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STABILITY OF GENERALIZED CUBIC FUNCTIONAL EQUATION IN RANDOM NORMED SPACE

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ABSTRACT

In this paper we obtain the Hyers-Ulam stability for the generalized cubic fuctional equation $4f(x+my)+4f(x-my)+m^2f(2x)=8f(x)+4m^2f(x+y)+4m^2f(x-y)$ for a positive integer $m \geq 1$ in random normed space.

INTRODUCTION

A question in the theory of functional equations is the following "When is it true that a function which approximately satisfies a functional equation $\in$ must be close to an exact solution $\in$?" If the problem accepts a solution, we say that the equation $\in$ is stable.

In 1940, S.M.Ulam [13] gave a wide ranging talk before the Mathematics Club of the University of Wisconsin in which he discussed a number of important unsolved problems. Among those was the following question concerning the stability of homomorphism:

Let $(G_1, *)$ be a group and $(G_2, \circ, d)$ be a metric group with the metric d. Given $\in > 0$, does there exists a $\delta_\in > 0$ such that if a mapping h: $G_1 \to G_2$ satisfies the inequality $d(h(x*y), h(x) \circ h(y)) < \delta_\in$ $\forall$ x, y $\in G_1$, then there is a mapping H: $G_1 \to G_2$ such that for each x, y $\in G_1$ $H(x*y) = H(x) \circ H(y)$ and $d(h(x), H(x)) < \in$?

In the next year, D.H.Hyers [5], gave answer to the above question for additive groups under the assumption that groups are Banach spaces. In 1978, T.M.Rassias [12] proved a generalization of Hyers' theorem for additive mapping as a special case in the form of following result.

Suppose that E and F are real normed spaces with F a complete normed space, f: E $\to$ F is a mapping such that for each fixed $x \in E$ the mapping $t \to f(tx)$ is continuous on R, and let there exist $\in \geq 0$ and $p \in [0,1)$ s.t

$$\|f(x+y) - f(x) - f(y)\| \leq \varepsilon(\|x\|^p + \|y\|^p) \quad x, y \in E.$$

Then there exists a unique linear mapping T: E $\to$ F s.t $\|f(x) - T(x)\| \leq \varepsilon \dfrac{\|x\|^p}{(1-2^{p-1})}$, $x \in E$.

In this paper, we discuss the generalized cubic functional equation

$$4f(x+my)+4f(x-my)+m^2f(2x)=8f(x)+4m^2f(x+y)+4m^2f(x-y), \tag{1}$$

Since the cubic function $f(x) = cx^3$ is its solution and easy to check $f(0) = 0$ and $f(2x) = 8f(x)$. We prove the stability of (1) in norm and random normed linear space.

2. STABILITY OF GENERALIZED CUBIC FUNCTIONAL EQUATION

Throughout in this section, let X and Y be normed vector space and Banach space respectively.

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Theorem 2.1: Let $\phi : X \times X \to [0,\infty)$ be a function such that $\sum_{i=0}^{\infty}\left(\frac{1}{8}\right)^i \varphi\left(2^i x, 2^i y\right) < \infty$ for all x, y $\in$ X. Suppose that f: X $\to$ Y satisfies the inequality

$$\|4f(x+my)+4f(x-my)+m^2f(2x)-8f(x)-4m^2f(x+y)-4m^2f(x-y)\| \le \phi(x,y) \qquad (2.1)$$

for all x, y $\in$ X. Then there exists a unique cubic mapping g: X $\to$ Y such that

$$\|f(x)-g(x)\| \le \frac{1}{8m^2}\phi(x,x) \text{ for all } x \in X \text{ and } m \ge 1. \qquad (2.2)$$

Proof: Putting y = 0 and f(0) = 0 in (2.1), we get

$$\|m^2f(2x)-8m^2f(x)\| \le \phi(x,0)$$

$$\|f(x)-\frac{1}{8}f(2x)\| \le \frac{1}{8m^2}\phi(x,0) \qquad (2.3)$$

Replacing x by 2x in (2.3), we get

$$\|f(2x)-\frac{1}{8}f(2^2x)\| \le \frac{1}{8m^2}\phi(2x,0) \qquad (2.4)$$

Combine (2.3) and (2.4),

$$\|f(x)-\left(\frac{1}{8}\right)^2 f(2^2x)\| \le \|f(x)-\frac{1}{8}f(2x)\| + \|\frac{1}{8}f(2x)-\left(\frac{1}{8}\right)^2 f(2^2x)\| \le \frac{1}{8m^2}[\phi(x,0)+\frac{1}{8}\phi(2x,0)]$$

Continue in this way, we have

$$\|f(x)-\left(\frac{1}{8}\right)^t f(2^tx)\| \le \frac{1}{8m^2}\sum_{i=0}^{t-1}\left(\frac{1}{8}\right)^i \phi(2^i x,0) \qquad (2.5)$$

Dividing by 8^n and replacing x by 2^nx, we get

$$\|\left(\frac{1}{8}\right)^n f(2^nx)-\left(\frac{1}{8}\right)^{n+t} f(2^{n+t}x)\| \le \frac{1}{8m^2}\sum_{i=0}^{t-1}\left(\frac{1}{8}\right)^{i+n} \phi(2^{n+i} x,0) \qquad (2.6)$$

For all x $\in$ X. This shows that $\{8^{-t}f(2^tx)\}$ is a Cauchy sequence in Y by taking the limit $n \to \infty$. Since Y is a Banach space, it follows that the sequence $\{8^{-t}f(2^tx)\}$ converges. We define g: X $\to$ Y by g(x) = $\lim_{t\to\infty} 8^{-t}f(2^tx)$ for all x $\in$ X. Then

$$\|4g(x+my)+4g(x-my)+m^2 g(2x)-8g(x)-4m^2 g(x+y)-4m^2 g(x-y)\|$$

$$= \lim_{t\to\infty}\left(\frac{1}{8}\right)^t \|4f(2^t(x+my))+4f(2^t(x-my))+m^2f(2^t 2x)-8f(2^t x)-4m^2f(2^t(x+y))-4m^2f(2^t(x-y))\|$$

$$\le \lim_{t\to\infty}\left(\frac{1}{8}\right)^t \phi(2^t x, 2^t y) = 0$$

For all x, y $\in$ X. Thus g: X $\to$ Y is cubic. Now, prove that the function g is unique. Let h: X $\to$ Y be another cubic function satisfying (2.2). Then

$$\|g(x)-h(x)\| = \left(\frac{1}{8}\right)^t \|g(2^tx)-h(2^tx)\|$$

$$\le \left(\frac{1}{8}\right)^t (\|g(2^tx)-f(2^tx)\| + \|f(2^tx)-h(2^tx)\|)$$

$$\le \left(\frac{1}{8}\right)^t \frac{1}{8}\phi(2^t x,0)$$

For all x $\in$ X. As $t \to \infty$, we can conclude that g(x) = h(x) for all x $\in$ X. Thus g is unique.

Now, we will investigate the stability of the given cubic functional equation (1) using the alternative fixed point. Before proceeding the proof, we will state the theorem, the alternative of fixed point.

Theorem 2.2: (The alternative of fixed point [7], [2]). Suppose that we are given a complete generalized metric space (Ω, d) and a strictly contractive mapping T: $\Omega \rightarrow \Omega$ with Lipschitz constant L. Then for each given $x \in \Omega$, either d(T^n x, T^{n+1} x) = ∞ for all n ≥ 0, or there exists a natural number n_0 such that

1. d(Tnx, Tn+1x) < ∞ for all n $\geq$ n_0
2. The sequence (T^n x) is convergent to a fixed point y^* of T;
3. y^* is the unique fixed point of T in the set Δ = {y $\in \Omega$, d($T^n{}_0$ x, y) < ∞};
4. d(y, y^*) $\leq \dfrac{1}{1-L}$ d(y, Ty) for all y $\in \Delta$.

Now, let ϕ : X × X $\rightarrow$ [0, ∞) be a function such that $\lim\limits_{n\to\infty} \dfrac{\phi(\lambda_i{}^n x, \lambda_i{}^n y)}{\lambda_i{}^{3n}}$ =0,

for all x, y $\in$ X , where λ_i = 2 if i = 0 and λ_i = 1/2 if i = 1.

Theorem 2.3: Suppose that a function f: X$\rightarrow$Y satisfies the functional inequality

$$\|4f(x + my) + 4f(x - my) + m^2f(2x) - 8f(x) - 4m^2f(x + y) - 4m^2f(x - y)\| \leq \phi(x, y) \quad (3.1)$$

for all x, y $\in$ X . If there exists L < 1 such that the function x$\rightarrow \psi(x) = \phi(2x,0)$ has the property

$$\psi(x) \leq 8L\,\psi(2x) \quad (3.2)$$

for all x $\in$ X , then there exists a unique cubic function C: X $\rightarrow$ Y such that the inequality

$$\| f(x) - C(x)\| \leq \frac{L}{1-L}\psi(x) \quad (3.3)$$

holds for all x $\in$ X .

Proof: Consider the set Ω = {g|g: X$\rightarrow$Y} and introduce the generalized metric on,
d(g, h) = inf{K $\in$ (0,∞), || g(x)−h(x)|| $\leq$ K ψ (x) , x $\in$X} .

It is easy to show that (Ω, d) is complete. Now we define a function T: $\Omega \rightarrow \Omega$ by
Tg(x) =8g(2x) for all x ∈ X. Note that for all g, h $\in \Omega$,

d(g, h) < K

$\Rightarrow$|| g(x) − h(x) || $\leq$ K ψ (x) , for all x$\in$X,

$\Rightarrow$|| $\frac{1}{8}$ g(2x) - $\frac{1}{8}$ h(2x)||$\leq$ $\frac{1}{8}$ K ψ (2x), for all x$\in$X,

$\Rightarrow$|| $\frac{1}{8}$ g(2x) - $\frac{1}{8}$ h(2x)||$\leq$ Lψ (x), for all x$\in$X,

$\Rightarrow$d(Tg, Th) $\leq$ LK .

Hence we have that
d(Tg, Th) $\leq$ Ld(g, h), for all g, h$\in \Omega$, that is, T is a strictly self-mapping of Ω with the Lipschitz constant L. By setting y = 0, we have the equation (2.3) as in the proof of Theorem 2.1 and we use the equation (3.2), which is reduced

to ||f(x)- $\frac{1}{8}$ f(2x)|| $\leq \dfrac{1}{8m^2}\phi(x,0)$

||f(x)- $\frac{1}{8}$ f(2x)|| $\leq \dfrac{1}{8m^2}\psi(\dfrac{x}{2})$

||f(x)- $\frac{1}{8}$ f(2x)||$\leq L\psi(x)$

for all $x \in X$, that is, $d(f, Tf) \leq L < 1$. Now, we can apply the fixed point alternative and since $\lim_{n\to\infty} d(T^n f, C) = 0$, there exists a fixed point C of T in such that

$$C(x) = \lim_{n\to\infty} \frac{f(2^n x)}{8^n}$$

for all $x \in X$. Letting $x = 2^n x$ and $y = 2^n y$ in the equation (4.1) and dividing by 8^n

$$\|4C(x+my)+4C(x-my)+m^2 C(2x)-8C(x)-4m^2 C(x+y)-4m^2 C(x-y)\|$$

$$= \lim_{n\to\infty} \left(\frac{1}{8}\right)^n \|4f(2^n (x+my))+4f(2^n (x-my))+m^2 f(2^n 2x)-8f(2^n x)-4m^2 f(2^n (x+y))-4m^2 f(2^n (x-y))\|$$

$$\leq \lim_{n\to\infty} \left(\frac{1}{8}\right)^n \phi(2^n x, 2^n y) = 0$$

for all $x, y \in X$; that is it satisfies the equation ( ). Thus C is cubic. Also, the fixed point alternative guarantees that such a C is the unique function such that

$\| f(x) - C(x)\| \leq K\psi(x)$ for all $x \in X$ and some $K > 0$. Again using the fixed point alternative, we have

$$d(f, C) \leq \frac{L}{1-L}\psi(x)$$

3. STABILITY OF CUBIC FUNCTIONAL EQUATION IN RN-SPACES

In this section, we will use the usual terminology, notations and conventions of the theory of random normed spaces. The space of all probability distribution functions is denoted by $\Delta^+ = \{F : R \cup \{-\infty, +\infty\} \rightarrow [0,1]$: F is Left-continuous and non decreasing on R and F(0)=0, $F(+\infty)=1\}$ and the subset $D^+ \subseteq \Delta^+$ is the set $D^+=\{F \in \Delta^+, \; l^- F(\infty) =1\}$, where $l^- F(x)$ denotes the left limit of the function f at the point x. The space Δ^+ is partially ordered by the usual point wise ordering of functions, that is $F \leq G$, if and only if $F(t) \leq G(t)$ for all t in R. The maximal element for Δ^+ in this order is the distribution function given by

$$\varepsilon_0 = \begin{cases} 0 & if\ t \leq 0 \\ 1 & if\ t > 0 \end{cases}$$

Definition 3.1: A mapping $T:[0,1]x[0,1] \rightarrow [0,1]$ is a continuous triangular norm (briefly, a t-norm) if satisfies the following conditions:

(i) T is commutative and associative;
(ii) T is continuous;
(iii) T(a,1)=a for all $a \in [0,1]$;
(iv) $T(a, b) \leq T(c, d)$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Three typical examples of continuous t-norm are T(a, b)=ab, T(a, b)=max{a+b-1, 0} and T(a, b)=min(a, b)

Definition 3.2: A random normed space (briefly, RN-space) is a triple (X, μ , T), where X is a vector space, T is a continuous -norm, and μ is a mapping from X into D^+ such that, the following conditions hold:

(RN1) $\mu_x(t) = \varepsilon_0(t)$ for all t>0 if and only if x=0;

(RN2) $\mu_{ax}(t) = \mu_x(t/|a|)$ for all,; (RN3) for all x in X, $a \neq 0$ and all $t \geq 0$

(RN3) $\mu_{x+y}(t+s) \geq T(\mu_x(t), \mu_y(t))$ for all x ,y in X and all $t, s \geq 0$

Definition 3.3[2]: Let (X, μ , T) be an RN-space. Consider the following.

(1) A sequence $\{x_n\}$ in X is said to be convergent to x in X if, for every t>0 and ε >0, there exists a positive integer N such that $\mu_{x_n - x}(t) > 1-\varepsilon$ whenever $n \geq N$.
(2) A sequence $\{x_n\}$ in X is called Cauchy if, for every t > 0 and ε > 0, there exists a positive integer N such that $\mu_{x_n - x_m}(t) > 1-\varepsilon$ whenever $n, m \geq N$.
(3) An RN-space (X, μ , T) is said to be complete if every Cauchy sequence in X is convergent to a point in X.

Theorem 3.4 (see [2]): If (X, μ ,T) is an RN-space and{x_n} is a sequence such that $x_n \to x$, then $\lim_{n\to\infty}\mu_{x_n}(t)=\mu_x(t)$

Theorem 3.5[2]: Let (X, μ , min) be an RN-space and define $E_{\lambda,\mu}(x)$=inf{t>0; $\mu_x(t)>1-\lambda$ }, $\forall\lambda\in$[0,1], x$\in$X. Then $E_{\lambda,\mu}(x_1-x_n)\le E_{\lambda,\mu}(x_1-x_2)+......+E_{\lambda,\mu}(x_{n-1}-x_n)$ for all $x_1,x_2,\ldots\ldots,x_n\in$ X and the sequence {x_n} is convergent to x with respect to random norm μ if and only if $E_{\lambda,\mu}(x_n-x)\to 0 as\, n\to\infty$. Also the sequence {x_n} is a Cauchy sequence with respect to random norm μ if and only if it is a Cauchy sequence with $E_{\lambda,\mu}$

Now prove the stability of cubic mappings in RN-Spaces

Theorem 3.6: Let X be linear space, (Z, μ', min) an RN-space, and ϕ: X x X$\to$Z a function such that for some 0< α <8

$$\mu'_{\varphi(2x,0)}(t)\ge\mu'_{\alpha\varphi(x,0)}(t)\qquad\forall x\in X,t>0 \tag{3.1}$$

F(0)=0 and $\lim_{n\to\infty}\mu'_{\phi(2^n x,2^n y)}(8^n t)=1$ for all x, y in X and all t>0. Let (Y, μ , min) be a complete RN-space. If f: X$\to$Y is a mapping such that

$$\mu_{4f(x+my)+4f(x-my)+m^2f(2x)-8f(x)-4m^2f(x+y)-4m^2f(x-y)}(t)\ge\mu'_{\phi(x,y)}(t)\quad\forall x,y\in X\text{ , t > 0} \tag{3.2}$$

Then there exists a unique cubic mapping C: X$\to$Y such that

$$\mu_{f(x)-C(x)}(t)\ge\mu'_{\phi(x,0)}(m^2(8-\alpha)t \tag{3.3}$$

Proof: From (3.2) it follows that

$$\begin{aligned}&E_{\lambda,\mu}(4f(x + my) + 4f(x-my) + m^2f(2x) - 8f(x) - 4m^2f(x + y) - 4m^2f(x - y))\\ &\quad=\inf\{t>0:\ \mu_{4f(x+my)+4f(x-my)+m^2f(2x)-8f(x)-4m^2f(x+y)-4m^2f(x-y)}(t)>1-\lambda\}\\ &\quad\le\inf\{t>0,\mu'_{\phi(x,y)}(t)>1-\lambda\}\\ &\quad=E_{\lambda,\mu'}(\phi(x,y)),\qquad\forall x,y\in X,\lambda\in(0,1).\end{aligned} \tag{3.4}$$

Putting y = 0 in (4), we get

$$E_{\lambda,\mu}(m^2f(2x)-8m^2f(x))\le E_{\lambda,\mu'}(\phi(x,0))$$

$$E_{\lambda,\mu}\ (\frac{1}{8}f(2x)-f(x)\|\le\frac{1}{8m^2}E_{\lambda,\mu'}(\phi(x,0)) \tag{3.5}$$

Replacing x by 2^n x in (3.5), we get

$$\begin{aligned}E_{\lambda,\mu}\ (\frac{1}{8^{n+1}}f(2^{n+1}x)-\frac{1}{8^n}f(2^n x)\| &\le\frac{1}{8^{n+1}m^2}E_{\lambda,\mu'}(\phi(2^n x,0))\\ &\le\frac{\alpha^n}{8^{n+1}m^2}E_{\lambda,\mu'}(\phi(x,0))\end{aligned} \tag{3.6}$$

$$\begin{aligned}E_{\lambda,\mu}(\frac{f(2^n x)}{8^n}-f(x))&\le E_{\lambda,\mu}\left(\sum_{k=0}^{n-1}\left(\frac{f(2^{k+1}x)}{8^{k+1}}-\frac{f(2^k x)}{8^k}\right)\right)\\ &\le\sum_{k=0}^{n-1}\left(E_{\lambda,\mu}\left(\frac{f(2^{k+1}x)}{8^{k+1}}-\frac{f(2^k x)}{8^k}\right)\right)\\ &\le\sum_{k=0}^{n-1}\frac{1}{8^{k+1}m^2}\left(E_{\lambda,\mu'}\left(\phi(2^k x,0\right)\right)\\ &\le\sum_{k=0}^{n-1}\frac{\alpha^k}{8^{k+1}m^2}\left(E_{\lambda,\mu'}\left(\phi(x,0\right)\right)\end{aligned} \tag{3.7}$$

Replacing x by 2^m x in (3.7), we get

$$E_{\lambda,\mu}\left(\frac{f(2^{n+m}x)}{8^{n+m}}-\frac{f(2^m x)}{8^m}\right)\le\sum_{k=0}^{n-1}\frac{\alpha^k}{8^{k+m+1}m^2}\left(E_{\lambda,\mu'}\left(\phi(2^m x,0)\right)\right)$$
$$\le\sum_{k=0}^{n-1}\frac{\alpha^{k+m}}{8^{k+m+1}m^2}\left(E_{\lambda,\mu'}\left(\phi(x,0)\right)\right)$$
$$\le\sum_{k=m}^{n+m-1}\frac{\alpha^k}{8^{k+1}m^2}\left(E_{\lambda,\mu'}\left(\phi(x,0)\right)\right)$$
$$=\frac{E_{\lambda,\mu'}(\phi(x,0))}{8m^2}\sum_{k=m}^{n+m-1}\left(\frac{\alpha}{8}\right)^k \tag{3.8}$$

Thus $\{f(2^n x)/8^n\}$ is a Cauchy sequence in (Y, μ ,min). Since (Y, μ , min) is complete RN-space, this sequence converges to some point C(x)in Y. From (3.7), we get

$$E_{\lambda,\mu}(C(x)-f(x))\le E_{\lambda,\mu}\left(C(x)-\frac{f(2^n x)}{8^n}\right)+E_{\lambda,\mu}\left(\frac{f(2^n x)}{8^n}-f(x)\right)$$
$$\le E_{\lambda,\mu}\left(C(x)-\frac{f(2^n x)}{8^n}\right)+\frac{E_{\lambda,\mu'}(\varphi(x,0))}{8m^2}\sum_{k=0}^{n-1}\left(\frac{\alpha}{8}\right)^k \tag{3.9}$$

Taking the limit as n $\rightarrow\infty$ and using (3.9), we have

$$E_{\lambda,\mu}(C(x)-f(x))\le E_{\lambda,\mu'}(\phi(x,0)).\frac{1}{m^2(8-\alpha)}$$

i.e.,

Inf {t>0; $\mu_{C(x)-f(x)}(t)>1-\lambda$ } $\le\inf\{t>0;\mu'_{\phi(x,0)}(m^2(8-\alpha)t)>1-\lambda\}$ (3.10)

Thus we have,

$$\mu_{C(x)-f(x)}(t)\ge\mu'_{\phi(x,0)}(m^2(8-\alpha)t)$$

Replacing x, y by 2^nx, 2^ny in (3.2) respectively, we get

$$\mu_{4f(2^n(x+my))/8^n+4f(2^n(x-my))/8^n+m^2f(2^{n+1}x)/8^n-8f(2^n x)/8^n-4m^2f(2^n(x+y))/8^n-4m^2f(2^n(x-y))/8^n}(t)$$
$$\ge\mu'_{\varphi(2^n x,2^n y)}(8^n t)$$

Since, $\lim_{n\to\infty}\mu'_{\phi(2^n x,2^n y)}(8^n t)=1$ we conclude that C satisfies the cubic equation.

To, Prove the uniqueness of the cubic mapping C, assume that there exists a cubic mapping D: X $\rightarrow$ Y which satisfies (3). Since $C(2^n x) = 8^n C(x)$ and $D(2^n x) = 8^n D(x)$.

$$\mu_{C(x)-D(x)}(t)=\lim_{n\to\infty}\mu_{C(2^n x)/8^n)-D(2^n x)/8^n)}(t)$$

$$\mu_{C(2^n x)/8^n)-D(2^n x)/8^n)}(t)\ge\min\left\{\mu_{C(2^n x)/8^n)-f(2^n x)/8^n)}\left(\frac{t}{2}\right),\mu_{f(2^n x)/8^n)-D(2^n x)/8^n)}\left(\frac{t}{2}\right)\right\}$$
$$\ge\mu'_{\phi(2^n x,o)}(8^n m^2(8-\alpha)t/2)$$
$$\ge\mu'_{\phi(x,o)}(8^n m^2(8-\alpha)t/2\alpha^n)$$

Since $\lim_{n\to\infty}(8^n m^2(8-\alpha)t/2\alpha^n)=\infty$, we get $\lim_{n\to\infty}\mu'_{\phi(x,o)}(8^n m^2(8-\alpha)t/2\alpha^n)=1$.thus $\mu_{C(x)-D(x)}(t)=1$ for all t > 0 and so C(x) = D(x).

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