



**FORMATION OF CERTAIN SUMMATION FORMULAE
USING HYPERGEOMETRIC FUNCTION**

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ABSTRACT

The main objective of the present paper is to establish certain new results in the aspects of Hypergeometric function.

Key words: Gaussian Hypergeometric function, Recurrence relation, Bailey summation theorem.

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I. INTRODUCTION

Generalized Gaussian Hypergeometric function of one variable is defined by:

$${}_A F_B(a_1, a_2, \dots, a_A; b_1, b_2, \dots, b_B; Z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \dots (a_A)_k}{(b_1)_k (b_2)_k \dots (b_B)_k} \frac{Z^k}{k!} \tag{1}$$

or

$${}_A F_B((a_A); (b_B); Z) \equiv {}_A F_B((a_j)_{j=1}^A; (b_j)_{j=1}^B; Z) = \sum_{k=0}^{\infty} \frac{((a_A))_k}{((b_B))_k} \frac{Z^k}{k!} \tag{2}$$

where the parameters b_1, b_2, \dots, b_B are neither zero nor negative integers and A, B are non negative integers.

Contiguous Relation is defined by:

[Andrews p.363 (9.16), E.D. p.51 (10), H.T.F.I. p.103 (32)]

$$(a-b) {}_2F_1(a, b; c; z) = a {}_2F_1(a+1, b; c; z) - b {}_2F_1(a, b+1; c; z) \tag{3}$$

Recurrence relation is defined by

$$\Gamma(\xi+1) = \xi \Gamma(\xi) \tag{4}$$

Bailey summation theorem[Prud, p.491(7.3,7.8)

$${}_2F_1(a, 1-a; c; \frac{1}{2}) = \frac{\Gamma(\frac{c}{2})\Gamma(\frac{c+1}{2})}{\Gamma(\frac{c+a}{2})\Gamma(\frac{c+1-a}{2})} = \frac{\sqrt{\pi}\Gamma(c)}{2^{c-1}\Gamma(\frac{c+a}{2})\Gamma(\frac{c+1-a}{2})} \tag{5}$$

II. MAIN SUMMATION FORMULAE

$$\begin{aligned} {}_2F_1(a, -a-27; c; \frac{1}{2}) &= \frac{\sqrt{\pi}\Gamma(c)}{2^{c+27}} \left[\frac{4(32382376266240000 - 50190021585849600a + 32382376266240000)}{\Gamma(\frac{c+a+27}{2})\Gamma(\frac{c-a}{2})} \right. \\ &+ \frac{4(-50190021585849600a + 19980916647138720a^2 - 2061868476959928a^3 - 128011784992364a^4)}{\Gamma(\frac{c+a+27}{2})\Gamma(\frac{c-a}{2})} \\ &+ \frac{4(15900567072390a^5 + 779078476085a^6 - 25186957614a^7 - 1913811627a^8 - 21793590a^9 + 568295a^{10})}{\Gamma(\frac{c+a+27}{2})\Gamma(\frac{c-a}{2})} \\ &+ \frac{4(14742a^{11} + 91a^{12} + 73325125620633600c - 75678852771682560ac + 20312273366353440a^2c)}{\Gamma(\frac{c+a+27}{2})\Gamma(\frac{c-a}{2})} \\ &+ \left. \frac{4(-972831670483608a^3c - 149077115547292a^4c + 4808539043550a^5c + 545414759177a^6c)}{\Gamma(\frac{c+a+27}{2})\Gamma(\frac{c-a}{2})} \right] \end{aligned}$$

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$$\begin{aligned}
 & \frac{4(3128128794a^7c - 514268055a^8c - 11897550a^9c - 31997a^{10}c + 1134a^{11}c + 7a^{12}c + 63353116248514560c^2)}{\Gamma\left(\frac{c+a+27}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{4(-46178257152089088ac^2 + 8266280659503360a^2c^2 - 43821110885760a^3c^2 - 52542802852800a^4c^2)}{\Gamma\left(\frac{c+a+27}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{4(-272386164960a^5c^2 + 107819248992a^6c^2 + 2497884480a^7c^2 - 24635520a^8c^2 - 1179360a^9c^2 - 8736a^{10}c^2)}{\Gamma\left(\frac{c+a+27}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{4(29483261324894208c^3 - 15463134566931456ac^3 + 1767382071708928a^2c^3 + 53898399561600a^3c^3)}{\Gamma\left(\frac{c+a+27}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{4(-8322079071680a^4c^3 - 237082114080a^5c^3 + 7651048608a^6c^3 + 309355200a^7c^3 + 1639680a^8c^3)}{\Gamma\left(\frac{c+a+27}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{4(-30240a^9c^3 - 224a^{10}c^3 + 8514589137793024c^4 - 3210969790978560ac^4 + 213380851382400a^2c^4)}{\Gamma\left(\frac{c+a+27}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{4(14232851418240a^3c^4 - 628120365600a^4c^4 - 32408812800a^5c^4 + 45689280a^6c^4 + 14152320a^7c^4)}{\Gamma\left(\frac{c+a+27}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{4(131040a^8c^4 + 1642556290625536c^5 - 439204166576640ac^5 + 13600766617728a^2c^5 + 1695027503232a^3c^5)}{\Gamma\left(\frac{c+a+27}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{4(-15445364256a^4c^5 - 1970438400a^5c^5 - 17467968a^6c^5 + 217728a^7c^5 + 2016a^8c^5 + 220549142528000c^6)}{\Gamma\left(\frac{c+a+27}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{4(-40722345990144ac^6 + 201591656448a^2c^6 + 112897774080a^3c^6 + 816990720a^4c^6 - 56609280a^5c^6)}{\Gamma\left(\frac{c+a+27}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} + \\
 & + \frac{4(-698880a^6c^6 + 21059726065664c^7 - 2575977043968ac^7 - 34985170944a^2c^7 + 4324492800a^3c^7)}{\Gamma\left(\frac{c+a+27}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{4(66086400a^4c^7 - 622080a^5c^7 - 7680a^6c^7 + 1439841841152c^8 - 109506608640ac^8 - 2854874880a^2c^8)}{\Gamma\left(\frac{c+a+27}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{4(88957440a^3c^8 + 1647360a^4c^8 + 69983367168c^9 - 2991859200ac^9 - 100545280a^2c^9 + 760320a^3c^9)}{\Gamma\left(\frac{c+a+27}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{4(14080a^4c^9 + 2360483840c^{10} - 47443968ac^{10} - 1757184a^2c^{10} + 52502528c^{11} - 331776ac^{11})}{\Gamma\left(\frac{c+a+27}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{4(-12288a^2c^{11} + 692224c^{12} + 4096c^{13})}{\Gamma\left(\frac{c+a+27}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} + \frac{(-417888285765350400a + 350152564982094720a^2)}{\Gamma\left(\frac{c+a+27}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(-417888285765350400a + 350152564982094720a^2 - 74313803935308672a^3 + 337588509650280a^4)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} + \\
 & + \frac{(758681199716076a^5 + 5482726057634a^6 - 2834464989951a^7 - 94914320765a^8 + 1465660413a^9)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} + \\
 & + \frac{(120338547a^{10} + 1853523a^{11} + 385a^{12} - 189a^{13} - a^{14} + 417888291992371200c - 1014622876799400960ac)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(485092884314473344a^2c - 57141989747230752a^3c - 3102171304352848a^4c + 447337013596200a^5c)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(20808510233420a^6c - 728363478024a^7c - 53355081780a^8c - 599213160a^9c + 15993796a^{10}c)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(412776a^{11}c + 2548a^{12}c + 664470331620065280c^2 - 856997920721843712ac^2 + 260077723907408064a^2c^2)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(-14319474881043024a^3c^2 - 1971084604702920a^4c^2 + 70574418546660a^5c^2 + 7528122438718a^6c^2)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(39469942764a^7c^2 - 7222642434a^8c^2 - 166142340a^9c^2 - 444822a^{10}c^2 + 15876a^{11}c^2 + 98a^{12}c^2)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(446218866939396096c^3 - 373677414087204864ac^3 + 73183109349855232a^2c^3 - 667335257971200a^3c^3)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(-478621838589440a^4c^3 - 1941004840320a^5c^3 + 1005414463872a^6c^3 + 23049411840a^7c^3 - 232377600a^8c^3)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(-11007360a^9c^3 - 81536a^{10}c^3 + 17040411731130368c^4 - 98285771101077504ac^4 + 12050410462809600a^2c^4)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(338231928908160a^3c^4 - 57886618012160a^4c^4 - 1613669027040a^5c^4 + 53964048096a^6c^4 + 2160406080a^7c^4)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(11430720a^8c^4 - 211680a^9c^4 - 1568a^{10}c^4 + 41765636694605824c^5 - 16861155187968000ac^5)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(1188769000054272a^2c^5 + 76559103548928a^3c^5 - 3540048430464a^4c^5 - 179904291840a^5c^5)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(275428608a^6c^5 + 79252992a^7c^5 + 733824a^8c^5 + 6996348184297472c^6 - 1966290770820096ac^6)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{(64585542617344a^2c^6 + 7769070215424a^3c^6 - 74233325376a^4c^6 - 9175057920a^5c^6 - 81266304a^6c^6)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(1016064a^7c^6 + 9408a^8c^6 + 829364007600128c^7 - 158819735273472ac^7 + 912259805184a^2c^7)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(448270018560a^3c^7 + 3206461440a^4c^7 - 226437120a^5c^7 - 2795520a^6c^7 + 70798002356224c^8)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(-8890440394752ac^8 - 118231819776a^2c^8 + 15103791360a^3c^8 + 230711040a^4c^8 - 2177280a^5c^8)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(-26880a^6c^8 + 4369849516032c^9 - 338473175040ac^9 - 8799831040a^2c^9 + 276756480a^3c^9 + 5125120a^4c^9)}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(193262125056c^{10} - 8360174592ac^{10} - 280896000a^2c^{10} + 2128896a^3c^{10} + 39424a^4c^{10} + 5969739776c^{11})}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{(-120766464ac^{11} - 4472832a^2c^{11} + 122257408c^{12} - 774144ac^{12} - 28672a^2c^{12} + 1490944c^{13} + 8192c^{14})}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \quad (6)
 \end{aligned}$$

$$\begin{aligned}
 {}_2F_1(a, -a-28; \frac{1}{2}) &= \frac{\sqrt{\pi}\Gamma(c)}{2c+28} \left[\frac{-835776577757721600a + 732560266788865920a^2}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \right. \\
 & + \frac{-175421799959309568a^3 + 6354743845373448a^4 + 1435874126626940a^5 - 34847676651046a^6}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-5714447085179a^7 - 56439882581a^8 + 6367545345a^9 + 190140027a^{10} + 798847a^{11}}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-31367a^{12} - 385a^{13} - a^{14} + 835776583984742400c - 2061500910226237440ac + 1045656760631258880a^2c}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-149298043145421312a^3c - 2114036683694256a^4c + 1051093994098664a^5c + 16863114492748a^6c}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-2335033062658a^7c - 76476289260a^8c + 447156990a^9c + 45742452a^{10}c + 544810a^{11}c + 476a^{12}c}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-14a^{13}c + 1328940663240130560c^2 - 1762672667954024448ac^2 + 580003878004867584a^2c^2}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-45958652827578240a^3c^2 - 2978527723457984a^4c^2 + 240236849574240a^5c^2 + 11493621526928a^6c^2}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-204033147360a^7c^2 - 15364896624a^8c^2 - 157016160a^9c^2 + 1995056a^{10}c^2 + 36960a^{11}c^2 + 112a^{12}c^2}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{892437733878792192c^3 - 777868781853327360ac^3 + 170377061643280384a^2c^3 - 5742422644760576a^3c^3}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-901847768741120a^4c^3 + 17602227024640a^5c^3 + 2042288291904a^6c^3 + 14785735104a^7c^3}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-1006481280a^8c^3 - 17915520a^9c^3 - 39872a^{10}c^3 + 448a^{11}c^3 + 340808223462260736c^4}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-207271509690142720ac^4 + 29801589430232576a^2c^4 + 32678415219840a^3c^4 - 128945425048320a^4c^4}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-1007496016800a^5c^4 + 155517640992a^6c^4 + 3001494720a^7c^4 - 14683200a^8c^4 - 554400a^9c^4}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-2016a^{10}c^4 + 83531273389211648c^5 - 36089578411937792ac^5 + 3243021818591232a^2c^5}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{97872099217152a^3c^5 - 9857984683008a^4c^5 - 248044966848a^5c^5 + 4816939008a^6c^5 + 153379968a^7c^5}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{580608a^8c^5 - 4032a^9c^5 + 13992696368594944c^6 - 4283965206308864ac^6 + 216554424231424a^2c^6}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{12969805094400a^3c^6 - 383804281728a^4c^6 - 16475182080a^5c^6 - 2983680a^6c^6 + 2956800a^7c^6}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{13440a^8c^6 + 1658728015200256c^7 - 353781343682560ac^7 + 7779215966208a^2c^7 + 863035250688a^3c^7}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-4266777600a^4c^7 - 507909120a^5c^7 - 3056640a^6c^7 + 15360a^7c^7 + 141596004712448c^8}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-20391369123840ac^8 + 35696871936a^2c^8 + 32912605440a^3c^8 + 198654720a^4c^8 - 6969600a^5c^8 - 42240a^6c^8}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{8739699032064c^9 - 808981426176ac^9 - 9512504320a^2c^9 + 705943040a^3c^9 + 7152640a^4c^9 - 28160a^5c^9}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{386524250112c^{10} - 21299007488ac^{10} - 416294912a^2c^{10} + 7434240a^3c^{10} + 67584a^4c^{10} + 11939479552c^{11}}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-345210880ac^{11} - 7593984a^2c^{11} + 24576a^3c^{11} + 244514816c^{12} - 2928640ac^{12} - 53248a^2c^{12} + 2981888c^{13}}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-8192ac^{13} + 16384c^{14}}{\Gamma\left(\frac{c+a+28}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} + \frac{3497296636753920000 - 5425964184667622400a + 2160400288275861120a^2}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-217398644527744512a^3 - 16312160038933416a^4 + 1875455701951748a^5 + 108750300435014a^6}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-3300007197221a^7 - 309431991713a^8 - 4315391241a^9 + 122905167a^{10} + 4214833a^{11} + 37429a^{12}}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-7a^{13} - a^{14} + 8048643072093388800c - 8380471399943178240ac + 2264802907727160576a^2c}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-10202944682330304a^3c - 18981724956747216a^4c + 568559997177496a^5c + 79725356491220a^6c}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{638143186690a^7c - 93789891636a^8c - 2679957630a^9c - 9533076a^{10}c + 471254a^{11}c + 5572a^{12}c + 14a^{13}c}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{7135437057322106880c^2 - 5287724776998254592ac^2 + 959448693198163968a^2c^2 - 1179557366997120a^3c^2}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-7013817637769024a^4c^2 - 57049475593440a^5c^2 + 17237922585872a^6c^2 + 479168903136a^7c^2}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-5206796784a^8c^2 - 337186080a^9c^2 - 3593296a^{10}c^2 + 672a^{11}c^2 + 112a^{12}c^2 + 3437604688082632704c^3}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-1849243134517395456ac^3 + 215633610585030656a^2c^3 + 7980759161409536a^3c^3 - 1193608956693760a^4c^3}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-40377636885760a^5c^3 + 1376931162432a^6c^3 + 69506934336a^7c^3 + 478853760a^8c^3 - 12566400a^9c^3}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-177856a^{10}c^3 - 448a^{11}c^3 + 1037508672181223424c^4 - 405600833229395968ac^4 + 27620912747150336a^2c^4}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{2157478012786560a^3c^4 - 98668285127040a^4c^4 - 6139374914400a^5c^4 + 6974164512a^6c^4 + 4046226240a^7c^4}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{53901120a^8c^4 - 10080a^9c^4 - 2016a^{10}c^5 + 211454435938729984c^5 - 59390641702862848ac^5}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{1870431843068928a^2c^5 + 280667136592128a^3c^5 - 2604772568832a^4c^5 - 441943774272a^5c^5}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-5065148928a^6c^5 + 90475392a^7c^5 + 1596672a^8c^5 + 4032a^9c^5 + 3038953255526144c^6}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-5993521873135616ac^6 + 23503394094592a^2c^6 + 21243279235584a^3c^6 + 194132921472a^4c^6}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-16184878080a^5c^6 - 287481600a^6c^6 + 53760a^7c^6 + 13440a^8c^6 + 3156646985203712c^7}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-421599979962368ac^7 - 7023564939264a^2c^7 + 968666867712a^3c^7 + 19093017600a^4c^7 - 258493440a^5c^7}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-6067200a^6c^7 - 15360a^7c^7 + 239741823107072c^8 - 20518843766784ac^8 - 653969631744a^2c^8}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{25433337600a^3c^8 + 677656320a^4c^8 - 126720a^5c^8 - 42240a^6c^8 + 13317571018752c^9 - 669494267904ac^9}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-28984017920a^2c^9 + 315927040a^3c^9 + 11095040a^4c^9 + 28160a^5c^9 + 534865723392c^{10} - 13564424192ac^{10}}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-722855936a^2c^{10} + 135168a^3c^{10} + 67584a^4c^{10} + 15112208384c^{11} - 137854976ac^{11} - 9658368a^2c^{11}}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-24576a^3c^{11} + 284770304c^{12} - 53248ac^{12} - 53248a^2c^{12} + 3211264c^{13} + 8192ac^{13} + 16384c^{14}}{\Gamma\left(\frac{c+a+29}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}] \tag{7}
 \end{aligned}$$

III Derivations of formula (5):

Putting $b = -a-27$, $z = \frac{1}{2}$ in known formula (2), we get

$$(2a+27) {}_2F_1\left(a, -a-27; c; \frac{1}{2}\right) = a {}_2F_1\left(a+1, -a-27; c; \frac{1}{2}\right) + (a+27) {}_2F_1\left(a, -a-26; c; \frac{1}{2}\right)$$

Now applying the same method of Ref [3], we get both the results.

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