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A STUDY ON TWO PARAMETER DISCRETE QUASI LINDLEY DISTRIBUTION AND ITS DERIVED DISTRIBUTIONS

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ABSTRACT

Modeling count data is one of the most important issues in statistical research. In this paper, a new probability mass function is introduced by discretizing the continuous failure model of the Quasi- Lindley distribution. The discrete Quasi Lindley (DQL) distribution has been derived and further certain properties of the distribution have been discussed. Properties such as the recurrence relations for probabilities, factorial moments and index of dispersion of this distribution are also investigated. Estimation of parameters of DQL distributions have been discussed. The size-biased, Zero- truncated and Zero- modified forms of DQL distribution have also been investigated. To test its goodness of fit, DQL distributions have been fitted to some of well known data sets where discrete Poisson- Lindley distribution and discrete gamma distributions have earlier been fitted by others. The results show that, the two parameter DQL distribution can provide a better fit than the other derived distributions. It is noted that DL is a particular case of DQL distribution.

Key words: *Discrete Quasi Lindley distribution, Zero-Modified distribution, Recurrence relations, Index of dispersion, Parameter Estimation and Goodness of fit.*

1. INTRODUCTION

Discrete distributions obtained by discretizing a continuous failure time model have appeared in the statistical literature. Discrete geometric distribution can be obtained by discretizing the exponential continuous distribution. Some of those works are by Nakagawa and Osaki (1975), where the discrete Weibull distribution is obtained; Roy (2004) studied the discrete Rayleigh distribution; in Kemp (2008) the discrete half-normal distribution is examined, in Krishna and Pundir (2008) the Burr discrete distribution and the Pareto discrete distribution as a particular case of the former are analyzed and more recently, Gómez-Déniz *et. al* (2011) derived a new generalization of the geometric distribution obtained from the generalized exponential distribution of Marshall and Olkin (1997). If the underlying continuous failure time X has the survival function $S(x)$, the probability mass function $Pr(X > x)$ of the discrete random variable associated with that continuous distribution can be written as

$$Pr(X = x) = S(x) - S(x + 1), x = 0,1,2,\ldots \quad (1.1)$$

A two-parameter Quasi Lindley distribution (QLD), of which the Lindley distribution (LD) is a particular case, has been introduced by Shanker and Mishra (2013). In this paper we deal with the derivation discrete Quasi- Lindley (DQL) distribution which takes values in {0, 1, . . .}. and the study of certain properties of distribution. This new distribution is generated by discretizing the continuous survival function of the QL distribution, with parameter α *and* θ which is given by

$$f(x:\alpha,\theta) = \frac{\theta}{\alpha+1}(\alpha+\theta x)\, e^{-\theta x}.\ x > 0. \theta > 0, \alpha > -1 \quad (1.2)$$

The corresponding cumulative distribution function (cdf) is given by

$$F(x) = 1 - \frac{1+\alpha+\theta x}{\alpha+1}\, e^{-\theta x},\quad x > 0,\ \theta > 0,\ \alpha > -1 \quad (1.3)$$

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Sankaran (1970) introduced the discrete Poisson-Lindley distribution by combining the Poisson and Lindley distributions. Ghitany *et al.* (2008, 2009) investigated most of the statistical properties of the Lindley distribution, showing that it may provide a better fitting than the exponential distribution. Mahmoudi and Zakerzadeh (2010) proposed an extended version of the compound Poisson distribution which was obtained by compounding the Poisson distribution with the generalized Lindley distribution which is obtained and analyzed by Zakerzadeh and Dolati (2009). A new extension of the Lindley distribution, called extended Lindley (EL) distribution, which offers a more flexible model for lifetime data was introduced by Bakouch *et al.* (2012).

Recently, Gómez-Déniz and Calderín-Ojeda (2011) proposed a discretization of the continuous Lindley distribution. Zakerzadeh and Dolati (2009) obtained discrete Lindley distribution by discretizing continuous failure rate model. This discrete distribution has been proved to be useful in both modeling count data and the collective risk model as an alternative to compound Poisson and compound negative binomial models.

Let us consider the survival function of Quasi Lindley distribution

$$S(x) = \frac{1+\alpha+\theta x}{\alpha+1}\, e^{-\theta x},\ \ x > 0.\theta > 0,\ \alpha > -1 \tag{1.4}$$

where $\lambda = e^{-\theta}$ and $log\lambda = -\theta.$

The probability mass function (pmf) of two parameter discrete quasi Lindley (DQL) distribution may be obtained by discretizing the survival function of quasi Lindley distribution

$$p_x = Pr(X = x)$$
$$= \frac{\lambda^x}{(\alpha+1)}\{(\alpha+1)(1-\lambda) + \{(\lambda-1)\,x + \lambda\}\log\lambda\}\ \text{, for x = 0, 1,…,}$$
$$\text{either}\ \ \alpha > \max\left(-1 + nlog\lambda - \frac{\lambda log\lambda}{1-\lambda},\ -1\right) or\ \ \alpha < \min\left(-1 + nlog\lambda - \frac{\lambda log\lambda}{1-\lambda},\ -1\right)$$
$$where\ 0 < \lambda < 1 \tag{1.5}$$

It is reduced to the pmf of discrete Lindley (DL) distribution as

$$p_x = \frac{\lambda^x}{1-log\,\lambda}[\lambda log\lambda + (1-\lambda)(1 - log\lambda^{x+1})]\ \text{, x=0, 1,…} \tag{1.6}$$

putting $\alpha = -log\lambda\ in\ (5)$, See Gómez-Déniz and Calderín-Ojeda (2011).

One of the advantages of the DQL model is that it is over-dispersed (variance is greater than the mean) being, therefore, more flexible than the Poisson distribution to model actuarial data that commonly include the over-dispersion phenomenon.

Proposition 1: The probability generating function (pgf) of a discrete random variable following the DQL distribution (1.5) is given by

$$G(t)=\frac{[(1-\lambda)(\alpha+1)+\lambda log\lambda\,](1-\lambda t)-(1-\lambda)\lambda t log\lambda}{(\alpha+1)(1-\lambda t)^2}\ . \tag{1.7}$$

Proposition 2: The cumulative distribution function (cdf) of a discrete random variable following the pmf (1.5) is given by

$$F(x) = \frac{1}{\alpha+1}[1 - \lambda^{x+1} + \alpha(1-\lambda^{x+1}) + (x+1)\lambda^{x+1}log\lambda] \tag{1.8}$$

The survival function of DQL distribution can be obtained from the distribution function as

$$S_{DQL}(x) = 1 - F(x)$$
$$= \frac{\lambda^{x+1}\{1+\alpha-(x+1)log\lambda\}}{\alpha+1} \tag{1.9}$$

The failure or hazard rate may be obtained as

$$r(x) = P(X < x|X < x-1) = \frac{P(X=x)}{P(X>x-1)}$$
$$= \frac{(\alpha+1)(1-\lambda)+\{(\lambda-1)x+\lambda\}log\lambda}{1+\alpha-xlog\lambda}, \tag{1.10}$$

The reversed failure rate may be obtained as

$$r^*(x) = \frac{P(X=x)}{P(X\le x)}$$
$$= \frac{\lambda^x[((\alpha+1)(1-\lambda)+\{(\lambda-1)x+\lambda\}log\lambda\,]}{\{1-\lambda^{x+1}+\alpha(1-\lambda^{x+1})+(x+1)\lambda^{x+1}log\lambda\,\}} \tag{1.11}$$

The second rate of failure is obtained as

$$r^{**}(x) = log\left[\frac{S(x)}{S(x+1)}\right]$$
$$= log\left[\frac{\{(x+2)log\lambda - 1\}}{\lambda\{(x+3)log\lambda - 1\}}\right] = log\left[\frac{1}{\lambda}\left\{\frac{(x+2)log\lambda - 1}{(x+3)log\lambda - 1}\right\}\right]. \quad (1.12)$$

The survival function S(x), failure rate $r(x)$, reversed hazard rate $r^*(x)$ and second rate of failure $r^{**}(x)$ of one parameter discrete Lindley distribution may be obtained by putting $\alpha = -log\lambda$ in the equation (1.9), (1.10),(1.11), and (1.12) respectively.

2. RECURRENCE RELATIONS OF TWO- PARAMETER DQL DISTRIBUTION

Proposition 3: The proportion of probabilities of DQL distribution may be given as

$$\frac{P_{x+1}}{P_x} = \lambda\left[1 + \frac{(2\lambda-1)log\lambda}{(\alpha+1)(1-\lambda)+\{(\lambda-1)x+\lambda\}log\lambda}\right],$$

either $\alpha > \max\left(-1 + nlog\lambda - \frac{\lambda log\lambda}{1-\lambda},\ -1\right)$ or $\alpha < \min\left(-1 + nlog\lambda - \frac{\lambda log\lambda}{1-\lambda},\ -1\right)$

where $0 < \lambda < 1$. (2.1)

(A) Recurrence Relation for Probabilities

The probability recurrence relation for two parameter DQL distribution can also be obtained as

$$p_{r+2} = \lambda(2p_{r+1} - \lambda p_r),\ r \geq 0 \quad (2.2)$$

where $p_0 = \frac{(\alpha+1)(1-\lambda)+\lambda\ log\lambda}{\alpha+1}$ and $p_1 = \lambda\frac{(\alpha+1)(1-\lambda)+(2\lambda-1)log\lambda}{\alpha+1}$,

The higher order probabilities may be computed using the recurrence relation (2.2). Similarly, the higher ordered probabilities of one parameter DL distribution can also be computed, putting $\alpha = -log\lambda$ in recurrence relation (2.2).

(B) Factorial Moment Recurrence Relation

Proposition 4: The factorial moment generating function $M_x(t)$ and r^{th} ordered factorial moment $\mu_{[r]}$ for two parameter DQL distribution can be written as

$$M_x(t) = \frac{[(\alpha+1)(1-\lambda)+\lambda log\lambda](1-\lambda-\lambda t)-(1-\lambda)\lambda(1+t)\log\lambda)}{(1-\lambda-\lambda t)^2\ (\alpha+1)}, \text{ and} \quad (2.3)$$

$$\mu_{[r]} = \frac{r!\lambda^r[(\alpha+1)(1-\lambda)-rlog\lambda]}{(1-\lambda)^{r+1}(\alpha+1)} \text{ respectively.}$$

The recurrence relation for factorial moment can be obtained as

$$\mu_{[r+2]} = \frac{\lambda(r+2)}{(1-\lambda)^2}[2(1-\lambda)\mu_{[r+1]} - \lambda(r+1)\mu_{[r]}], \quad (2.4)$$

where

$$\mu_{[1]} = \frac{\lambda[(\alpha+1)(1-\lambda)-log\lambda]}{(1-\lambda)^2(\alpha+1)},$$

$$\mu_{[2]} = \frac{2\lambda^2[(\alpha+1)(1-\lambda)-2log\lambda]}{(1-\lambda)^3(\alpha+1)},\quad \mu_{[3]} = \frac{6\lambda^3[(\alpha+1)(1-\lambda)-3log\lambda]}{(1-\lambda)^4(\alpha+1)}, \text{ etc.}$$

From the above factorial moments, the mean μ and variance σ^2 can be derived as

$$\mu = \frac{\lambda[(\alpha+1)(1-\lambda)-log\lambda]}{(1-\lambda)^2(\alpha+1)} \quad (2.5)$$

$$\sigma^2 = \frac{\lambda[(\alpha+1)^2(1-\lambda)^2-(\alpha+1)(1-\lambda)(1+\lambda)log\lambda-\lambda(log\lambda)^2]}{(1-\lambda)^4(\alpha+1)^2} \quad (2.6)$$

Table-1: Mean of the DQL distribution for different values of the parameters α and λ.

$\alpha\backslash\lambda$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	.395	.753	1.166	1.685	2.386	3.416	5.107	8.463	18.482
0.2	.348	.669	1.043	1.515	2.155	3.096	4.645	7.719	16.902
0.4	.314	.609	.955	1.394	1.990	2.868	4.315	7.188	15.773
0.6	.289	.564	.889	1.303	1.866	2.697	4.067	6.789	14.927
0.8	.269	.529	.838	1.232	1.770	2.564	3.875	6.479	14.268
1.0	.253	.501	.797	1.176	1.693	2.458	3.720	6.231	13.741
1.2	.240	.479	.764	1.129	1.630	2.371	3.594	6.029	13.310
1.4	.230	.460	.736	1.091	1.578	2.298	3.489	5.860	12.951
1.6	.220	.443	.712	1.058	1.533	2.237	3.400	5.716	12.647

Table-2: Variance of the DQL distribution for different values of the parameters α and λ.

$\alpha\backslash\lambda$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	.39	.81	1.44	2.45	4.24	7.74	15.80	40.25	180.25
0.2	.36	.77	1.38	2.37	4.13	7.59	15.53	39.64	177.70
0.4	.33	.72	1.31	2.28	3.99	7.35	15.08	38.53	172.81
0.6	.31	.69	1.26	2.19	3.85	7.11	14.60	37.32	167.48
0.8	.29	.65	1.21	2.11	3.72	6.87	14.14	36.17	162.34
1.0	.28	.63	1.16	2.04	3.60	6.66	13.71	35.10	157.60
1.2	.26	.60	1.12	1.98	3.49	6.47	13.33	34.14	153.32
1.4	.25	.58	1.09	1.92	3.40	6.31	12.99	33.28	149.46
1.6	.25	.57	1.06	1.87	3.32	6.15	12.69	32.50	145.99

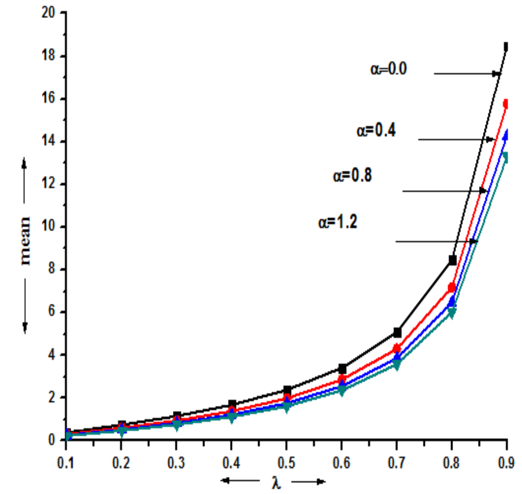

Figure 1(a) : mean increases as λ increase

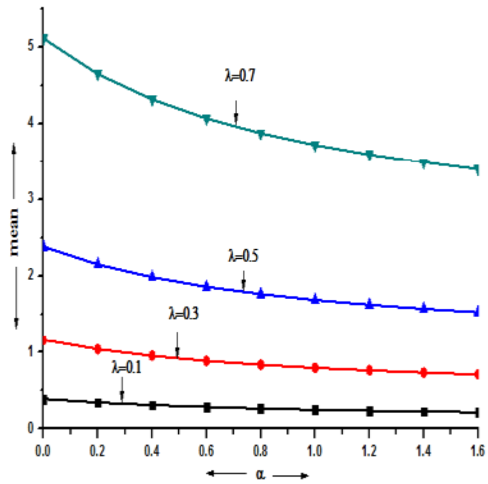

Figure 1(b) : mean decreases as α increase

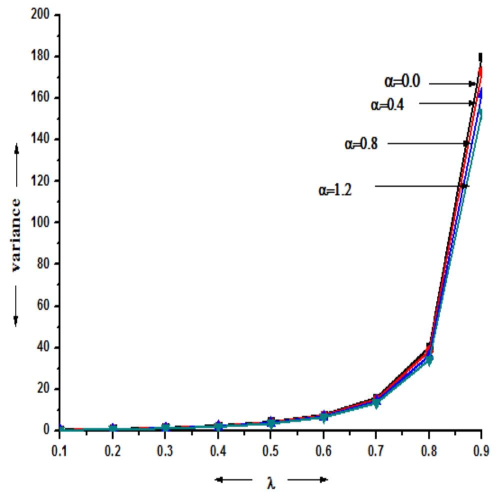

Figure 2(a) : variance increases as λ increase

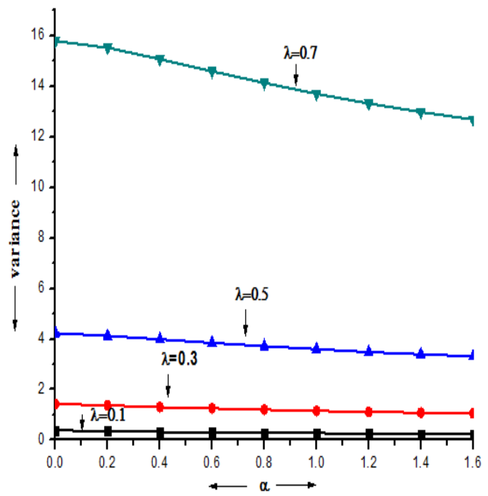

Figure 2(b) : variance decreases as α increase

It is noted that the mean and variance increases rapidly as λ increased but it decreases slowly as α increased.

3. SIZE –BIASED DISCRETE QUASI LINDLEY (SBDQL) DISTRIBUTION

Size biased distribution arises naturally in practice when observations from a sample are recorded with unequal probabilities, having probability proportional to size (PPS). It is a more general form known as weighted distributions. Fisher(1934) first introduced these distributions to model ascertainment bias which were formalized by Rao (1965) in a unifying theory. If the random variable X has pmf $f(x;\theta)$, with unknown parameter θ, then the corresponding weighted distribution is of the form $f^w(x;\theta) = \frac{w(x)f(x;\theta)}{E[w(x)]}$, where $w(x)$ is a non-negative weight function such that $E[w(x)]$.

$$f_s(x,\alpha) = \frac{xp_x}{\mu} = x\lambda^{x-1}\frac{[(\alpha+1)(1-\lambda)+\{(\lambda-1)\,x+\lambda\}\log\lambda](1-\lambda)^2}{(\alpha+1)(1-\lambda)-\log\lambda} \qquad x=1,2,\ldots \qquad (3.1)$$

A special case of interest arise when the weight function $w(x) = x^{\alpha}$. Such distributions are known as sized biased distributions of order α. The most common case of size-biased distribution occur when $\alpha = 1$ and $\alpha = 2$, these special cases may be termed as length (size) and area biased respectively. If a random variable X has pmf $f(x;\theta)$, then the pmf of size-biased QL distribution may obtained as

$$f_s(x,\alpha) = \frac{x p_x}{\mu},\ \mu \text{ denotes the mean of the DQL distribution}$$

$$= x\lambda^{x-1}\frac{[(\alpha+1)(1-\lambda)+\{(\lambda-1)\,x+\lambda\}\log\lambda](1-\lambda)^2}{(\alpha+1)(1-\lambda)-\log\lambda}. \qquad x=1, 2, \ldots \tag{3.2}$$

A size- biased quasi Poisson-Lindley distribution, of which the size-biased Poisson-Lindley distribution of Ghitany and Al-Mutairi (2008) is a particular case.

Proposition 3: The probability generating function of a discrete random variable following the DQL distribution (5) is given by

$$G_s(t) = \frac{t[(1-\lambda)(\alpha+1)+\lambda log\lambda\,](1-\lambda)^2(1-\lambda t)-t(1-\lambda)^3(1+\lambda t)log\lambda}{[(\alpha+1)(1-\lambda)-\log\lambda](1-\lambda t)^3}. \tag{3.3}$$

Probability recurrence relation of the DQL distribution is obtained as

$$p_r = 3\lambda p_{r-1} - 3\lambda^2 p_{r-2} + \lambda^3 p_{r-3} \quad \text{for } r > 3, \text{ and} \tag{3.4}$$

$$where \quad p_1 = \frac{[(\alpha+1)(1-\lambda)+(2\lambda-1)\log\lambda](1-\lambda)^2}{(\alpha+1)(1-\lambda)-\log\lambda}, \qquad p_2 = 2\lambda\frac{[(\alpha+1)(1-\lambda)+(3\lambda-2)\log\lambda](1-\lambda)^2}{(\alpha+1)(1-\lambda)-\log\lambda}$$

$$p_3 = 3\lambda^2\frac{[(\alpha+1)(1-\lambda)+(4\lambda-3)\log\lambda](1-\lambda)^2}{(\alpha+1)(1-\lambda)-\log\lambda}.$$

The factorial moments of size biased discrete quasi Lindley (SBDQL) distribution may be obtained as

$$\mu_{[r]} = r!\,\lambda^{r-1}\frac{[(\alpha+1)(1-\lambda)(r+\lambda)-\{r^2+(2r+1)\lambda\}\log\lambda]}{[(\alpha+1)(1-\lambda)-\log\lambda](1-\lambda)^r} \tag{3.5}$$

from its factorial moment generating function (fmgf)

$$M_s(t) = \frac{(1+t)[(1-\lambda)(\alpha+1)+\lambda log\lambda](1-\lambda)^2(1-\lambda-\lambda t)-(1-\lambda)^3(1+\lambda+\lambda t)(1+t)log\lambda}{[(\alpha+1)(1-\lambda)-\log\lambda](1-\lambda-\lambda t)^3}$$

Factorial recurrence relation may also be obtained as

$$\mu_{[r]} = \frac{1}{(1-\lambda)^3}[3(1-\lambda)^2\lambda r\mu_{[r-1]} - 3(1-\lambda)\lambda^2 r(r-1)\mu_{[r-2]} + \lambda^3 r(\mathrm{r}-1)(\mathrm{r}-2)\mu_{[r-3]}] \quad \text{for } r > 3, \tag{3.6}$$

where

$$\mu_{[1]} = \frac{[(\alpha+1)(1-\lambda)(1+\lambda)-(7+3\lambda)\log\lambda]}{[(\alpha+1)(1-\lambda)-\log\lambda](1-\lambda)}$$

$$\mu_{[2]} = 2\lambda\frac{[(\alpha+1)(1-\lambda)(2+\lambda)-(4+5\lambda)\log\lambda]}{[(\alpha+1)(1-\lambda)-\log\lambda](1-\lambda)^2}$$

$$\mu_{[3]} = 6\lambda^2\frac{[(\alpha+1)(1-\lambda)(3+\lambda)-(9+7\lambda)\log\lambda]}{[(\alpha+1)(1-\lambda)-\log\lambda](1-\lambda)^3}$$

4. ZERO- TRUNCATED DISCRETE QUASI LINDLEY (ZTDQL) DISTRIBUTION

When the data to be modeled originate from a generating mechanism that structurally excludes zero counts, discrete quasi–Lindley distribution must be adjusted to count for the missing zeros. In this paper we consider the zero-truncated quasi–Lindley (ZTDQL) distribution with the pmf

$$P_Z(X=x) = \lambda^{x-1}\frac{[(\alpha+1)(1-\lambda)+\{(\lambda-1)\,x+\lambda\}\log\lambda]}{(\alpha+1)-\log\lambda} \quad x = 1, 2, \ldots \tag{4.1}$$

where $P_Z(X=0) = \frac{[(\alpha+1)(1-\lambda)+\lambda\log\lambda]}{(\alpha+1)}$.

Proposition 4: The probability generating function of a discrete random variable following the ZTDQL distribution (5) is given by

$$G_z(t) = \frac{[\{(1-\lambda)(\alpha+1)+\lambda log\lambda\,\}(1-\lambda t)-\lambda t\,(1-\lambda)log\lambda\,]}{[(\alpha+1)-\log\lambda](1-\lambda t)^2} \tag{4.2}$$

5. ZERO- MODIFIED DISCRETE QUASI LINDLEY (ZMDQL) DISTRIBUTION

In recent years there has been considerable and growing interest in modeling zero-modified count data. Zero-modified DQL model address the problem, that the data display a higher fraction of zeros, or non occurrences, than can be possibly explained through any fitted standard count model. The zero-modified distributions are appropriate alternatives for modeling clustered samples when the population consists of two sub-populations, one containing only zeros, while in the other, counts from a discrete distribution are observed.

$$P_z[X=0] = \omega + (1-\omega)P_0$$
$$= \omega + (1-\omega)\left[\frac{(\alpha+1)(1-\lambda)+\lambda log\lambda}{\alpha+1}\right]$$

$$P_z[X=x] = (1-\omega)\lambda^x \frac{[(\alpha+1)(1-\lambda)+\{(\lambda-1)\,x+\lambda\}\log\lambda]}{(\alpha+1)} \qquad x=1, 2, ...$$
$$\alpha \geq 0, \qquad 0<\lambda<1, \qquad \omega \geq \frac{-P_0}{1-P_0}$$

$where\ P_z[X=x]$ denotes the probability of ZMDQL distribution**.**

6. ESTIMATION OF PARAMETER OF DQL DISTRIBUTION ESTIMATION OF λ IN TERMS OF MEAN AND VARIANCE OF DQL DISTRIBUTION

Form $\mu = \frac{\lambda[(\alpha+1)(1-\lambda)-log\lambda]}{(1-\lambda)^2(\alpha+1)}$ the mean of DQL distribution, the value of $\lambda log\lambda\ may\ be\ expressed\ as\ (1-\lambda)(\alpha+1)-(\lambda-(1-\lambda)\mu)$. Now putting the value of $\lambda log\lambda$ in

$\sigma^2 = \frac{\lambda[(\alpha+1)^2(1-\lambda)^2-(\alpha+1)(1-\lambda)(1+\lambda)log\lambda-\lambda(log\lambda)^2]}{(1-\lambda)^4(\alpha+1)^2}$ the variance of DQL distribution, the quadratic equation in λ may be obtained as

$$\lambda^2 A - 2\lambda B + C = 0 \qquad (6.1)$$

Given a random sample $x_1, x_2, \ldots x_n$ of size n from the DQL distribution with the pmf (1), the moment estimate $\hat{\lambda}$ of DQL distribution may be obtained from the quadratic equation (6.1) as

$$\hat{\lambda} = \frac{B \pm \sqrt{B^2-AC}}{A}, \qquad (6.2)$$

where $A = \sigma^2 + \mu^2 + 3\mu + 2$, $B = \sigma^2 + \mu^2 + \mu$ and $C = \sigma^2 + \mu^2 - \mu$.

7. AN APPLICATION

In the last two decades, standard discrete distributions such as geometric and negative binomials have been used to model lifetime data. However, there is a need to find more plausible discrete lifetime distributions to fit different types of lifetime data. In this article, discrete Quasi-Lindley distribution has been investigated by discretizing the continuous Quasi- Lindley distribution. Two sets of real data from Sankaran (1970) and one set of data on distribution of number of European red mites on apple leaves have been considered for the fitting of QPL distribution. The first set of data represents the mistakes in copying groups of random digits and the second set are the number of accidents to 647 women working on high explosive shells in 5 weeks. ###

8. GOODNESS OF FIT

The fittings of the two-parameter DQL distribution based on three data-sets have been presented in the following tables. The expected frequencies according to the one parameter Poisson- Lindley with parameter θ in Table 3 presented by Sankaran (1970), two parameter Poisson- Lindley distributions with parameter θ and α in Table 4 presented by Shanker *et al.* (2012) and two parameter discrete gamma with parameter k and θ in Table 5 presented by Chakraborty an Chakravarty (2012) have also been given for ready comparison with DQL distribution. The estimates of the parameters have been obtained by the method of moments.

Table-3: Observed and expected frequencies for mistakes in copying groups of random digits.

No. of errors per group	Observed frequencies	Expected frequencies		
		Poisson- Lindley (θ)	Poisson- Lindley (θ, α)	DQL (α, λ)
0	35	33.1	32.4	31.34
1	11	15.3	15.8	15.77
2	8	6.8	7.0	7.72
3	4	2.9	2.9	3.63
4	2	1.2	1.9	1.54
	60	60	60	60
		$\hat{\theta}$ =1.743	$\hat{\alpha} = 2.61204$	$\hat{\alpha} = -3.89024$
			$\hat{\theta} = 5.22337$	$\lambda = 0.586021$
	χ^2	2.20	2.11	2.01
	$P\ value$	0.1380	.3482	0.366

Table-4: Observed and expected frequencies for distribution of Pyrausta nublilalis in 1937.

No. of accidents	Observed frequencies	Expected frequencies		
		Poisson-Lindley (θ)	Poisson- Lindley (θ, α)	DQL (α, λ)
0	33	31.49	31.9	30.74
1	12	14.16	13.8	13.98
2	6	6.09	5.9	6.37
3	3	2.54	2.5	2.93
4	1	1.04	1.1	1.37
≥5	1	0.42	0.8	0.61
56		56 $\hat{\theta} = 1.8082$	56 $\hat{\alpha} = 0.2573$ $\hat{\theta} = 0.39249$	56 $\hat{\alpha} = -24.3726$ $\hat{\lambda} = 0.466893$
	χ^2 $P\ value$	4.82 0.1855	0.36 0.8353	0.46 0.7945

Table-5: Distribution of number of European red mites on apple leaves

European red mites	Observed frequencies	Expected frequencies		
		$d\gamma(k, \theta)$	NBD(r,p)	DQL (α, λ)
0	70	69.67	69.49	66.63
1	38	37.49	37.6	37.96
2	17	20.02	20.1	21.25
3	10	10.67	10.7	11.81
4	9	5.69	5.69	6.39
5	3	3.03	3.02	3.26
6	2	1.61	1.6	1.69
7	1	.86	0.85	0.79
8	0	.96	0.95	0.23
150		150 $\hat{k} = 1.0078$ $\hat{\theta} = 1.5830$	150 $\hat{r} = 1.0245$ $\hat{p} = 0.5281$	150 $\hat{\alpha} = -4.6625$ $\hat{\lambda} = 0.63461$
	χ^2 $P\ value$	2.89 0.7169	2.91 0.7139	2.36 0.7974

The fitting of two parameter DQL distribution along with χ^2 and p- values has been presented to three data-sets. From the above tables it is observed that DQL distribution provides closer fits.

9. CONCLUSION

Two-parameter DQL distribution has been introduced, of which the one-parameter DL is a particular case, for modeling waiting and survival time's data. Several properties of the two-parameter DQL, such as moments, failure rate function, mean residual life function, estimation of parameters by the method of maximum likelihood and the method of moments have been discussed. The properties of size- biased and Zero- truncated version of DQL distribution have also been investigated. Finally, the proposed distribution has been fitted to a number of data sets relating to waiting and survival times to test its goodness of fit to which earlier the one-parameter DL has been fitted. It is observed that two-parameter DQL provides better fits than those by the DL and hence it should be preferred to the DQL while modeling count data-sets.

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