(H_k) |\, d_e = 3\} \Rightarrow |VL_3| = |E_5| = 12(k-1)$
$VL_4 = \{e \in V(L(H_k))\ ore \in\ E(H_k) |\, d_e = 4\} \Rightarrow |VL_4| = |E_6| = 9k^2 - 15k + 6$
$EL_5 = \{\mu = ef \epsilon E(L(H_k)) | d_e = 2, d_f = 3\} \Rightarrow |EL_5| = 2|VL_2| = 12$
$EL_6 = \{\mu = ef \epsilon E(L(H_k)) | d_e = d_f = 3\} \Rightarrow |EL_6| = |VL_3| - |VL_2| = 6(2k-3)$
$EL_7 = \{\mu = ef \epsilon E(L(H_k)) | d_e = 3, d_f = 3\} \Rightarrow |EL_7| = |VL_3| - |VL_2| = 12(k-1)$
$EL_8 = \{\mu = ef \epsilon E(L(H_k)) | d_e = d_f = 4\} \Rightarrow |EL_8| = E(L(H_k)) - |EL_7| - |EL_6| - |EL_5| = 18k^2 - 36k + 18$
$= 18(k-1)^2$

In Figure 3 all edges belong to EL_5, EL_6, EL_7 *and* EL_8 *are* marked by red, green, and blue black colors, respectively.

Hence

$$ISI_e(H_k) = \sum_{e=uv \in E(L(H_k))} \frac{d_u \times d_v}{d_u + d_v}$$
$$= \sum_{ef \in EL_5} \frac{2 \times 3}{2+3} + \sum_{ef \in EL_6} \frac{3 \times 3}{3+3} + \sum_{ef \in EL_7} \frac{3 \times 4}{3+4} + \sum_{ef \in EL_8} \frac{4 \times 4}{4+4}$$

$$= \frac{6}{5}|EL_5| + \frac{9}{6}|EL_6| + \frac{12}{7}|EL_7| + \frac{16}{8}|EL_8|$$
$$= \frac{6}{5}(12) + \frac{3}{2}(12k - 18) + \frac{12}{7}(12k - 12) + 2(18k^2 - 36k + 18)$$

$$ISI_e\ (H_k) = \frac{(1260\,k^2 - 1170\,k + 99)}{35}$$

Example 1: Let H_3be the Circumcoronene. Then the number of edges $e_5, e_6, e_7\ and\ e_8$ in line graph H_3 are equal to 12, 18, 24 and 72, respectively (see Figure 4). So ISI_e index ofH_3 is equal to

$$ISI_e\ (H_k) = ISI(L(H_3)) = 226.5428$$

REFERENCES

1. H. Wiener, Structural determination of paraffin boiling points, J. Amer. Chem. Soc, (1947), 69, 7-20.
2. A. Iranmanesh, I. Gutman, O. Khormali, A. Mahmiani, MATCH Commun. Math. Comput. Chem. 61(3), (2009), 663.
3. M.R. Farahani, Eccentricity Version of Atom-Bond Connectivity Index of Ben- zenoid Family ABC5 (Hk), World Applied Sciences Journal, (2013), 21(9), 1260-1265.
4. B. Furtula, A. Graovac, D. Vukicevic, Disc. Appl. Math. 157, (2009). 2828
5. D. Vukicevic, B. Furtula, Topological index based on the ratios of geometrical and arithmetical means of end-vertex degrees of edges, J. Mathematical Chemistry. 46, (2009). 1369.
6. M.R. Farahani A New Version of Atom-Bond Connectivity Index of Circum- coronene Series of Benzenoid, J. Math. Nano Science. 2(1), (2012), 15-20.
7. J. Asadpour Some topological polynomial indices of nanostructures, Optoelec- tron. Adv. Mater. - Rapid Commun. 5(7), 2011, 769 - 772.
8. A. Madanshekaf, M. Ghaneeei, Computing two topological indices of nanostars dendrimer, Optoelectron. Adv. Mater.-Rapid Commun. 4(12), (2010), 2200-2202.
9. V. Chepoi S. Klavzar, Distances in benzenoid systems: Further developments, Discrete Math. 192, (1998) 27-39.
10. M.V. Diudea, Studia Univ. Babes-Bolyai, 4, (2003) 3-21.
11. M.R. Farahani, The Edge Version of Geometric-Arithmetic Index of Benzenoid Graph, Romanian Academy Seri B. 15(2). (2013). .
12. S. Klavzar, I. Gutman, B. Mohar, Labeling of Benzenoid Systems which Reects the Vertex-Distance Relations, J. Chem. Int Comput. Sci. 35, (1995) 590-593.
13. S. Klavzar, A Bird's Eye View of The Cut Method And A Survey of Its Appli- cations In Chemical Graph Theory, MATCH Commun. Math. Comput. Chem. 60, (2008), 255-274.
14. A. Soncini, E. Steiner, P.W. Fowler, R.W.A. Havenith, L.W. Jenneskens. Perimeter E_ects on Ring Currents in Polycyclic Aromatic Hydrocarbons: Circum- coronene and Two Hexa benzocoronenes, Chem. Eur. J. 9, (2003) 2974-2981.
15. P. Zigert, S. Klavzar, I. Gutman. Calculating the hyper-Wiener index of ben- zenoid hydrocarbons, ACH Models Chem. 137, (2000) 83-94.
16. M.R. Farahani, Computing Randic, Geometric-Arithmetic and Atom-Bond Connectivity indices of Circumcoronene Series of Benzenoid, Int. J. Chem. Model. 5(5), (2013).
17. M.R. Farahani,The edge version of atom-bond connectivity index of connected graphs, Int. J. Chem. Model., (36/2013), 277-284.
18. M.R. Farahani and M.P. Vlad. On the Schultz, Modified Schultz and Hosoya polynomials and Derived Indices of Capra-designed planar Benzenoids. *Studia Univ. Babes-Bolyai*. 57(4) In press, (2012).
19. M.R. Farahani, On the Schultz polynomial, Modified Schultz polynomial, Hosoya polynomial and Wiener index of circumcoronene series of benzenoid, *J. Applied Mathe. & Informatics.* 31(3) in press, (2013).
20. Gutman and S. Klavžar, M. Petkovsek and P. Zigert. On Hosoya polynomials of benzenoid. graphs, *MATCH Commun. Math. Comput. Chem.* 43, 49–66 (2001).
21. Gutman and S. Klavžar. A method for calculationg Wiener numbers of benzenoid hydrocarbons and phenylenes. *ACH Models Chem.* 133, 389-399. (1996).

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