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INTUITIONISTIC Q-FUZZY HX GROUP

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ABSTRACT

In this paper, we define the concept of an intuitionistic Q-fuzzy HX group and define a new algebraic structure of intuitionistic Q-fuzzy HX group and some related properties are investigated. We also define the level subsets of an intuitionistic Q-fuzzy HX group and discussed some of its properties.

Keywords: *Intuitionistic fuzzy set, intuitionistic Q-fuzzy set, intuitionistic fuzzy subgroup, intuitionistic Q-fuzzy subgroup, intuitionistic Q-fuzzy HX subgroup and level sub HX groups.*

AMS Subject Classification (2000): *20N25, 03E72, 03F055, 06F35, 03G25.*

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1. INTRODUCTION

K.H.Kim [2] introduced the concept of intuitionistic Q-fuzzy semi prime ideals in semi groups and Osman kazanci, sultan yamark and serife yilmaz [10] introduced the concept of intuitionistic Q-fuzzy R-subgroups of near rings. A.Solairaju and R.Nagarajan [13][14] introduced and defined a new algebraic structure of Q-fuzzy groups. Li Hongxing [5] introduced the concept of HX group and the authors Luo Chengzhong , Mi Honghai , Li Hongxing [6] introduced the concept of fuzzy HX group. In this paper we define a new concept of intuitionistic Q-fuzzy HX group and its level sub HX groups and study some of their related properties.

2. PRELIMINARIES

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, G = (G , *) is a group, e is the identity element of G, and xy, we mean x * y.

2.1 Definition [1]: An intuitionistic fuzzy subset (IFS) λ in a set X is defined as an object of the form $\lambda = \{ \langle x , \mu_\lambda(x), \nu_\lambda(x) \rangle / x \in X \}$, where $\mu_\lambda : X \to [0,1]$ and $\nu_\lambda : X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively to be in λ and for every $x \in X$, $0 \leq \mu_\lambda(x) + \nu_\lambda(x) \leq 1$.

2.2 Definition [2]: An intuitionistic Q-fuzzy subset λ in a set X is defined as an object of the form $\lambda = \{ \langle (x,q), \mu_\lambda(x,q), \nu_\lambda(x,q) \rangle / x \in X \text{ and } q \in Q \}$, where $\mu_\lambda : X \times Q \to [0,1]$ and $\nu_\lambda : X \times Q \to [0,1]$ define the degree of membership and

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the degree of non-membership of the element $(x,q) \in X \times Q$ respectively and for every $(x,q) \in X \times Q$, $0 \leq \mu_\lambda(x,q) + \nu_\lambda(x,q) \leq 1$.

2.3 Definition [5]: In 2^G-{ϕ}, a nonempty set $\vartheta \subset 2^G$-{ϕ} is called a HX group on G, if ϑ is a group with respect to the algebraic operation defined by AB = {ab / a $\in$ A and b $\in$ B}, and its unit element is denoted by E.

3. INTUITIONISTIC FUZZY HX GROUPS

3.1 Definition: Let G be a group and a non-empty set $\vartheta \subset 2^G$-{ϕ} is a HX group on G. An intuitionistic fuzzy subset $\lambda = \langle A, \mu_\lambda(A), \nu_\lambda(A) \rangle$ of a HX group ϑ is said to be an intuitionistic fuzzy HX subgroup of a HX group ϑ if the following conditions are satisfied. For all A and B $\in \vartheta$,

(i) $\mu_\lambda(AB) \geq \min\{ \mu_\lambda(A), \mu_\lambda(B) \}$,
(ii) $\nu_\lambda(AB) \leq \max\{ \nu_\lambda(A), \nu_\lambda(B)) \}$.
(iii) $\mu_\lambda(A^{-1}) = \mu_\lambda(A)$,
(iv) $\nu_\lambda(A^{-1}) = \nu_\lambda(A)$.

3.1 Example: Let G = { 1 , -1 } ,then (G , •) is a group and ϑ = { {1} ,{-1} }, then (ϑ , •) is a Hx group. Define

$\mu_\lambda : \vartheta \rightarrow [0,1]$ by μ_λ ({1}) = 0.8 and μ_λ ({-1}) = 0.5.
$\nu_\lambda : \vartheta \rightarrow [0,1]$ by ν_λ ({1}) = 0.1 and ν_λ ({-1}) = 0.3.

Clearly λ is an intuitionistic fuzzy HX group of a HX group ϑ.

3.2 Definition: An intuitionistic Q-fuzzy subset $\lambda = \{\langle (A,q) , \mu_\lambda(A,q) , \nu_\lambda(A,q) \rangle / A \in \vartheta$ and $q \in Q \}$ of a HX group ϑ is said to be an intuitionistic Q-fuzzy HX subgroup of a HX group ϑ if the following conditions are satisfied. For all A and B $\in \vartheta$ and $q \in Q$,

(i) $\mu_\lambda(AB,q) \geq \min\{ \mu_\lambda(A,q), \mu_\lambda(B,q) \}$,
(ii) $\nu_\lambda(AB,q) \leq \max\{ \nu_\lambda(A,q), \nu_\lambda(B,q)) \}$,
(iii) $\mu_\lambda(A^{-1},q) = \mu_\lambda(A,q)$,
(iv) $\nu_\lambda(A^{-1},q) = \nu_\lambda(A,q)$.

4. PROPERTIES OF INTUITIONISTIC Q-FUZZY HX SUBGROUP

In this section, we discuss some of the properties of intuitionistic Q-fuzzy HX subgroup.

4.1 Theorem: Let λ be an intuitionistic Q-fuzzy HX subgroup of a HX group ϑ then

(i) $\mu_\lambda (A, q) \leq \mu_\lambda (E, q)$ and $\nu_\lambda (A, q) \geq \nu_\lambda (E, q)$ for all $A \in \vartheta$ and $q \in Q$ and E is the identity element of ϑ.

(ii) The subset H = {$A \in \vartheta$ / $\mu_\lambda (A, q) = \mu_\lambda (E, q)$ and $\nu_\lambda (A, q) = \nu_\lambda (E, q)$} is a sub HX group of ϑ.

Proof:
(i) Let $A \in \vartheta$ and $q \in Q$,

$$\begin{aligned}\mu_\lambda (A, q) &= \min \{ \mu_\lambda (A, q) , \mu_\lambda (A, q) \} \\ &= \min \{ \mu_\lambda(A, q) , \mu_\lambda(A^{-1} , q) \} \\ &\leq \mu_\lambda (AA^{-1} , q) \\ &= \mu_\lambda (E, q).\end{aligned}$$

$\mu_\lambda (A, q) \leq \mu_\lambda (E , q)$ for all $A \in \vartheta$ and $q \in Q$.

$$\begin{aligned}\nu_\lambda (A, q) &= \max \{ \nu_\lambda (A, q) , \nu_\lambda (A, q) \} \\ &= \max \{ \nu_\lambda (A, q) , \nu_\lambda (A^{-1} , q) \} \\ &\geq \nu_\lambda (AA^{-1} , q) \\ &= \nu_\lambda (E, q).\end{aligned}$$

$\nu_\lambda (A, q) \geq \nu_\lambda (E , q)$ for all $A \in \vartheta$ and $q \in Q$.

(ii) Let H = {$A \in \vartheta$ / $\mu_\lambda (A, q) = \mu_\lambda (E, q)$ and $\nu_\lambda (A, q) = \nu_\lambda (E, q)$}.

Clearly H is non-empty as $E \in H$.

Let A, B $\in$ H and q $\in$ Q,

Then, $\mu_\lambda (A, q) = \mu_\lambda (B, q) = \mu_\lambda (E, q)$ and $\nu_\lambda (A, q) = \nu_\lambda (B, q) = \nu_\lambda (E, q)$.

$$\begin{aligned} \mu_\lambda (AB^{-1}, q) &\geq \min \{\mu_\lambda (A, q), \mu_\lambda (B^{-1}, q)\} \\ &= \min \{\mu_\lambda (A, q), \mu_\lambda (B, q)\} \\ &= \min \{\mu_\lambda(E, q), \mu_\lambda (E, q)\} \\ &= \mu_\lambda (E, q). \end{aligned}$$

That is, $\mu_\lambda (AB^{-1}, q) \geq \mu_\lambda (E, q)$ and by part i, $\mu_\lambda (AB^{-1}, q) \leq \mu_\lambda (E, q)$.

Hence, $\mu_\lambda (AB^{-1}, q) = \mu_\lambda (E, q)$.

Also, $$\begin{aligned} \nu_\lambda (AB^{-1}, q) &\leq \max \{\nu_\lambda (A, q), \nu_\lambda (B^{-1}, q)\} \\ &= \max \{\nu_\lambda (A, q), \nu_\lambda (B, q)\} \\ &= \max \{\nu_\lambda (E, q), \nu_\lambda (E, q)\} \\ &= \nu_\lambda (E, q). \end{aligned}$$

That is, $\nu_\lambda (AB^{-1}, q) \leq \nu_\lambda (E, q)$ and by part i, $\nu_\lambda (AB^{-1}, q) \geq \nu_\lambda (E, q)$.

Hence, $\nu_\lambda (AB^{-1}, q) = \nu_\lambda (E, q)$.

Hence, $\mu_\lambda (AB^{-1}, q) = \mu_\lambda (E, q)$ and $\nu_\lambda (AB^{-1}, q) = \nu_\lambda (E, q)$ then $AB^{-1} \in H$. Clearly, H is a sub HX group of ϑ.

4.2 Theorem: Let λ be any intuitionistic Q-fuzzy HX subgroup of a HX group ϑ with identity E. Then

$\mu_\lambda (AB^{-1},q) = \mu_\lambda (E, q) \Rightarrow \mu_\lambda (A, q) = \mu_\lambda (B, q)$ and
$\nu_\lambda (AB^{-1},q) = \nu_\lambda (E, q) \Rightarrow \nu_\lambda (A, q) = \nu_\lambda (B, q)$, for all A,B in ϑ and $q \in Q$.

Proof: Given λ is an intuitionistic Q-fuzzy HX subgroup of ϑ and $\mu_\lambda (AB^{-1}, q) = \mu_\lambda (E, q)$ and
$\nu_\lambda (AB^{-1},q) = \nu_\lambda (E, q)$.

Then for all A , B in ϑ and $q \in Q$,

$$\begin{aligned} \mu_\lambda (A, q) &= \mu_\lambda (A(B^{-1}B), q) \\ &= \mu_\lambda ((AB^{-1})B, q) \\ &\geq \min \{ \mu_\lambda (AB^{-1}, q), \mu_\lambda (B, q)\} \\ &= \min \{ \mu_\lambda (E, q), \mu_\lambda (B, q)\} \\ &= \mu_\lambda (B, q). \end{aligned}$$

That is, $\mu_\lambda (A, q) \geq \mu_\lambda (B, q)$.

That is, $\mu_\lambda (AB^{-1}, q) = \mu_\lambda (E, q) \Rightarrow \mu_\lambda (A, q) \geq \mu_\lambda (B, q)$.

Since, $$\begin{aligned} \mu_\lambda (BA^{-1}, q) &= \mu_\lambda ((BA^{-1})^{-1}, q) \\ &= \mu_\lambda (AB^{-1}, q) \\ &= \mu_\lambda (E, q). \end{aligned}$$

That is, $\mu_\lambda (BA^{-1},q) = \mu_\lambda (E, q)$, then $\mu_\lambda (B, q) \geq \mu_\lambda (A, q)$.

Hence, $\mu_\lambda (A, q) = \mu_\lambda (B, q)$.

$$\begin{aligned} \nu_\lambda (A, q) &= \nu_\lambda (A(B^{-1}B), q) \\ &= \nu_\lambda ((AB^{-1})B, q) \\ &\leq \max \{ \nu_\lambda (AB^{-1}, q), \nu_\lambda (B, q)\} \\ &= \max \{ \nu_\lambda (E, q), \nu_\lambda (B, q)\} \\ &= \nu_\lambda (B, q). \end{aligned}$$

That is, $\nu_\lambda (A, q) \leq \nu_\lambda (B, q)$.

That is, $\nu_\lambda(AB^{-1}, q) = \nu_\lambda(E, q) \Rightarrow \nu_\lambda(A, q) \le \nu_\lambda(B, q)$.

Since, $\nu_\lambda(BA^{-1}, q) = \nu_\lambda((BA^{-1})^{-1}, q)$
$= \nu_\lambda(AB^{-1}, q)$
$= \nu_\lambda(E, q)$.

That is, $\nu_\lambda(BA^{-1}, q) = \nu_\lambda(E, q)$, then $\nu_\lambda(B, q) \le \nu_\lambda(A, q)$.

Hence, $\nu_\lambda(A, q) = \nu_\lambda(B, q)$.

4.3 Theorem: If λ is an intuitionistic Q-fuzzy HX subgroup of a HX group then for all A, B in ϑ and $q \in Q$,

(i) $\mu_\lambda(AB^{-1}, q) \ge \min\{\mu_\lambda(A, q), \mu_\lambda(B, q)\}$,

(ii) $\nu_\lambda(AB^{-1}, q) \le \max\{\nu_\lambda(A, q), \nu_\lambda(B, q)\}$.

Proof: Let λ be an intuitionistic Q-fuzzy HX subgroup of a HX group ϑ. Then for all A, B in ϑ and $q \in Q$,

Now, $\mu_\lambda(AB^{-1}, q) \ge \min\{\mu_\lambda(A, q), \mu_\lambda(B^{-1}, q)\}$.
$= \min\{\mu_\lambda(A, q), \mu_\lambda(B, q)\}$

$\mu_\lambda(AB^{-1}, q) \ge \min\{\mu_\lambda(A, q), \mu_\lambda(B, q)\}$.

Now, $\nu_\lambda(AB^{-1}, q) \le \max\{\nu_\lambda(A, q), \nu_\lambda(B^{-1}, q)\}$.
$= \max\{\nu_\lambda(A, q), \nu_\lambda(B, q)\}$
$\nu_\lambda(AB^{-1}, q) \le \max\{\nu_\lambda(A, q), \nu_\lambda(B, q)\}$.

4.4 Theorem: If λ is an intuitionistic Q-fuzzy subset of a HX group and for all A, B in ϑ, $q \in Q$,

(i) $\mu_\lambda(AB^{-1}, q) \ge \min\{\mu_\lambda(A, q), \mu_\lambda(B, q)\}$,

(ii) $\nu_\lambda(AB^{-1}, q) \le \max\{\nu_\lambda(A, q), \nu_\lambda(B, q)\}$, then λ is an intuitionistic Q-fuzzy HX subgroup of a HX group.

Proof: λ is an intuitionistic Q-fuzzy subset of a HX group and for all A, B in ϑ, $q \in Q$,

(i) $\mu_\lambda(AB^{-1}, q) \ge \min\{\mu_\lambda(A, q), \mu_\lambda(B, q)\}$,

Now, $\mu_\lambda(A, q) = \mu_\lambda(E(A^{-1})^{-1}, q) \ge \min\{\mu_\lambda(E, q), \mu_\lambda(A^{-1}, q)\} = \mu_\lambda(A^{-1}, q)$

Also, $\mu_\lambda(A^{-1}, q) = \mu_\lambda(EA^{-1}, q) \ge \min\{\mu_\lambda(E, q), \mu_\lambda(A, q)\} = \mu_\lambda(A, q)$.

Hence, $\mu_\lambda(A, q) = \mu_\lambda(A^{-1}, q)$.

Also, $\mu_\lambda(AB, q) = \mu_\lambda(A(B^{-1})^{-1}, q)$
$\ge \min\{\mu_\lambda(A, q), \mu_\lambda(B^{-1}, q)\}$
$\ge \min\{\mu_\lambda(A, q), \mu_\lambda(B, q)\}$.

That is, $\mu_\lambda(AB, q) \ge \min\{\mu_\lambda(A, q), \mu_\lambda(B, q)\}$.

(ii) $\nu_\lambda(AB^{-1}, q) \le \max\{\nu_\lambda(A, q), \nu_\lambda(B, q)\}$.

Now, $\nu_\lambda(A, q) = \nu_\lambda(E(A^{-1})^{-1}, q) \le \max\{\nu_\lambda(E, q), \nu_\lambda(A^{-1}, q)\} = \nu_\lambda(A^{-1}, q)$.

Also, $\nu_\lambda(A^{-1}, q) = \nu_\lambda(EA^{-1}, q) \le \max\{\nu_\lambda(E, q), \nu_\lambda(A, q)\} = \nu_\lambda(A, q)$.

Hence, $\nu_\lambda(A, q) = \nu_\lambda(A^{-1}, q)$.

Also, $\nu_\lambda(AB, q) = \nu_\lambda(A(B^{-1})^{-1}, q)$
$\le \max\{\nu_\lambda(A, q), \nu_\lambda(B^{-1}, q)\}$
$\le \max\{\nu_\lambda(A, q), \nu_\lambda(B, q)\}$.

That is, $\nu_\lambda (AB, q) \leq \max \{\nu_\lambda (A, q), \nu_\lambda (B, q)\}$.

Hence, λ is an intuitionistic Q-fuzzy HX subgroup of a HX group ϑ.

5. PROPERTIES OF LEVEL SUBSETS OF AN INTUITIONISTIC Q-FUZZY HX SUBGROUP

In this section, we introduce the concept of level subset of an intuitionistic Q-fuzzy HX subgroup and discuss some of its properties.

5.1 Definition: Let λ be an intuitionistic Q-fuzzy HX subgroup of a HX group ϑ. For any α, $\beta \in [0,1]$, we define the set, $\lambda_{<\alpha,\beta>} = \{ A \in \vartheta / \mu_\lambda(A, q) \geq \alpha$ and $\nu_\lambda (A, q) \leq \beta$, for some $q \in Q \}$, is called the level subset of λ.

5.1 Theorem: Let λ be an intuitionistic Q-fuzzy HX subgroup of a HX group ϑ and $\lambda_{<\alpha,\beta>}$ is non- empty. Then , $\lambda_{<\alpha,\beta>}$ is a sub HX group of ϑ.

Proof: For all A, B $\in \lambda_{<\alpha,\beta>}$ and $q \in Q$, we have,

$\mu_\lambda (A, q) \geq \alpha$ and $\nu_\lambda(A, q) \leq \beta$ and
$\mu_\lambda (B, q) \geq \alpha$ and $\nu_\lambda(B, q) \leq \beta$.

Now, $\mu_\lambda (AB^{-1}, q) \geq \min \{\mu_\lambda (A, q), \mu_\lambda (B, q)\}$.
$\mu_\lambda (AB^{-1}, q) \geq \min \{\alpha, \alpha\} = \alpha$.
$\nu_\lambda (AB^{-1}, q) \leq \max \{\nu_\lambda (A, q), \nu_\lambda (B, q)\}$.
$\nu_\lambda (AB^{-1}, q) \leq \max \{\beta, \beta\} = \beta$.
$AB^{-1} \in \lambda_{<\alpha,\beta>}$.

Hence $\lambda_{<\alpha,\beta>}$ is a sub HX group of ϑ.

5.2 Theorem: Let ϑ be a HX group and λ be a intuitionistic Q-fuzzy subset of ϑ such that $\lambda_{<\alpha,\beta>}$ is a sub HX group of ϑ for all α, $\beta \in [0,1]$ then λ is an intuitionistic Q-fuzzy HX subgroup of ϑ.

Proof: Clearly λ is non-zero.

Let A, B $\in \vartheta$ and $q \in Q$.

Let $\mu_\lambda (A, q) = \alpha_1$ and $\nu_\lambda(A, q) = \beta_1$ and

$\mu_\lambda (B, q) = \alpha_2$ and $\nu_\lambda(B, q) = \beta_2$.

Suppose let , $\alpha_1 < \alpha_2$ and $\beta_1 > \beta_2$, then A, B $\in \lambda_{<\alpha 1,\beta 1>}$.

As $\lambda_{<\alpha 1,\beta 1>}$ is a sub HX group of ϑ, $AB^{-1} \in \lambda_{<\alpha 1,\beta 1>}$.

Hence, $\mu_\lambda (AB^{-1}, q) \geq \alpha_1 = \min \{\alpha_1, \alpha_2\}$
$\geq \min \{\mu_\lambda (A, q), \mu_\lambda (B, q)\}$

That is, $\mu_\lambda (AB^{-1}, q) \geq \min\{\mu_\lambda (A, q), \mu_\lambda (B, q)\}$.

$\nu_\lambda (AB^{-1}, q) \leq \beta_1 = \max\{\beta_1, \beta_2\}$
$\leq \max \{\nu_\lambda (A, q), \nu_\lambda (B, q)\}$

That is, $\nu_\lambda (AB^{-1}, q) \leq \max \{\nu_\lambda (A, q), \nu_\lambda (B, q)\}$.

By Theorem 4.4, λ is an intuitionistic Q-fuzzy HX subgroup of ϑ.

5.2 Definition: Let λ be an intuitionistic Q-fuzzy HX subgroup of a HX group ϑ. The sub HX groups $\lambda_{<\alpha,\beta>}$, α, $\beta \in [0,1]$ are called level sub HX groups of λ.

5.3 Theorem: An intuitionistic Q-fuzzy subset λ of ϑ is an intuitionistic Q-fuzzy HX subgroup of a HX group ϑ if and only if the level subsets $\lambda_{<\alpha,\beta>}$, $\alpha, \beta \in$ Image λ, are sub HX groups of ϑ.

Proof: It is clear.

5.4 Theorem: λ is an intuitionistic Q-fuzzy HX subgroup of a HX group ϑ. The sequence of level sub HX groups

$\{\lambda_{<\alpha n, \beta n>} / n \in N\}$ is a nested family if α_n is decreasing and β_n is an increasing, where N={0,1,2,3,…}.

Proof: The level sub HX groups of an intuitionistic Q-fuzzy HX subgroup λ of a HX group ϑ form a chain. Since $\mu_\lambda(E, q) \geq \mu_\lambda(A, q)$ and $\nu_\lambda(E, q) \leq \nu_\lambda(A, q)$ for all A in ϑ and $q \in Q$, therefore $\lambda_{<\alpha 0, \beta 0>}$, where $\mu_\lambda(E, q) = \alpha_0$ and $\nu_\lambda(E, q) = \beta_0$ is the smallest and we have the chain :

$\{E\} \subseteq \lambda_{<\alpha 0, \beta 0>} \subseteq \lambda_{<\alpha 1, \beta 1>} \subseteq \lambda_{<\alpha 2, \beta 2>} \subseteq \ldots \subseteq \lambda_{<\alpha n, \beta n>} = \vartheta$, where $\alpha_0 > \alpha_1 > \alpha_2 > \ldots\ldots > \alpha_n$ and $\beta_0 < \beta_1 < \beta_2 < \ldots\ldots < \beta_n$.

5.5 Theorem: Any sub HX group H of a HX group ϑ can be realized as a level sub HX group of some intuitionistic Q-fuzzy HX subgroup of ϑ.

Proof: Let λ be an intuitionistic Q-fuzzy subset and $A \in \vartheta$ and $q \in Q$. Define,

$$\mu_\lambda(A, q) = \begin{cases} 0 & \text{if } A \notin H \\ \alpha & \text{if } A \in H, \text{ where } \alpha \in (0,1]. \end{cases}$$

$$\nu_\lambda(A, q) = \begin{cases} 0 & \text{if } A \in H \\ \beta & \text{if } A \notin H, \text{ where } \beta \in (0,1]. \end{cases}$$

We shall prove that λ is an intuitionistic Q-fuzzy HX subgroup of ϑ.

Let $A, B \in \vartheta$ and $q \in Q$,

(i) Suppose $A, B \in H$, then $AB \in H$ and $AB^{-1} \in H$.

$\mu_\lambda(A, q) = \alpha$, $\mu_\lambda(B, q) = \alpha$, and $\mu_\lambda(AB^{-1}, q) = \alpha$.

$\nu_\lambda(A, q) = 0$, $\nu_\lambda(B, q) = 0$, and $\lambda(AB^{-1}, q) = 0$.

Hence $\mu_\lambda(AB^{-1}, q) \geq \min\{\mu_\lambda(A, q), \mu_\lambda(B, q)\}$ and $\nu_\lambda(AB^{-1}, q) \leq \max\{\nu_\lambda(A, q), \nu_\lambda(B, q)\}$.

(ii) Suppose $A \in H$ and $B \notin H$, then $AB \notin H$ and $AB^{-1} \notin H$.

$\mu_\lambda(A, q) = \alpha$, $\mu_\lambda(B, q) = 0$ and $\mu_\lambda(AB^{-1}, q) = 0$.

$\nu_\lambda(A, q) = 0$, $\nu_\lambda(B, q) = \beta$ and $\nu_\lambda(AB^{-1}, q) = \beta$.

Hence $\mu_\lambda(AB^{-1}, q) \geq \min\{\mu_\lambda(A, q), \mu_\lambda(B, q)\}$ and $\nu_\lambda(AB^{-1}, q) \leq \max\{\nu_\lambda(A, q), \nu_\lambda(B, q)\}$.

(iii) Suppose $A, B \notin H$, then $AB^{-1} \in H$ or $AB^{-1} \notin H$.

$\mu_\lambda(A, q) = 0$, $\mu_\lambda(B, q) = 0$ and $\mu_\lambda(AB^{-1}, q) = \alpha$ or 0.

$\nu_\lambda(A, q) = \beta$, $\lambda(B, q) = \beta$ and $\nu_\lambda(AB^{-1}, q) = 0$ or β.

Hence $\mu_\lambda(AB^{-1}, q) \geq \min\{\mu_\lambda(A, q), \mu_\lambda(B, q)\}$ and $\nu_\lambda(AB^{-1}, q) \leq \max\{\nu_\lambda(A, q), \nu_\lambda(B, q)\}$.

Thus in all cases, λ is an intuitionistic Q-fuzzy HX subgroup of ϑ.

For this intuitionistic Q-fuzzy HX subgroup, $\lambda_{<\alpha, \beta>} = H$.

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