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THE EFFECT OF HEAT TRANSFER ON MHD OSCILLATORY FLOW OF A JEFFREY FLUID IN A CHANNEL

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ABSTRACT

We studied the effect of heat transfer on MHD oscillatory flow of Jeffrey fluid in a channel. The expressions for the velocity and temperature are obtained analytically. It is found that, the velocity u *increases with increasing* λ_1, *N''*, Gr, Pe *and* λ *, while it decreases with increasing* M, Re *and* ω *. Also, it is observed that the temperature* θ *increases with increasing N and* Pe, *while it decreases with increasing* ω *. Further, it is found that, the velocity is more for Jeffrey fluid than that of Newtonian fluid.*

2. INTRODUCTION

The flow of an electrically conducting fluid has important applications in many branches of engineering science such as magnetohydrodynamics (MHD) generators, plasma studies, nuclear reactor, geothermal energy extraction, electromagnetic propulsion, the boundary layer control in the field of aerodynamics and so on. Heat transfer effect on laminar flow between parallel plates under the action of transverse magnetic field was studied by Nigam and Singh [60]. Soundalgekar and Bhat [86] have investigated the MHD oscillatory flow of a Newtonian fluid in a channel with heat transfer. MHD flow of viscous fluid between two parallel plates with heat transfer was discussed by Attia, and Kotb [10]. Raptis *et al.* [74] have analyzed the hydromagnetic free convection flow through a porous medium between two parallel plates. Aldoss *et al.* [3] have studied mixed convection flow from a vertical plate embedded in a porous medium in the presence of a magnetic field. Makinde and Mhone [54] have considered heat transfer to MHD oscillatory flow in a channel filled with porous medium. Mostafa [58] have studied thermal radiation effect on unsteady MHD free convection flow past a vertical plate with temperature dependent viscosity. Unsteady heat transfer to MHD oscillatory flow through a porous medium under slip condition was investigated by Hamza *et al.* [41].

Moreover the non-Newtonian fluids are more appropriate than Newtonian fluids in many practical applications. Examples of such fluids include certain oils, lubricants, mud, shampoo, ketchup, blood at low shear rate, cosmetic products, polymers and many others. Unlike the viscous fluids, all the non-Newtonian fluids (in terms of their diverse characteristics) cannot be described by a single constitutive relationship. Hence, several models of non-Newtonian fluids are proposed in the literature: Al Khatib and Wilson (4) have studied the Poiseuifle flow of a yield stress fluid in a channel. Flow of a visco-plastic fluid in a channel of slowly varying width was studied by Frigaard and Ryan [35].

Aamir Ali and Saleem Asghar [1] have analyzed by oscillatory channel flow for non-Newtonian fluid. Rita and Jyoti Das [79] have studied the effect of heat transfer on MHD oscillatory viscoelastic fluid flow in a channel through a porous medium.

In view of these, we studied the effect of heat transfer on MHD oscillatory flow of a Jeffrey fluid in a channel. The expressions are obtained for velocity and temperature analytically. The effects of various emerging parameters on the velocity and temperature are discussed through graphs in detail.

3. MATHEMATICAL FORMULATION

We consider the flow of a Jeffrey fluid in a channel of width *h* under the influence of electrically applied magnetic field and radiative heat transfer as depicted in Fig.l. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. We choose the Cartesian coordinate system (x, y) , where x - is taken along center of the channel and the y - axis is taken normal to the flow direction.

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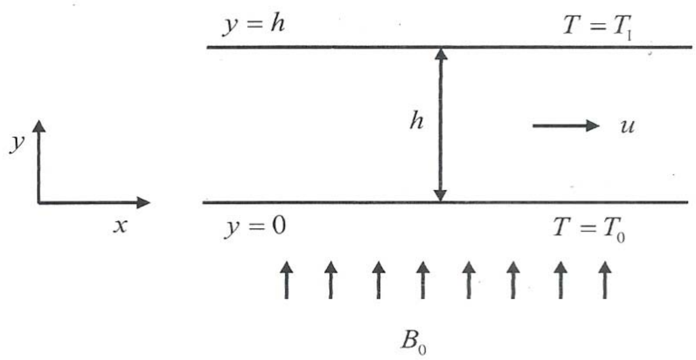

Fig. 1 Physical model of the problem

The constitute equation of S for Jeffrey fluid is

$$S = \frac{\mu}{1+\lambda_1}(r + \lambda_2 r) \tag{2.1}$$

where u is the dynamic viscosity, \ is the ratio of relaxation to retardation times, X, is the retardation time, y is the shear rate and dots over the quantities denote differentiation with time.

The basic equations of momentum and energy governing such a flow, subject to the Boussinesq approximation, are:

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\mu}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 u + \rho g \beta\left(T - T_0\right) \tag{2.2}$$

$$\rho\frac{\partial T}{\partial t} = \frac{k}{c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{c_p}\frac{\partial q}{\partial y} \tag{2.3}$$

The boundary conditions are given by

$$u = 0, \qquad T = T_0 \qquad \text{at} \qquad y = 0 \tag{2.4}$$

$$u = 0, \qquad T = T_1 \qquad \text{at} \qquad y = h \tag{2.5}$$

where u is the axial velocity, T is the fluid temperature, p is the pressure, ρ is the fluid density, B_0 is the magnetic field strength, σ is the conductivity of the fluid, g is the acceleration due to gravity, β is the coefficient of volume expansion due to temperature, c_p is the specific heat at constant pressure, k is the thermal conductivity and q is the radiative heat flux. Following Cogley *et al.* (1968), it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q}{\partial y} = 4\alpha^2\left(T_0 - T\right) \tag{2.6}$$

here α is the mean radiation absorption coefficient.

Introducing the following non-dimensional variables

$$\bar{x} = \frac{x}{h},\ \bar{y} = \frac{y}{h},\ \bar{u} = \frac{u}{U},\ \theta = \frac{T - T_0}{T_1 - T_0},\ \bar{t} = \frac{tU}{h},\ \bar{p} - \frac{pa}{\mu U},\ M^2 = \frac{\sigma h^2 B_0^2}{\mu},$$

$$Gr = \frac{\rho g \beta\left(T_1 - T_0\right)}{U\mu},\ \mathrm{Re} = \frac{\rho h U}{\mu}, pe = \frac{\rho h U c_p}{k}, N^2 = \frac{4\alpha^2 h^2}{k}$$

here U is the mean flow velocity, into the equations (2.2) and (2.3), we get (after dropping bars)

$$\mathrm{Re}\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2} - M^2 u + Gr\theta \tag{2.7}$$

$$pe\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2} + N^2\theta \tag{2.8}$$

where Re is the Reynolds nuber, M is the Hartmann number, Gr is the Grashof number, pe is the peclet number and N is the radiation parameter.

The corresponding non-dimensional boundary conditions are

$$u = 0, \qquad \theta = 0 \qquad \text{at} \qquad y = 0 \tag{2.9}$$

$$u = 0, \qquad \theta = 1 \qquad \text{at} \qquad y = 1 \tag{2.10}$$

4. SOLUTION

In order to solve equations (2.7) – (2.10) for purely oscillatory flow, let

$$-\frac{\partial p}{\partial x} = \lambda e^{iwt} \tag{3.1}$$

$$u(y-t) = u_0(y)e^{i\omega t} \tag{3.2}$$

$$\theta(y,t) = \theta_0(y)e^{i\omega t} \tag{3.3}$$

where λ is a real constant and ω is the frequency of the oscillation.

Substituting the equations (3.1) – (3.3) in to the equations (2.7) – (2.10), we get

$$\frac{d^2u_0}{dy^2} - m_2^2 u_0 = -\lambda(1+\lambda_1) - Gr(1+\lambda_1)\theta_0 \tag{3.4}$$

$$\frac{d^2\theta_0}{dy^2} + m_1^2\theta_0 = 0 \tag{3.5}$$

with the boundary conditions

$$u_0 = 0, \qquad \theta_0 = 0 \qquad \text{at} \qquad y = 0 \tag{3.6}$$

$$u_0 = 0, \qquad \theta_0 = 1 \qquad \text{at} \qquad y = 1 \tag{3.7}$$

in which $m_1 = \sqrt{N^2 - i\omega pe}$ and $m_2 = \sqrt{M^2 + i\omega \mathrm{Re}}$.

Solving equations (3.4) and (3.5) using the boundary conditions (3.6) and (3.7), we obtain

$$u_0(y) = -A\cosh m_2 y + C\frac{\sinh m_2 y}{\sinh m_2} + A + B\frac{\sin m_1 y}{\sin m_1} \tag{3.8}$$

and $$\theta_0(y) = \frac{\sin m_1 y}{\sin m} \tag{3.9}$$

where $A = \dfrac{\lambda(1+\lambda_1)}{m_2^2}, B = \dfrac{Gr(1+\lambda_1)}{(m_1^2+m_2^2)}$ and $C = (A\cosh m_2 - A - B)$

Therefore, the fluid velocity and temperature are given as

$$u(y,t) = \left(-A\cosh m_2 y + C\frac{\sinh m_2 y}{\sinh m_2} + A + B\frac{\sin m_1 y}{\sin m_1}\right)e^{i\omega t} \tag{3.10}$$

and $$\theta(y,t) = \frac{\sin m_1 y}{\sin m}e^{i\omega t} \tag{3.11}$$

The rate of heat transfer coefficient in terms of Nusselt number Nu at the plate $y = 0$ of the channel is given by

$$Nu = \left.\frac{\partial\theta}{\partial y}\right|_{y=0} = \frac{m_1}{\sin m_1}e^{i\omega t} \tag{3.12}$$

When $M \to 0$, the Eq. (3.10) becomes

$$u(y,t) = \left(-A_1\cosh m_3 y + C_1\frac{\sinh m_3 y}{\sinh m_3} + A_1 + B_1\frac{\sin m_1 y}{\sin m_1}\right)e^{i\omega t} \tag{3.13}$$

where $m_3 = \sqrt{i\omega \mathrm{Re}}$ $A_1 = \dfrac{\lambda(1+\lambda_1)}{m_3^2}, B_1 = \dfrac{Gr(1+\lambda_1)}{(m_1^2+m_3^2)}$ and $C = (A_1\cosh m_3 - A_1 - B_1)$

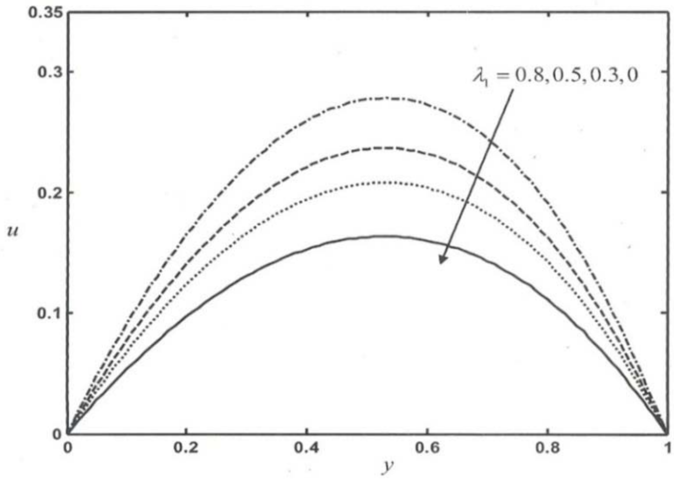

Fig. 2. Effects of material parameter λ_1 on velocity u
for Re = 1, Gr = 1, Pe = 0.71, λ = 1, ω = 1, t = 0.5, N = 1 and M = 1.

Above Fig. 2 shows the effect of material parameter λ_1 on velocity u for $\mathrm{Re}=1, Gr=1, pe=0.71, \lambda=1, \omega=1, t=0.5, N=1$ and $M=1$. It is observed that, the axial velocity u increases with increasing λ_1. Also, the maximum velocity occurs at the centerline of the channel while the minimum at the channel walls. Moreover, the velocity is more of Jeffrey fluid $(\lambda_1 > 0)$ than that of Newtonian fluid $(\lambda_1 \to 0)$.

Effect of Hartmann number M on velocity u for $\mathrm{Re}=1, Gr=1, pe=0.71$, $\lambda=1$, $\omega=1, t=0.5, N=1$ and $\lambda_1=0.3$ is shown in

5. DISCUSSION OF THE RESULTS

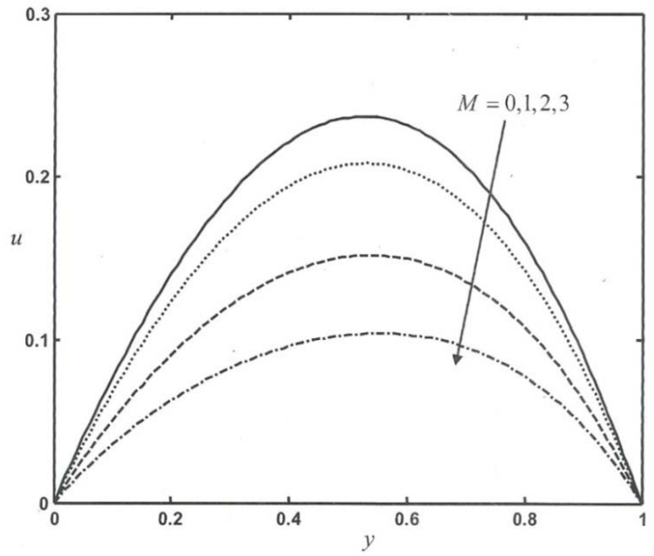

Fig. 3. Effect of Hartmann number M on velocity u for
Re = 1, Gr = 1, Pe = 0.71, λ = 1, ω = 1, t = 0.5, N = 1 and λ_1 = 0.3.

Above Fig. 3. It is found that, the axial velocity u decreases with increasing M. Further, it is found that, the velocity is more for non-conducting (magnetic) (i.e., $M \to 0$) Jeffrey fluid than that of conducting Jeffrey fluid.

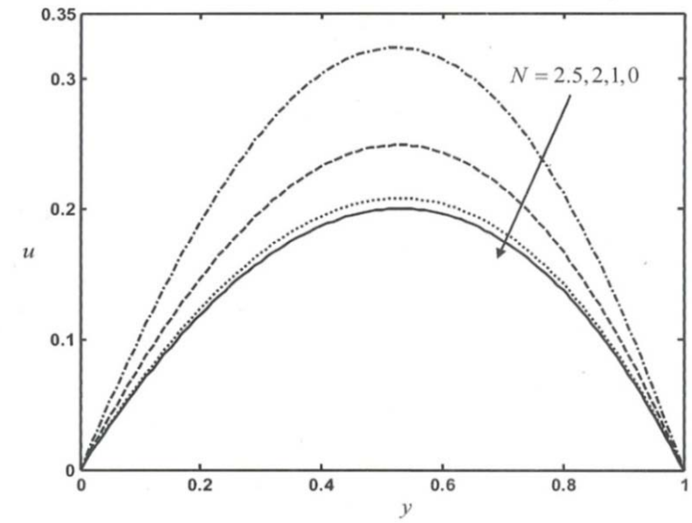

Fig. 4. Effect of radiation parameter N on velocity u for
Re = 1, Gr = 1, Pe = 0.71, λ = 1, ω = 1, t = 0.5, M = 1 and λ_1 = 0.3.

Above Fig. 4 depicts the effect of radiation parameter N on velocity u for Re = 1, Gr = 1, Pe = 0.71, λ = 1, ω = 1, t = 0.5, M - 1 and λ_1 = 0.3. It is noted that, the axial velocity u increases with an increase in N.

Effect of Grashof number Gr on velocity u for Re = 1, M = 1, = 0.71, λ = 1, ω= 1, t = 0.5, N = 1 and λ_1 =0.3 is depicted in

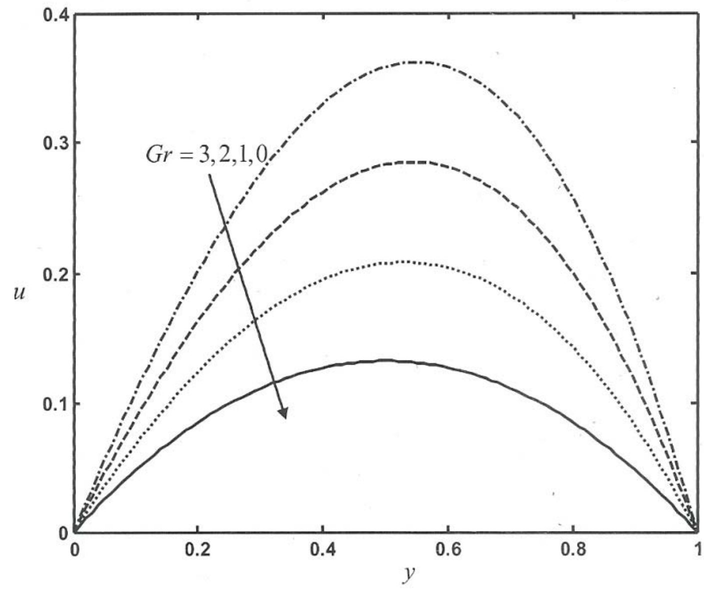

Fig. 5. Effect of Grashof number Gr on velocity u for
Re = 1, M = 1, Pe = 0.71, λ = 1, ω = 1, t = 0.5, N = 1 and λ_1 = 0.3.

Above Fig.5. It is observed that, the axial velocity u increases with increasing Gr.

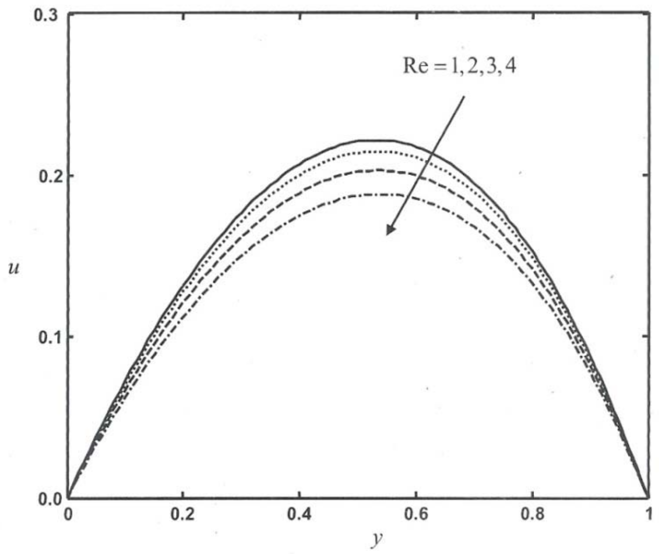

Fig. 6. Effect of Reynolds number *Re* on velocity *u* for
$M = 1$, $Gr = 1$, $Pe = 0.71$, $\lambda = 1$, $\omega = 1$, $t = 0.5$, $N = 1$ and $\lambda_1 = 0.3$.

Above Fig. 6 illustrates the effect of Reynolds number Re on velocity *u* for $M = 1$, Gr =1, $Pe = 0.71$, $\lambda = 1$, $\omega = 1$, $t = 0.5$, $N = 1$ and λ_1, = 0.3. It is found that, the axial velocity *u* decreases with decreasing Re.

Effect of λ on velocity *u* for Re = 1, $Gr = 1$, $Pe = 0.71$, $M = 1$, $\omega = 1$, $t = 0.5$, M= 1 and t = 0.3 is illustrated in

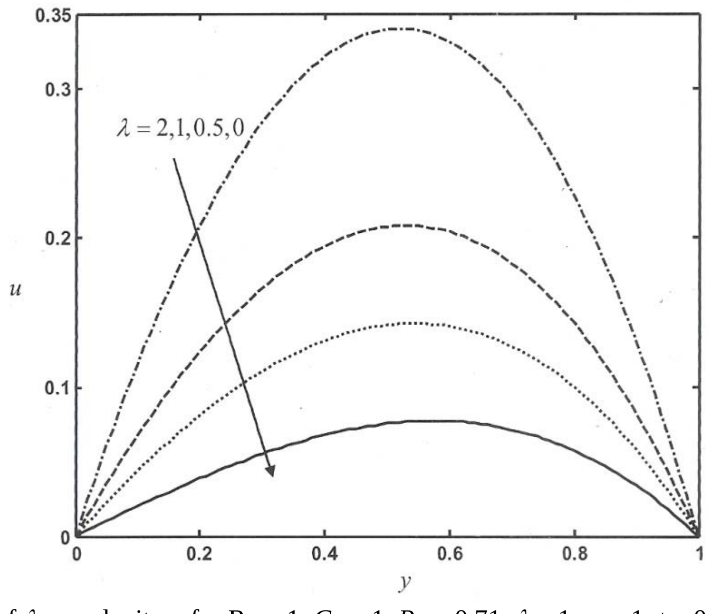

Fig. 7. Effect of λ on velocity *u* for Re = 1, $Gr = 1$, $Pe = 0.71$, $\lambda = 1$, $\omega = 1$, $t = 0.5$, $N = 1$ and $\lambda_1 = 0.3$.

Above Fig. 7. It is noted that, the axial velocity *u* increases with an increase in λ .

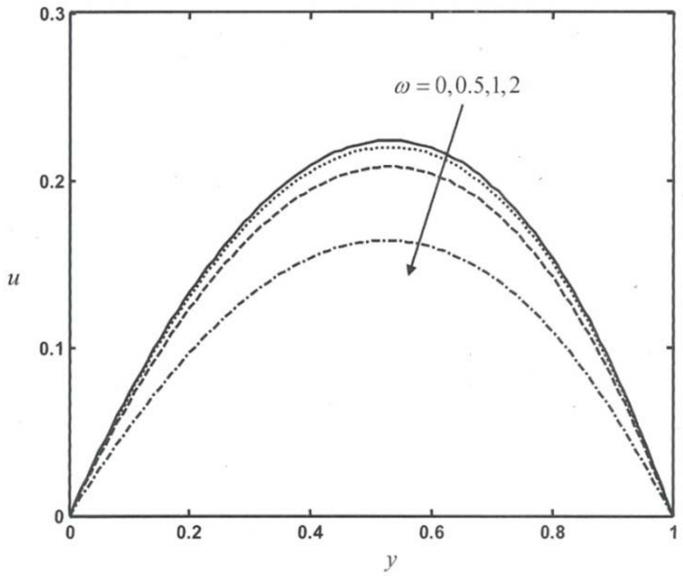

Fig. 8. Effect of ω on velocity u for Re = 1, Gr = 1, Pe = 0.71, λ = 1, M = 1, t = 0.5, N = 1 and λ_1 = 0.3.

Above Fig. 8 shows the effect of ω on velocity u for Re = 1, Gr — 1, Pe - 0.71, λ = 1, M = 1, t = 0.5, = 1 and λ_1, =0.3. It is observed that, the axial velocity u decreases with increasing ω.

Effect of TV on temperature θ for Pe = 0.71, ω= 1 and t = 0.5 is shown in

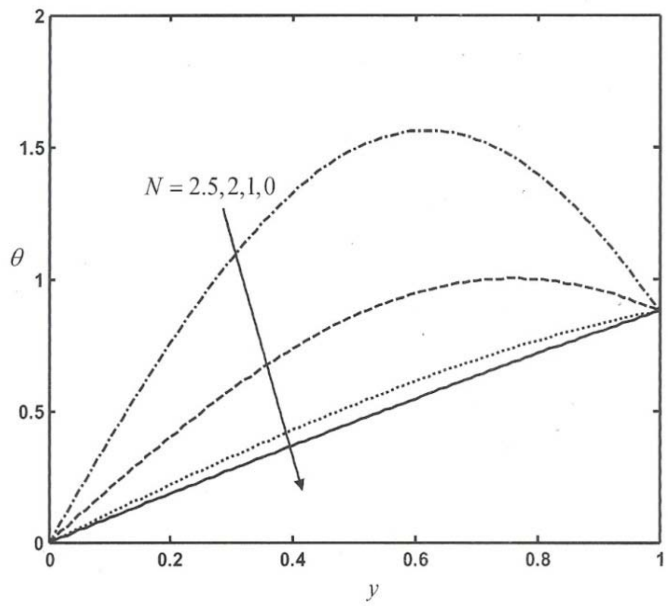

Fig. 9. Effect of N on temperature θ for Pe = 0.71, ω = 1, and t = 0.5.

Above Fig.9. It is noted that, the temperature θ increases with an increase in N.

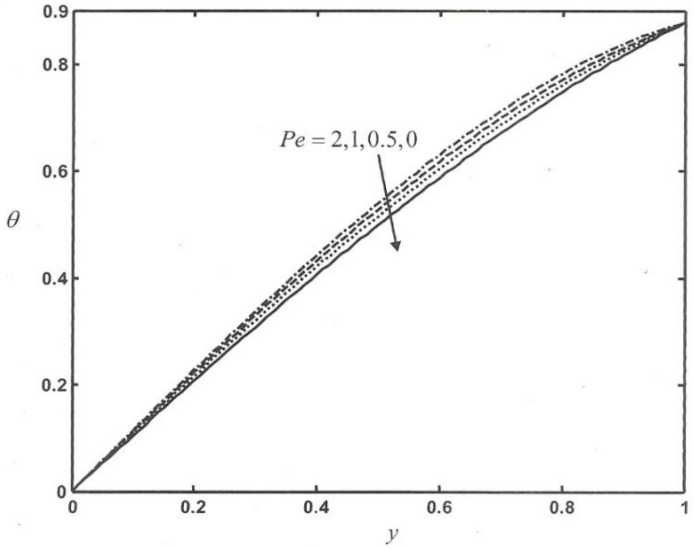

Fig. 10. Effect of Pe on temperature θ for $N = 1$, $\omega = 1$, and $t = 0.5$.

Above Fig. 10 depicts the effect of Pe on temperature θ for TV = 1, $\boldsymbol{\omega} = 1$ and $t = 0.5$. It is observed that, the temperature θ increases with increasing Pe .

Effect of ω on temperature θ for TV = 1, $Pe = 0.71$ and $t = 0.5$ is depicted in

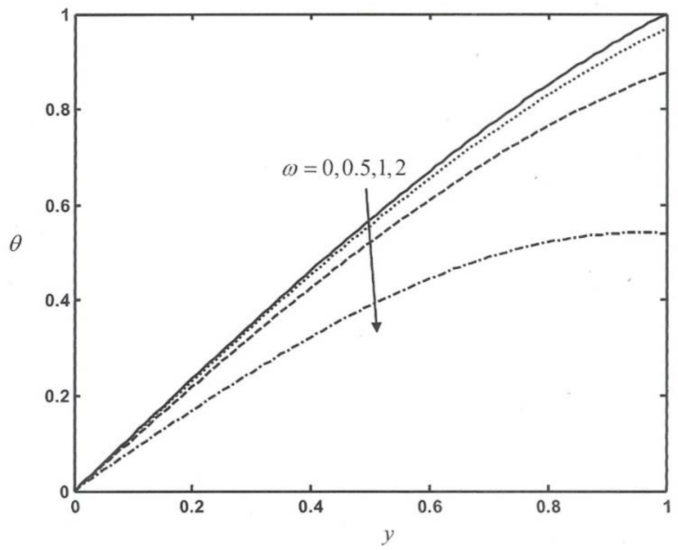

Fig. 11. Effect of ω on temperature θ for $N = 1$, $Pe = 0.71$, and $t = 0.5$.

Above Fig. 11. It is found that, the temperature θ decreases with increasing ω.

Table -1: The variation of velocity u with y for different values of Pe with Re=1, Gr=1, M=1, λ=1, ω=1, t = 0.5, N=1 and $\lambda_1 = 0.3$.

	Pe=0	Pe=0.5	Pe=1	Pe=2
0	0	0	0	0
0.1	0.0675	0.0679	0.0682	0.0683
0.2	0.1227	0.1236	0.1241	0.1243
0.3	0.1648	0.1660	0.1667	0.1670
0.4	0.1928	0.1942	0.1951	0.1954
0.5	0.2056	0.2070	0.2080	0.2084
0.6	0.2020	0.2035	0.2044	0.2049
0.7	0.1810	0.1822	0.1831	0.1835
0.8	0.1412	0.1421	0.1427	0.1431
0.9	0.0813	0.0818	0.0821	0.0824
1	0	0	0	0

From Above Table-1, it is found that the axial velocity *u* increases with increasing *Pe*.

Table – 2: Effect of N on Nusselt number Nu for ω = 1, *Pe* = 0.71 and *t* = 0.5.

N	*Nu*
0	0.9254
1	1.1029
2	2.0648
2.5	3.9649

Above Table-2 shows the effect of TV on Nusselt number *Nu* for ω= 1, *Pe* = 0.71 and *t* = 0.5. It is found that, the *Nu* increases with an increase in *N*.

Table – 3: Effect of *Pe* on Nusselt number *Nu* for ω = 1, *N=1* and *t = 0.5*

Pe	*Nu*
0	1.0429
0.5	1.0878
1	1.1199
2	1.1450

Above Table-3 depicts the effect of *Pe* on Nusselt number *Nu* for ω = 1, *N* = 1 and *t* = 0.5. It is observed that, the *Nu* increases with increasing Pe .

Table – 4: Effect of ω on Nusselt number *Nu* for *N=1, Pe=0.71* and *t = 0.5*

Pe	*Nu*
0	1.1884
0.5	1.1668
1	1.1029
2	0.8600

Above Table-4 illustrates the effect of ω on Nusselt number *Nu* for *N = 1* , *Pe* = 0.71 and *t* = *0*.5. It is noted that, the *Nu* decreases with increasing ω.

Table – 5: Effect of *t* on Nusselt number *Nu* for ω = *1, Pe=0.71* and *N = 1*

t	*Nu*
0	1.1748
0.5	1.1029
1	0.7609
2	-0.3526

Above Table-5 shows the effect of *t* on Nusselt number *Nu* for ω= 1, Pe = 0.71 and *N* = 1. It is observed that, the *Nu* decreases with increasing *t.*

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