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MIXED CONVECTION IN A VERTICAL PARALLEL PLATE MICROCHANNEL WITH ASYMMETRIC WALL HEAT FLUXES UNDER THE EFFECT OF A MAGNETIC FIELD

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ABSTRACT

The mixed convective heat transfer of a conducting Newtonian fluid in an open - ended vertical parallel plate microchannel with asymmetric wall heating at uniform heat fluxes by taking the velocity slip and temperature jump at the wall into account.

The velocity, temperature, skin friction and the rate of heat and mass transfer at the walls are obtained and discussed graphically.

Keywords: *Heat transfer, vertical parallel plate, skin friction and the rate of heat and mass transfer.*

INTRODUCTION

Microfluidic systems typically have characteristic lengths of the order of $1-100\mu m$. Rarefied gaseous flow in these microgeometries has frequently been observed. The Knudsen number *Kn* characterizing the effect of rarefaction is defined as the ratio of the molecular mean free path λ to the characteristic length. A classification of different flow regimes based on *Kn* is given in Schaaf and Chambre [14]. Here, we concern ourselves with a rarefied gas considered near the continuum region in the range a $10^{-2} < Kn < 10^{-1}$, the so-called slip flow. Using the Navier-Stokes equations, Arkilic et al. [1] and Liu et al. [12] found that the theoretical results for some microflows would fit the experimental data as the slip-flow condition induced by rarefaction effect is considered. Larrode et al. [11] and Yu and Ameel [16] considered the temperature jump condition and found that the effect of fluid wall interaction is also important. Aydin and Avci [5],[6] theoretically studied forced convective heat transfer in two different microgeometries, mainly microtube and microduct between two parallel plates, both for the fully developed and for the developing cases.

Chen and Weng [7] analytically studied the fully developed natural convection an open-ended vertical parallel plate microchannel with asymmetric wall temperature distributions. The effects of rarefaction and fluid - wall interaction were shown to increase the volume flow and to decrease the heat transfer rate. Khadrawi et al. [10] analytically investigated the transient hydrodynamics and thermal behaviors of fluid flow in an open-ended vertical parallel-plate microchannel under the effect of the hyperbolic heat conduction model. Haddad et al. [8] numerically investigated the developing hydrodynamical behaviors of free convection gas flow in a vertical open-ended parallel-plate microchannel filled with porous media. Recently, Aydin and Avci [2] studied mixed convection of rarefied gas in a vertical asymmetrically heated microchannel between two parallel plates. In that study, isothermal boundary conditions at walls were considered.The study of magnetohydrodynamic heat transfer flow in a microchannel is not yet found in the literature, but it has many applications in engineering problems such as plasma studies, nuclear reactors, oil exploration and geothermal energy extractions (Singh and Cowling [15]; Riley [13]). The problem of MHD mixed convection flow of Newtonian fluids over a horizontal plate has been studied by Ibrahim and Hady [9].

In view of these, we study the fully developed mixed convective heat transfer of a conducting Newtonian fluid in an open - ended vertical parallel plate microchannel with asymmetric wall heating at uniform heat fluxes by taking the velocity slip and temperature jump at the wall into account. The effects of the modified mixed parameter, Gr_q/Re, the Knudsen number, Kn, Hartmann number, M, and the ratio of wall heat flux, $r_q = q_1/q_2$, on the microchannel hydrodynamic and thermal behaviours are determined.

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MATHEMATICAL FORMULATION

Fig. 1 depicts the geometry considered, a microchannel between two parallel plates having asymmetric heat fluxes under the influence of magnetic field. Both hydrodynamically and thermally fully developed flow is assumed steady, laminar flow having constant properties is considered. A uniform magnetic field B_0 is applied transverse direction to the flow. It is assumed that the transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible. The axial heat conduction in the fluid and in the wall is assumed to be negligible. In a similar manner followed in Avci and Aydin [2], the usually continuum approach is applied by the continuum equations with the two main characteristics of the microscale phenomena, the velocity slip and the temperature jump. Velocity slip is defined as

$$u_s = -\frac{2-F}{F}\lambda\frac{\partial u}{\partial y}\Big|_{y=w} \tag{1}$$

where u_s is the slip velocity, λ - the molecular mean free path, and F - the tangential momentum accommodation co-efficient, and the temperature jump is define as

$$T_s - T_w = -\frac{2-F_i}{F_i}\frac{2\gamma}{\gamma+1}\frac{\lambda}{\Pr}\frac{\partial T}{\partial y}\Big|_{y=w} \tag{2}$$

where T_s is the temperature of the gas at the wall, T_w is the wall temperature, γ - specific heat ratio and F_i is the thermal accommodation co-efficient, which depends on the gas the surface material. Particularly for air, it assumes typical values near unity. For the rest of the analysis, F and F_i will be shown by F .

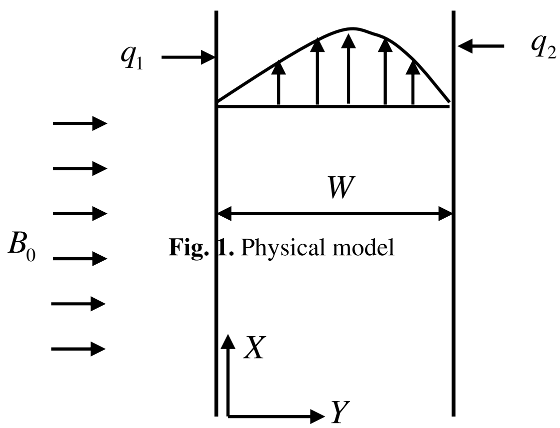

Fig. 1. Physical model

In a sufficient long channel, the velocity and the temperature profiles will cease to change with distance along the channel, i.e., a fully developed flow will exist. Assuming hydrodynamically fully developed flow, the transverse velocity becomes equal to zero. Then, the continuity equation drops to $\frac{\partial u}{\partial x} = 0$. Using the Boussinesq approximation, ignoring viscous dissipation (assuming a low-speed flow of a low-Prandtl fluid) fluid and substituting the above condition into the governing equations of the heat and fluid flow, we obtain

$$0 = -\frac{dp}{dx} + \mu\frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 u \pm \beta g \rho (T - T_{s,1}) \tag{3}$$

$$\mu\frac{\partial T}{\partial x} = \frac{k}{\rho c_P}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where p is the pressure, u - the velocity in the axial direction x, σ - the electrical conductivity, ρ - the density, g - gravitational acceleration, β -the thermal expansion coefficient, k -the thermal conductivity and c_P -the specific heat at constant pressure. The pressure p is thus measured relative to that which would exist at the same elevation in the stagnant fluid if it were at a uniform temperature of T_0. The positive sign in front of the buoyancy term applies to the buoyancy-assisted flow and the negative one applying to the buoyancy-opposed flow.

The boundary conditions are

$$u = u_{s1} \text{ at } y = 0$$

$$u = u_{s2} \text{ at } y = w$$

$$\frac{\partial T}{\partial y} = \frac{q_1}{k} \text{ at } y = 0$$

$$\frac{\partial T}{\partial y} = \frac{q_2}{k} \text{ at } y = w \tag{5}$$

For thermally developed flow,

$$\frac{\partial T}{\partial x} = \frac{q_1 + q_2}{\rho c_P u_m W} \tag{6}$$

By introducing the following non-dimensional quantities,

$$X = \frac{x}{D_h \operatorname{Re}}, Y = \frac{y}{D_h}, r_q = \frac{q_1}{q_2}, p = \frac{p + \rho g x}{\rho u_0^2}, \theta = \frac{T - T_{s,1}}{q_2 \frac{D_h}{k}}, U = \frac{u}{u_0}, Kn = \frac{\lambda}{D_h},$$

$$\Pr = \frac{\mu c_p}{k}, Gr_q = \frac{8\beta q_2 D_h^4}{k v^2}, \operatorname{Re} = \frac{u_0 D_h}{v}, M^2 = \frac{\sigma B_0^2 D_h^2}{\mu} (\text{Hartmann number}) \tag{7}$$

Equations (3) and (4) can be written as

$$-\frac{dp}{dx} + \frac{d^2U}{dy^2} - M^2 U + \frac{Gr_q}{\operatorname{Re}}\theta = 0 \tag{8}$$

$$\frac{d^2\theta}{dy^2} = 2\left(1 + \frac{q_1}{q_2}\right)U \tag{9}$$

In terms of the dimensionless variables introduced in Eq. (7), the boundary conditions given in Eq. (5) can be shown as

$$U = \beta_v Kn \frac{dU}{dY} \text{ at } Y = 0$$

$$U = -\beta_v Kn \frac{dU}{dY} \text{ at } Y = 0.5$$

$$U = \beta_v Kn \frac{d\theta}{dY} = \frac{q_1}{q_2} = -r_q \text{ at } Y = 0$$

$$\frac{d\theta}{dY} = 1 \text{ at } Y = 0.5 \tag{10}$$

Eq. (3.8) can be rewritten to give θ as

$$\theta = \frac{\operatorname{Re}}{Gr_q}\left(\frac{dp}{dX} - \frac{d^2U}{dY^2} + M^2 U\right) \tag{11}$$

By double-differentiating Eq. (8) with respect to Y and using Eq. (9), one obtains the fourth-order differential equation

$$\frac{d^4U}{dY^4} - M^2\frac{d^2U}{dY^2} + 2\frac{Gr_q}{\operatorname{Re}}\left(1 + \frac{q_1}{q_2}\right)U = 0 \tag{12}$$

Solution: Solving Eq. (12), we get

$$U = c_1 \cosh R_1 y + c_2 \sinh R_1 y + c_3 \cosh R_2 y + c_4 \sinh R_2 y \tag{3.13}$$

where

$$R_1 = \sqrt{\frac{M^2 + \sqrt{M^4 - 8\frac{Gr_q}{\mathrm{Re}}\left(1+\frac{q_1}{q_2}\right)}}{2}} \text{ and } R_2 = \sqrt{\frac{M^2 - \sqrt{M^4 - 8\frac{Gr_q}{\mathrm{Re}}\left(1+\frac{q_1}{q_2}\right)}}{2}}.$$

Substituting Eq. (3.13) into the energy Eq. (3.11), gives

$$\theta = \frac{\mathrm{Re}}{Gr_q}\left[\frac{dp}{dx} + \left(M^2 - R_1^2\right)c_1 \cosh R_1 y + \left(M^2 - R_1^2\right)c_2 \sinh R_1 y + \left(M^2 - R_2^2\right)c_3 \cosh R_2 y + \left(M^2 - R_2^2\right)c_4 \sinh R_2 y\right] \tag{14}$$

By applying boundary conditions given in Eq. (10) to the equations (13) and (14), we get

$$c_1 = (d_1 b_4 f_1 + a_1 f e_4 b_3 - b_4 e_2 f + b_2 f e_4 - b_2 d_2 f_1 + d_1 a_2 f_1 b_3 - a_2 e_2 f b_3 + a_2 e_3 B_2 f - a_1 d_2 f_1 b_3 - a_1 f e_3 b_4)/(d_1 e_4 b_3 - d_1 e_4 b_1 - e_2 d_2 b_3 + e_2 d_2 b_1 - e_3 d_1 b_4 + e_3 b_2 d_2 + e_1 d_1 b_4 + e_1 d_1 a_2 b_3 - e_1 a_1 d_2 b_3 - e_3 b_1 d_1 a_2 + e_3 b_1 a_1 d_2 - e_1 b_2 d_2),$$

$$c_2 = (-d_2 f_1 b_3 + d_2 f_1 b_1 + f e_4 b_3 - f e_4 b_1 - f e_3 b_4 + b_4 e_1 f + f e_1 a_2 b_3 - f e_3 b_1 a_2)/ (d_1 e_4 b_3 - d_1 e_4 b_1 - e_2 d_2 b_3 + e_2 d_2 b_1 - e_3 d_1 b_4 + e_3 b_2 d_2 + e_1 d_1 b_4 + e_1 d_1 a_2 B_3 - e_1 a_1 d_2 b_3 - e_3 b_1 d_1 a_2 + e_3 b_1 a_1 d_2 - e_1 b_2 d_2),$$

$$c_3 = (-b_1 a_1 f e_4 + b_1 a_1 d_2 f_1 + b_1 a_2 e_2 f - b_1 d_1 a_2 f_1 - d_1 b_4 f_1 - b_2 f e_1 a_2 + b_4 e_2 f - b_2 f e_4 + b_2 d_2 f_1 + b_4 e_1 a_1 f)/(d_1 e_4 b_3 - d_1 e_4 b_1 - e_2 d_2 b_3 + e_2 d_2 b_1 - e_3 d_1 b_4 + e_3 b_2 d_2 + e_1 d_1 B_4 + e_1 d_1 a_2 b_3 - e_1 a_1 d_2 b_3 - e_3 b_1 d_1 a_2 + e_3 b_1 a_1 d_2 - e_1 b_2 d_2),$$

$$c_4 = -(e_1 a_1 f b_3 - d_1 f_1 b3 + d_1 f_1 b_1 + e_2 f b_3 - e_2 f b_1 - e_3 b_2 f - e_3 b_1 a_1 f + e_1 b_2 f)/ (d_1 e_4 b_3 - d_1 e_4 b_1 - e_2 d_2 b_3 + e_2 d_2 b_1 - e_3 d_1 b_4 + e_3 b_2 d_2 + e_1 d_1 b_4 + e_1 d_1 a_2 b_3 - e_1 a_1 d_2 b_3 - e_3 b_1 d_1 a_2 + e_3 b_1 a_1 d_2 - e_1 b_2 d_2),$$

$$a_1 = \beta_v Kn R_1, \qquad a_2 = \beta_v Kn R_2,$$

$$b_1 = \cosh\left(\frac{R_1}{2}\right) + \beta_v Kn R_1 \sinh\left(\frac{R_1}{2}\right), \qquad b_2 = \sinh\left(\frac{R_1}{2}\right) + \beta_v Kn R_1 \cosh\left(\frac{R_1}{2}\right),$$

$$b_3 = \cosh\left(\frac{R_2}{2}\right) + \beta_v Kn R_2 \sinh\left(\frac{R_2}{2}\right), \qquad b_4 = \sinh\left(\frac{R_2}{2}\right) + \beta_v Kn R_1 \cosh\left(\frac{R_2}{2}\right),$$

$$d_1 = \left(M^2 - R_1^2\right)R_1, \qquad d_2 = \left(M^2 - R_2^2\right)R_2,$$

$$e_1 = \left(M^2 - R_1^2\right)R_1 \sinh\left(\frac{R_1}{2}\right), \qquad e_2 = \left(M^2 - R_1^2\right)R_1 \cosh\left(\frac{R_1}{2}\right),$$

$$e_3 = \left(M^2 - R_2^2\right)R_2 \sinh\left(\frac{R_2}{2}\right), \qquad e_4 = \left(M^2 - R_2^2\right)R_2 \cosh\left(\frac{R_2}{2}\right),$$

$$f = -r_q\left(\frac{Gr_q}{\mathrm{Re}}\right) - \frac{dp}{dx}, \qquad f_1 = \left(\frac{Gr_q}{\mathrm{Re}}\right) - \frac{dp}{dx}.$$

Taking the temperature jump into consideration, Eq. (14) becomes

$$\theta^* = \frac{T - T_1}{q_2 D_h / k} = \frac{T - T_{s,1}}{q_2 D_h / k} + \frac{T_{s,1} - T_1}{q_2 D_h / k} = \theta - \beta_t Kn\left(q_1 / q_2\right) \tag{15}$$

Using Eq. (15), the dimensionless mean/bulk temperature is defined as

$$\theta_m^* = \frac{T_m - T_1}{q_2 D_h / k} = \frac{\int_0^{0.5} U\theta^* dY}{\int_0^{0.5} U dY} \tag{16}$$

The convection heat transfer coefficients at the left-hand side and right-hand side wall can be obtained as follows

$$h_1 = \frac{-k\left(\partial T / \partial y\right)/_{y=0}}{T_1 - T_m} = -\frac{1}{\theta_m^*}\frac{k\left(q_1 / q_2\right)}{D_h}$$

$$h_2 = \frac{-k\left(\partial T / \partial y\right)/_{y=w}}{T_2 - T_m} = \frac{k\left(\partial T / \partial y\right)/_{y=w}}{(T_m - T_1) - (T_{s,2} - T_1) + (T_{s,2} - T_2)}$$

$$= \frac{-k / D_h}{\theta_m^* - \theta^*(0.5) - \beta_t Kn} \tag{17}$$

Similarly the Nusselt numbers at the left - hand side and right - hand side walls can be written as

$$Nu_1 = \frac{-\left(q_1 / q_2\right)}{\theta_m^*} \tag{18}$$

$$Nu_2 = \frac{-1}{\theta_m^* - \theta^*(0.5) - \beta_t Kn} \tag{19}$$

RESULTS AND DISCUSSION

In order to study the effects of Knudsen number Kn mixed convection number $\frac{Gr_q}{\mathrm{Re}}$ and Hartmann number M we have plotted Figs. 2 - 4. From Fig. 2, it is noted that an increasing in $\frac{Gr_q}{\mathrm{Re}}$ results in flow acceleration near the walls and consequently flow deceleration in the centerline of the channel.

For a constant value of the mixed convention parameter $\frac{Gr_q}{\mathrm{Re}} = 50$, the effect of Knudsen number on the velocity profile is illustrated in Fig. 3. It is observed that increasing Kn increases velocity slips at the walls and decreases the maximum velocity at the centerline.

Fig.4 shows the effect of the Hartman number and velocity profile with $Kn = 0.04$, $\frac{Gr_q}{\mathrm{Re}} = 50, \beta_n = 1, \frac{dp}{dx} = -1, r_q = 1$. It is observe that an increase in M results in flow acceleration near the walls and consequently flow deceleration in the centerline of the channel as $M \to 0$ our results consider with the results of Avci and Aydin (2007).

Fig.5 shows the effect of the Knudsen number Kn on the temperature θ. It is found that as Kn increases the temperature decreases.
To illustrate the effect of Hartman number M, we have plotted Fig.6. It is observed that the temperature decreases with an increase in M .

Fig. 7 represents the effect Knudsen number Kn on the Nusselt number Nu_1 at the left-hand side wall. It is noted that Nu_1 decreases with increasing r_q . Further it is observe that Nu_1 decreases with an increase in Kn .

The variation of Nu_1 with r_q for different values of M is presented in Fig. 8. It is found that Nu_1 decreases with increasing M .

Fig. 9 shows the effect of the Knudsen number Kn on the Nusselt number Nu_2 at the right hand side wall. It is observed that Nu_2 increases with increasing r_q .Further it is observed that Nu_2 decreases with an increasing Kn.

Fig.10 shows the variation of Nu_2 with r_q for different values of M . It observed that Nu_2 increases with an increase in M .

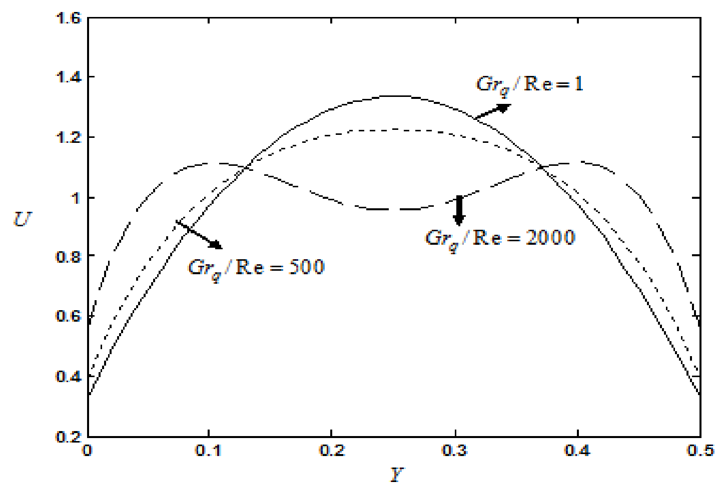

Fig. -2: Effects of the thermal buoyancy parameter Gr_q / Re on the velocity U with $Kn = 0.04, M = 1, r_q = 1,$ $dp/dx = -1$ and $\beta_v = 1$.

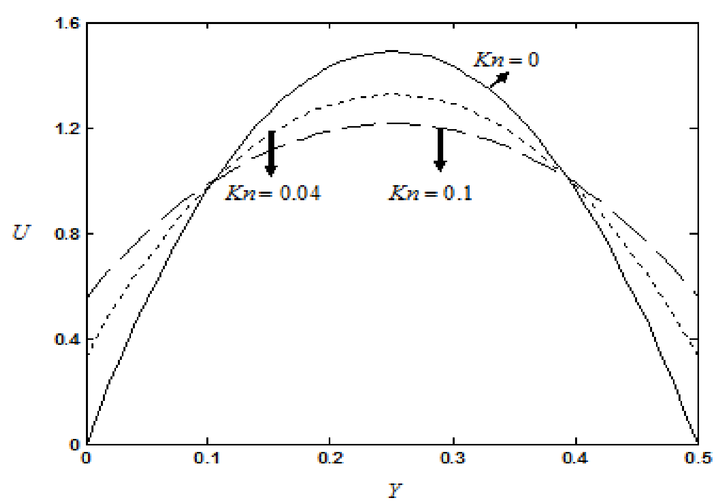

Fig. -3: Effects of the Knudsen number Kn on the velocity U with $Gr_q / \mathrm{Re} = 50$ $M = 1, r_q = 1, dp/dx = -1$ and $\beta_v = 1$

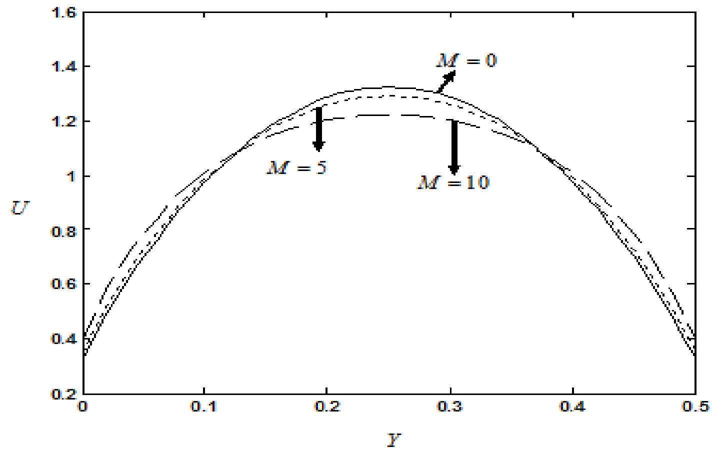

Fig. -4: Effects of the Magnetic parameter M on the velocity U with $Kn = 0.04,\ Gr_q / \mathrm{Re} = 50, r_q = 1,$ $dp/dx = -1$ and $\beta_v = 1$.

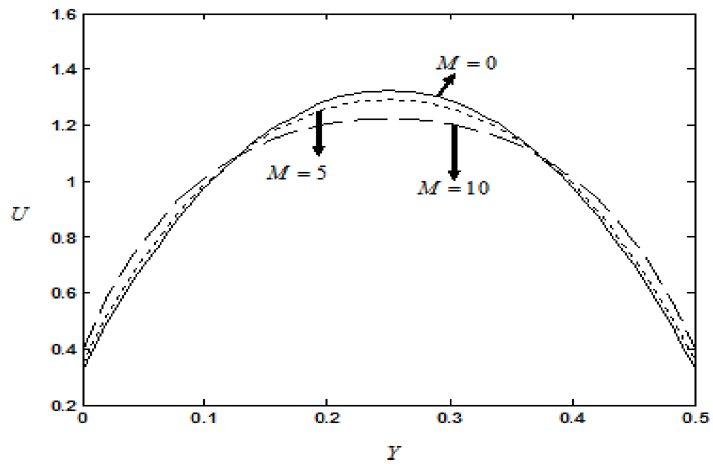

Fig. - 5: Effects of the Knudsen number Kn on the temperature θ with $Gr_q / \mathrm{Re} = 50\ \ M = 1, r_q = 1,$ $dp/dx = -1, \beta_t = 1.667$ and $\beta_v = 1$.

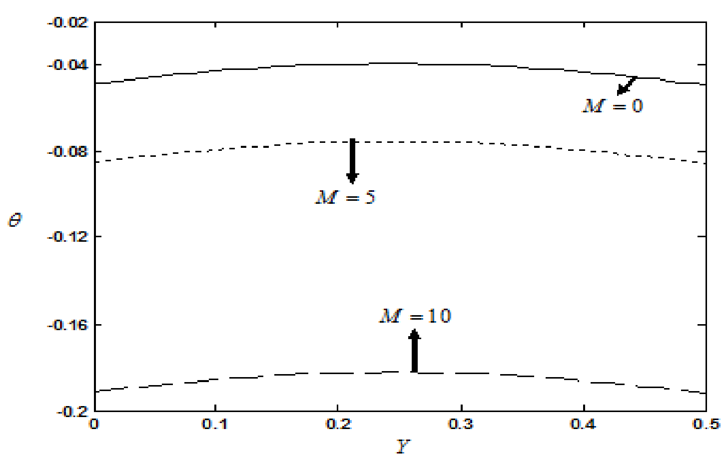

Fig. - 6: Effects of the Magnetic parameter M on the velocity U with $Kn = 0.04,\ Gr_q / \mathrm{Re} = 50, r_q = 1,$ $dp/dx = -1, \beta_t = 1.667$ and $\beta_v = 1$.

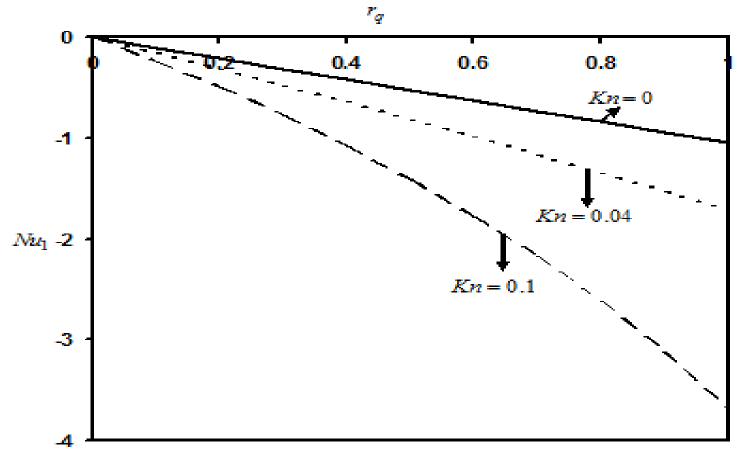

Fig.-7: Effects of the Knudsen number Kn on the Nusselt number at the left - hand side wall Nu_1 with $Gr_q/\mathrm{Re}=50\ \ M=1, r_q=1,\ \beta_t=1.667\ \ dp/dx=-1,$ and $\beta_v=1$.

r_q

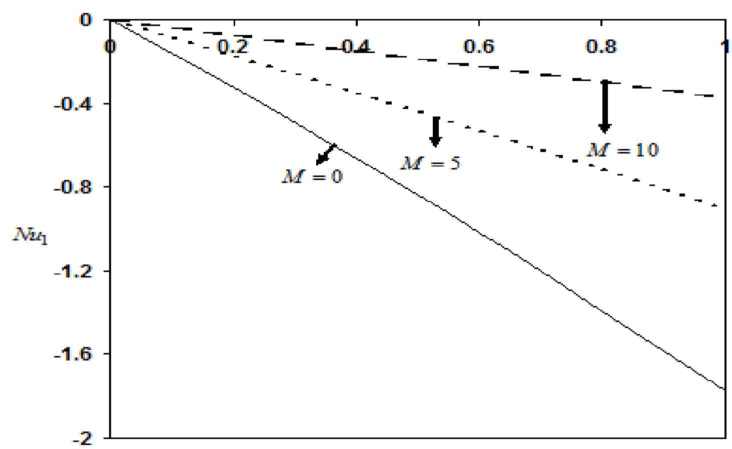

Fig. - 8: Effects of the Magnetic parameter M on the Nusselt number at the left - hand side wall Nu_1 with $Kn=0.04,\ Gr_q/\mathrm{Re}=50, r_q=1,\ dp/dx=-1, \beta_t=1.667$ and $\beta_v=1$.

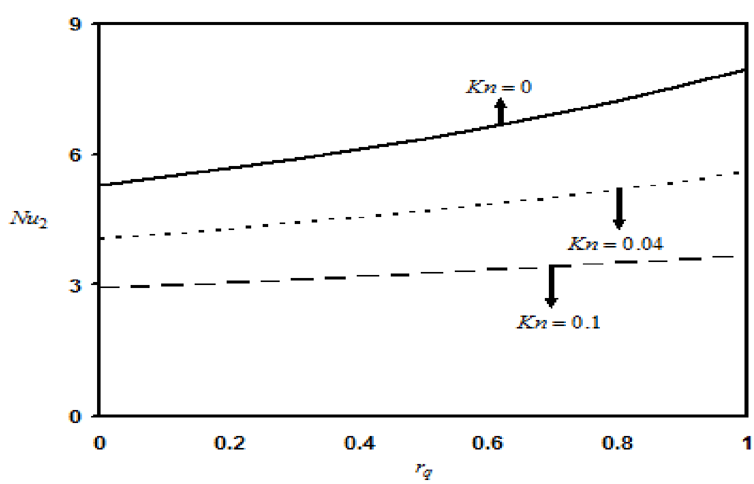

Fig.9: Effects of the Knudsen number Kn on the Nusselt number at the right - hand side wall Nu_2 with $Gr_q/\mathrm{Re}=50\ \ M=1, r_q=1,\ dp/dx=-1, \beta_t=1.667$ and $\beta_v=1$

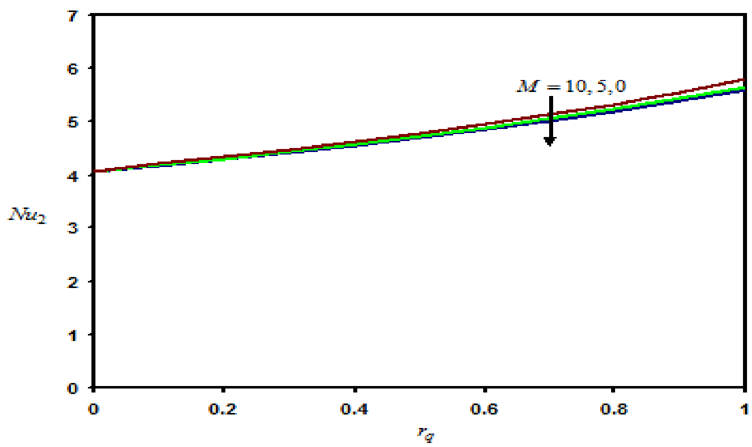

Fig. 10: Effects of the Magnetic parameter M on the Nusselt number at the right - hand side wall Nu_2 with $Kn = 0.04,\ Gr_q / \mathrm{Re} = 50, r_q = 1,\ dp/dx = -1, \beta_t = 1.667$

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