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RUBAN'S STRING COSMOLOGICAL MODEL IN GENERAL THEORY OF RELATIVITY

V. G. METE*, V. D. ELKAR[1], A. S. NIMKAR[2]

Department of Mathematics,
R. D. I. K. & K. D. College, Badnera-Amravati, India.

[1]Department of Mathematics,
J. D. Patil Sangludkar Mahavidyalaya, Daryapur, Dist. Amravati, India.

[2]Department of Mathematics,
Shri. Dr. R. G. Rathod Arts and Science College, Murtizapur Dist. Akola, India.

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ABSTRACT

In this paper we have taken an attempt to construct a Ruban's cosmological model in presence of cosmic string source in the framework of general theory of relativity. Exact solution of Einstein's field equations are obtained by using Nambu string and relation between metric coefficients. Also some physical and kinematical properties of the model are discussed.

Keywords: *Ruban's space time, cosmic string, variable* Λ.

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1. INTRODUCTION

The string theory is a developing branch of theoretical physics that combines quantum mechanics and general relativity into a quantum theory of gravity. The strings of string theory are one dimensional oscillating lines. These strings arise during the phase transition after the big bang explosion as the temperature drops down below some critical temperature as predicted by grand unified theories [1-5].

The concept of string theory was developed to describe events at the early stages of the evaluation of the universe. Vilenkin [6] believed that string may be one of the source of density perturbations that are required for the formation of large scale structure in the universe. Relativistic string models in the context of Bianchi space time have been obtained by Krori *et al.* [7], Sen *et al.* [8], Barros *et al.* [9]. Reddy [10] have studied several aspects of cosmic string in Brans-Dicke [11] scalar tensor theories of gravitations.

In recent years Reddy [12-13] have presented string cosmological models in alternative theories of gravitation, Raj Bali and Singh [14] have investigated string cosmological models in general theory of relativity, Adhav *et al.* [15-18] have studied several aspects of cosmic string in scalar tensor theories of gravitation and Barber's self creation cosmology.

Very recently Kandalkar *et al.*[19] have constructed Bianchi type-V string dust cosmological model with bulk viscous magnetic field, Raj Bali *et al.*[20] have studied LRS Bianchi type-II massive string cosmological models with magnetic field, D.R.K.Reddy *et al.*[21] have investigated anisotropic bulk viscous string cosmological model in scalar tensor theory of gravitation, Binaya K. Bishi and K.L.Mohanta [22] have investigated Bianchi type-V bulk viscous cosmic string in $f(R,T)$ gravity with time varying deceleration parameter. Reddy *et al.* [23-27], Naidu *et al.* [28-29], Kiran and Reddy [30], Hendi and Momeni [31] R. Venkateswarlu and J.Satish [32], V.U.M.Rao and Neelima [33] are some of the authors who have investigated several aspects of string theory. Lima and Nobre [34-36] studied the spatially inhomogeneous solutions of the Einstein's-Maxwell equations in the frame work of Ruban's metric.

In this paper we have obtained Ruban's cosmological model with variable Λ in presence of cosmic string. The physical behavior of the model is also discussed.

Corresponding Author: V. G. Mete*

2. THE METRIC AND FIELD EQUATIONS

We consider the space- time of Ruban's in the form

$$ds^2 = dt^2 - Q^2(x,t)dx^2 - R^2(t)(dy^2 + h^2 dz^2), \tag{1}$$

where

$$h(y) = \frac{\sin\sqrt{k}\,y}{\sqrt{k}} = \begin{array}{lll} \sin y & if & k=1 \\ y & if & k=0 \\ \sinh y & if & k=-1 \end{array}$$

and k is the curvature parameter of the homogeneous 2-spaces t and x constants.

The functions Q and R are free and will be determined.

The energy-momentum tensor for cloud of string given by

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \quad , \tag{2}$$

where ρ is the rest energy density of cloud of string with particle attached to them and λ is the tension density of the system of the string, u^i is describe the cloud four velocity and x^i represents the direction of anisotropy.

We have

$$u_i u^j = -x_i x^j = 1 \text{ and } u^i x_j = 0 \tag{3}$$

we consider

$$\rho = \rho_p + \lambda, \tag{4}$$

where ρ_p is the rest energy density of particles and x_i to be along *x*-axis.

The Einstein's field equations (EFE) for cosmological term Λ with $8\pi G = C = 1$ can be written as

$$R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = -T_{ij} \tag{5}$$

The field equations (5) for the metric (1) with matter distribution (2) yield

$$\left(\frac{\dot{R}}{R}\right)^2 + 2\frac{\ddot{R}}{R} + \frac{k}{R^2} - \Lambda = \lambda \tag{6}$$

$$\frac{\dot{R}\dot{Q}}{RQ} + \frac{\ddot{R}}{R} + \frac{\ddot{Q}}{Q} = \Lambda \tag{7}$$

$$2\frac{\dot{R}\dot{Q}}{RQ} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} - \Lambda = \rho \ , \tag{8}$$

where the over head dot(.) at the symbol Q and R denotes ordinary differentiation with respectively to time t.

The spatial volume for the model (1) is given by

$$V = hQR^2 \tag{9}$$

The scalar expansion θ, and shear scalar σ are define by

$$\theta = \frac{\dot{Q}}{Q} + 2\frac{\dot{R}}{R} \tag{10}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma ij \tag{11}$$

3. SOLUTION OF FIELD EQUATION

The field equations (6)-(8) are system of three equations with five unknowns parameters R, Q, λ, ρ and Λ. For complete determinacy of the system, two extra conditions are needed.

One way is to use an equation of state and the other alternative is to assume relation between metric coefficient and then to discuss the physical nature of the universe. In this paper we confine ourselves to the Nambu string

$$\rho = \lambda \text{ and } Q = (xR)^n \tag{12}$$

after solving the equations (6)-(8) with the help of (12) we get

$$\frac{\ddot{R}}{R} - n\frac{\dot{R}^2}{R^2} = 0 \tag{13}$$

Solving equation (13), we obtain

$$R = M(c_1 t + c_2)^{\frac{1}{1-n}} \text{ and } Q = Nx^n (c_1 t + c_2)^{\frac{n}{1-n}} , \tag{14}$$

where

$$M = (1-n)^{\frac{1}{1-n}} \text{ and } N = (1-n)^{\frac{n}{1-n}}$$

Using equation (14), the line element (1) becomes

$$ds^2 = dt^2 - N^2 x^{2n} (c_1 t + c_2)^{\frac{2n}{1-n}} dx^2 - M^2 (c_1 t + c_2)^{\frac{2}{1-n}} (dy^2 + h^2 dz^2) \tag{15}$$

This metric can be transformed through a proper choice of coordinates to the form

$$ds^2 = dT^2 - N^2 x^{2n} T^{\frac{2n}{1-n}} dx^2 - M^2 T^{\frac{2}{1-n}} (dy^2 + h^2 dz^2) , \tag{16}$$

where $T = c_1 t + c_2$

4. GEOMETRIC AND PHYSICAL SIGNIFICANCE OF THE MODEL

The energy density ρ, the string tension density λ, cosmological term Λ, for the model (16) are given by

$$\rho = \lambda = \frac{c_1^2 (1 + n - 2n^2)}{(1-n)^2 T^2} + \frac{k}{M^2} \frac{1}{T^{\frac{2}{1-n}}} \tag{17}$$

$$\Lambda = \frac{c_1^2 (n + 2n^2)}{(1-n)^2 T^2} \tag{18}$$

The model (16) contains the term *h*, it is related to the curvature parameter *k*.

For the value $k = -1, 0, 1$ equation (17) becomes

$$\rho = \lambda = \frac{c_1^2 (1 + n - 2n^2)}{(1-n)^2 T^2} - \frac{1}{M^2} \frac{1}{T^{\frac{2}{1-n}}} \tag{19}$$

$$\rho = \lambda = \frac{c_1^2 (1 + n - 2n^2)}{(1-n)^2 T^2} \tag{20}$$

$$\rho = \lambda = \frac{c_1^2 (1 + n - 2n^2)}{(1-n)^2 T^2} + \frac{1}{M^2} \frac{1}{T^{\frac{2}{1-n}}} \tag{21}$$

Graphically, the energy density, string tension density and time dependent Λ for the different values of parameter *k* are shows in figures:

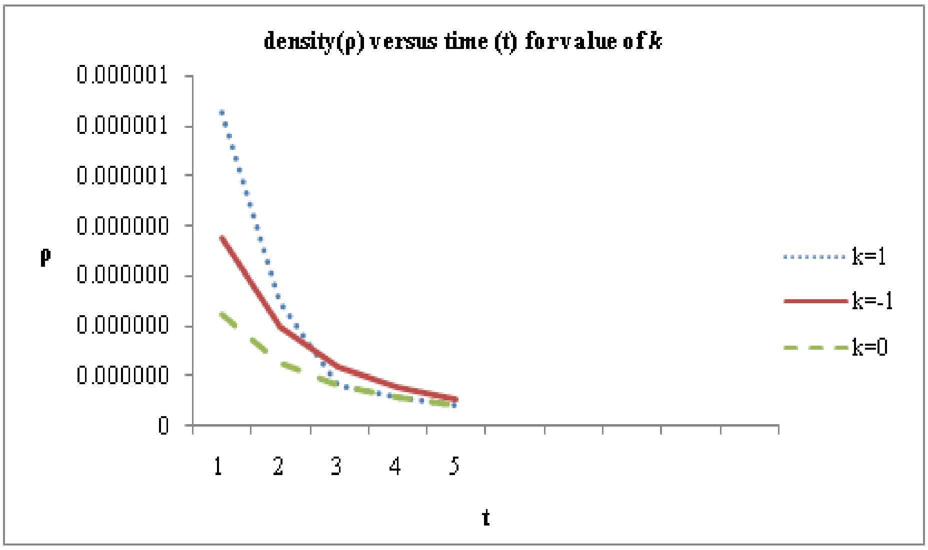

Fig.-1: density ρ versus time t for the value of k

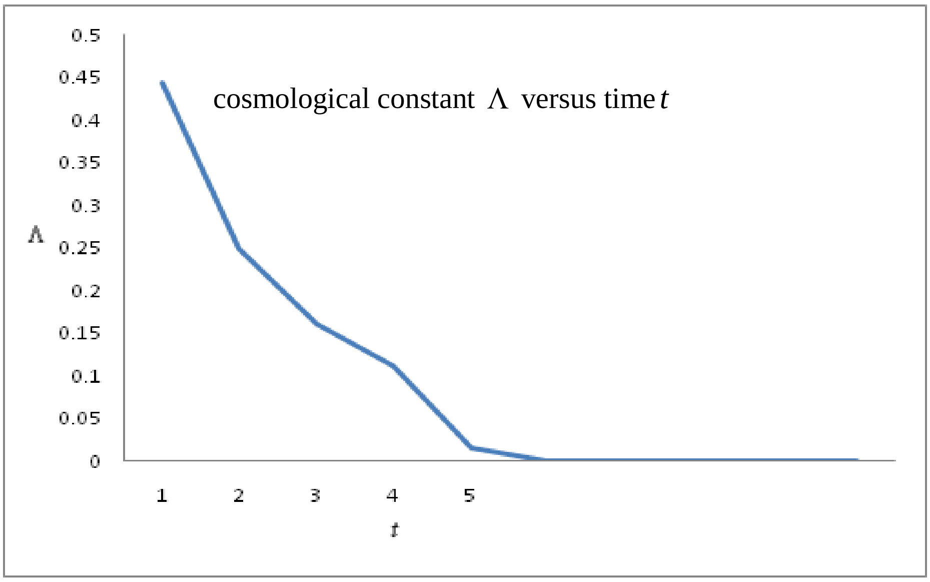

Fig.-2: cosmological constant Λ versus time t

From equations (9-12) the spatial volume, expansion scalar, shear scalar and deceleration parameter are given by

Spatial volume $$V = hx^n M^2 N T^{\frac{n+2}{1-n}} \tag{22}$$

Scalar Expansion $$\theta = \frac{3c_1}{(1-n)T} \tag{23}$$

Shear Scalar $$\sigma^2 = \frac{c_1^2}{3T^2} \tag{24}$$

Deceleration Parameter $$q = -\frac{(2n+1)}{(n+2)} \tag{25}$$

The model (16) has no initial singularity while the energy density ρ, string tension density λ given by (20) possess initial singularities. However, as T increases these singularities vanish. The spatial volume of the model given by (22) shows the anisotropic expansion of the universe with time.

For the model (16), the expansion scalar θ and shear scalar σ tends to zero as $T \to \infty$ $\lim\limits_{T\to\infty} \frac{\sigma}{\theta} = \frac{(1-n)}{3\sqrt{3}} \neq 0$ and hence the model does not approach isotropy the similar results are obtained for $k=1$ and $k=-1$.

CONCLUSION

In this paper, we have obtained Ruban's string cosmological model in the frame work of Einstein's general theory of relativity. It is observed that the model is expanding, shearing non rotating and has no initial singularity for $k = -1, 0, +1$.

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