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ANOTHER DECOMPOSITION OF FUZZY CONTINUITY

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ABSTRACT

There are various types of generalizations of fuzzy continuous functions in the development of fuzzy topology. Recently some decompositions of fuzzy continuity have been obtained by various authors with the help of generalized fuzzy continuous functions in fuzzy topological spaces. In this paper we obtain a decomposition of fuzzy continuity by using generalized fuzzy continuity in fuzzy topology.

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1. INTRODUCTION

The idea of fuzzy sets and fuzzy set operations were first introduced by L. A. Zadeh in his classical paper [6] in the year 1965. Subsequently many researchers have worked on various basic concepts from general topology using fuzzy sets and developed the theory of fuzzy topological spaces. The notion of fuzzy sets naturally plays a very significant role in the study of fuzzy topology introduced by C. L. Chang [2].

Different types of generalizations of fuzzy continuous functions were introduced and studied by various authors in the recent development of fuzzy topology. The decomposition of fuzzy continuity is one of the many problems in fuzzy topology. Tong [5] obtained a decomposition of fuzzy continuity by introducing two weak notions of fuzzy continuity namely, fuzzy strong semi-continuity and fuzzy precontinuity. Rajamani [4] obtained a decomposition of fuzzy continuity. In this paper, we obtain a decomposition of fuzzy continuity in fuzzy topological spaces by using generalized fuzzy continuity.

2. PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) (or X and Y) represent fuzzy topological spaces (fts, for short) in Chang's [2] sense.

We recall the following definitions which are useful in the sequel.

Definition: 2.1 [3, 6]
If X is a set, then any function $A : X \to [0, 1]$ (from X to the closed unit interval [0, 1]) is called a fuzzy set in X.

Definition: 2.2 [3]
(i) The complement of a fuzzy set A, denoted by A', is defined by $A'(x) = 1 - A(x), \forall x \in X$.

(ii) Union of two fuzzy sets A and B, denoted by $A \cup B$, is defined by $(A \cup B)(x) = \max \{ A(x), B(x) \}, \forall x \in X$.

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(iii) Intersection of two fuzzy sets A and B, denoted by $A \cap B$, is defined by $(A \cap B)(x) = \min\{A(x), B(x)\}$, $\forall x \in X$.

Definition: 2.3 [3, 6]
Let f: $X \to Y$ be a function from a set X into a set Y. Let A be a fuzzy subset in X and B be fuzzy subset in Y. Then the Zadeh's function f(A) and $f^{-1}(B)$ are defined by

(i) f(A) is a fuzzy subset of Y where

$$f(A) = \begin{cases} \sup\limits_{z \in f^{-1}(y)} A(z), & if\ \ f^{-1}(y) \neq \phi \\ 0, & otherwise \end{cases}$$

for each $y \in Y$.

(ii) $f^{-1}(B)$ is a fuzzy subset of X where $f^{-1}(B)(x) = B(f(x))$, for each $x \in X$.

Definition: 2.4 [2, 3]
Let X be a set and τ be a family of fuzzy sets in X. Then τ is called a fuzzy topology if τ satisfies the following conditions:

(i) $0, 1 \in \tau$.

(ii) If $A_i \in \tau$, $i \in I$ then $\cup_{i \in I} A_i \in \tau$ or $\vee_{i \in I} A_i \in \tau$.

(iii) If $A, B \in \tau$ then $A \cap B \in \tau$ or $A \wedge B \in \tau$.

The pair (X,τ) is called a fuzzy topological space (or fts). The elements of τ are called fuzzy open sets. Complements of fuzzy open sets are called fuzzy closed sets.

Definition: 2.5 [3]
Let A be a fuzzy set in a fts (X,τ). Then,

(i) the closure of A, denoted by cl(A), is defined by $cl(A) = \wedge \{F : A \leq F$ and F is a fuzzy closed $\}$;

(ii) the interior of A, denoted by int(A), is defined by $int(A) = \vee \{G: G \leq A$ and G is a fuzzy open $\}$.

Definition: 2.6 [1]
Let X be a fuzzy topological space. A fuzzy set λ in X is said to be a generalized fuzzy closed (briefly gf-closed) set if $cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open. The complement of gf-closed set is called gf-open.

Definition: 2.7 [1]
A function f: $X \to Y$ is said to be generalized fuzzy continuous (briefly gf-continuous) if for each fuzzy closed set μ of Y, $f^{-1}(\mu)$ is gf-closed in X.

Definition: 2.8 [2, 4]
A function f: $X \to Y$ is said to be fuzzy continuous if $f^{-1}(\mu)$ is fuzzy open in X for each fuzzy open set μ in Y.

Theorem: 2.9 [3]
Every fuzzy closed set is gf-closed but not conversely.

Theorem 2.10 [1]
Every fuzzy continuous function is gf-continuous but not conversely.

3. DECOMPOSITION OF FUZZY CONTINUITY

In this section, we obtain a decomposition of fuzzy continuity in fuzzy topological spaces by using gf-continuity.
To obtain a decomposition of fuzzy continuity, we first recall the notion of fuzzy LC-continuous function in fuzzy topological spaces and by using gf-continuity, prove that a function is fuzzy continuous if and only if it is both gf-continuous and fuzzy LC-continuous.

Definition: 3.1 [4]
A subset λ in a fts X is called fuzzy LC-set in X if $\lambda = \alpha \wedge \beta$, where α is fuzzy open and β is fuzzy closed in X.

Proposition: 3.2 [4]
Every fuzzy closed set is fuzzy LC-set but not conversely.

Remark: 3.3
gf-closed sets and fuzzy LC-sets are independent of each other.

Example: 3.4
Let X = {a, b} with $\tau = \{0_X, b_{0.7}, 1_X\}$. Then (X,τ) is a fts. Consider μ = 0.5. The fuzzy set μ is gf-closed but not fuzzy LC-set.

Example: 3.5
Let X = {a} with $\tau = \{0_X, a_{0.1}, a_{0.3}, 1_X\}$. Then (X,τ) is a fts. Consider $\mu = a_{0.3}$. The fuzzy set μ is fuzzy LC-set but not fuzzy gf-closed.

Proposition: 3.6
Let (X,τ) be a fuzzy topological space. Then a subset λ of (X,τ) is fuzzy closed if and only if it is both gf-closed and fuzzy LC-set.

Proof: Necessity is trivial. To prove the sufficiency, assume that λ is both gf-closed and fuzzy LC-set. Then $\lambda = \alpha \wedge \beta$, where α is fuzzy open and β is fuzzy closed in (X, τ). Therefore, $\lambda \leq \alpha$ and $\lambda \leq \beta$ and so by hypothesis, $cl(\lambda) \leq \alpha$ and $cl(\lambda) \leq \beta$. Thus $cl(\lambda) \leq \alpha \wedge \beta = \lambda$ and hence $cl(\lambda) = \lambda$ i.e., λ is fuzzy closed in (X, τ).

Definition: 3.7 [4]
A function f : (X, τ) → (Y, σ) is said to be fuzzy LC-continuous if for each fuzzy closed set μ of (Y, σ), $f^{-1}(\mu)$ is a fuzzy LC-set in (X, τ).

Example: 3.8
Let X = Y = {a, b, c} with $\tau = \{0, \chi_{\{a\}}, 1\}$ and $\sigma = \{0, \chi_{\{b, c\}}, 1\}$. Let f: (X,τ) → (Y,σ) be the identity fuzzy function. Then f is fuzzy LC-continuous function.

Remark: 3.9 [4]
Every fuzzy continuous function is fuzzy LC-continuous but not conversely.

Remark: 3.10
gf- continuity and fuzzy LC-continuity are independent of each other.

Example: 3.11
Let X = Y = {a, b}. Consider the fuzzy topology τ as in Example 3.4 and $\sigma = \{0_Y, b_{0.5}, 1_Y\}$. Let f: (X,τ) → (Y,σ) be the identity fuzzy function. Then f is gf-continuous but not fuzzy LC-continuous.

Example: 3.12
Let X = Y = {a}. Consider the fuzzy topology τ as in Example 3.5 and $\sigma = \{0_Y, a_{0.7}, 1_Y\}$. Let f: (X, τ) → (Y, σ) be the identity fuzzy function. Then f is fuzzy LC-continuous but not gf-continuous.

We have the following decomposition for fuzzy continuity.

Theorem: 3.13
A function f: (X,τ) → (Y,σ) is fuzzy continuous if and only if it is both gf-continuous and fuzzy LC-continuous.

Proof: Assume that f is fuzzy continuous. Then by Theorem 2.10 and Remark 3.9, f is both gf-continuous and fuzzy LC-continuous.

Conversely, assume that f is both gf-continuous and fuzzy LC-continuous. Let μ be a fuzzy closed subset of (Y, σ). Then $f^{-1}(\mu)$ is both gf-closed and fuzzy LC-set. By Proposition 3.6, $f^{-1}(\mu)$ is a fuzzy closed set in (X, τ) and so f is fuzzy continuous.

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