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THE OPEN EDGE NEIGHBORHOOD GRAPH AND THE COMMON EDGE NEIGHBORHOOD GRAPH OF A GRAPH

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ABSTRACT

Let $G = (V, E)$ be a graph. For any edge $e \in E$, the open edge neighborhood set $N(e)$ of e is the set of edges adjacent to e. The open edge neighborhood graph $N_{oe}(G)$ of a graph G is defined to be the intersection graph on the family of all open edge neighborhood sets of edges in G. The common edge neighborhood graph $N_{ce}(G)$ of G is the graph having the vertex set E and with the two vertices in $N_{ce}(G)$ are adjacent if there exists an open edge neighborhood set in G containing them. In this paper, we initiate a study of these new graphs.

Keywords: *open edge neighborhood set, open edge neighborhood graph, common edge neighborhood graph.*

Mathematics Subject Classification: *05C.*

1. INTRODUCTION

The graphs considered here are finite, undirected without loops or multiple edges. Throughout this paper, the graphs G have order p and size q. Any undefined term in this paper may be found in Kulli [1].

Let $G = (V, E)$ be a graph. For any edge $e \in E$, the open edge neighborhood $N(e)$ of e is the set of edges adjacent to e. We call $N(e)$ is the open edge neighborhood set of an edge e of G. Let $E = \{e_1, e_2, \ldots, e_q\}$. Let $S = \{N(e_1), N(e_2), \ldots, N(e_q)\}$ be the set of all open edge neighborhood sets of G.

Let X be a finite set and let $F = \{X_1, X_2, \ldots X_n\}$ be a partition of X. Then the intersection graph $\Omega(F)$ of F is the graph whose vertices are the subsets in F and in which two vertices X_i and X_j are adjacent if and only if $X_i \cap X_j \neq \phi$.

The minimal dominating graph $MD(G)$ of G is the intersection graph defined on the family of all minimal dominating sets of vertices in G. This concept was introduced by Kulli and Janakiram in [2]. Recently many new graph valued functions in graph theory were studied, for example, in [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17] and also graph valued functions in domination theory were studied, for example, in [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32].

The degree of an edge uv is defined to be deg u+deg v–2. An edge $e = uv$ is called an isolated edge if deg e=deg uv = 0. Let $\Delta_1(G)$ denote the maximum degree among the edges of G. We consider only graphs without isolated vertices and without isolated edges.

2. OPEN EDGE NEIGHBORHOOD GRAPHS

The purpose of this paper is to introduce a new class of intersection graphs in graph theory.

Definition 1: Let G be a graph without isolated vertices and without isolated edges. The open edge neighborhood graph $N_{oe}(G)$ of a graph G is the intersection graph defined on the family of all open edge neighborhood sets of edges in G.

Example 2: In Figure 1, a graph G and its open edge neighborhood graph $N_{oe}(G)$ are shown. For the graph G in Figure 1, $N(e_1)= \{e_2, e_3\}$, $N(e_2) = \{e_1, e_3\}$, $N(e_3) = \{e_1, e_2, e_4\}$, $N(e_4) = \{e_3\}$.

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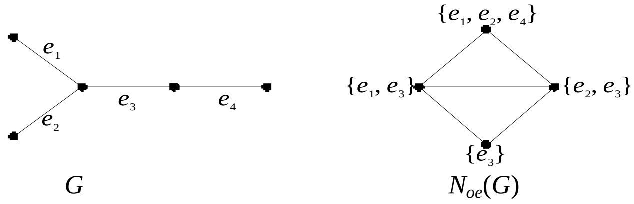


Figure-1

Theorem 3: If P_p is a path with $p \geq 2$ vertices, then

$$N_{oe}(P_p) = P_{\frac{p}{2}-1} \cup P_{\frac{p}{2}}, \quad \text{if } p \text{ is even,}$$
$$= 2P_{\left\lfloor \frac{p}{2} \right\rfloor}, \quad \text{if } p \text{ is odd.}$$

Proof: Let $E(P_p) = \{e_1, e_2, \ldots, e_{p-1}\}$, $p \geq 2$. Let $N(e_i) = X_i$, $1 \leq i \leq p-1$.

Then $V(N_{oe}(P_p) = \{x_1, x_2, \ldots, x_{p-1}\}$.

Consider $X_1 = N(e_1) = \{e_2\}$
$X_2 = N(e_2) = \{e_1, e_3\}$
. . .
. . .
. . .
$X_{p-2} = N(e_{p-2}\} = \{e_{p-3}, e_{p-1}\}$
$X_{p-1} = N(e_{p-1}\} = \{e_{p-2}\}$.

Clearly X_1 and X_{p-1} are endvertices in $N_{oe}(P_p)$. We have $X_1 \cap X_3 \neq \phi$, $X_3 \cap X_5 \neq \phi$, and so on, and $X_2 \cap X_4 \neq \phi$, $X_4 \cap X_6 \neq \phi$, … and so on, $X_{p-3} \cap X_{p-1} \neq \phi$,

We consider the following two cases:

Case-1: Suppose p is even. Then $p-1$ is odd. Then the adjacencies of the vertices in $N_{oe}(P_p)$ is as follows: $X_1 X_3 X_5 \ldots X_{p-1}$ which is a path with $\frac{p}{2}$ vertices and $X_2\ X_4\ X_6 \ldots X_{p-2}$ which is a path with $\frac{p}{2}-1$ vertices. These two paths are disjoint. Hence

$$N_{oe}\left(P_p\right) = P_{\frac{p}{2}-1} \cup P_{\frac{p}{2}}, \quad \text{if } p \text{ is even.}$$

Case-2: Suppose p is odd. Then $p-2$ is odd. Then the adjacencies of the vertices in $N_{oe}(P_p)$ is as follows:
$X_1\ X_3\ X_5 \ldots X_{p-4}\ X_{p-2}$ which is a path with $\left\lfloor \frac{p}{2} \right\rfloor$ vertices and $X_2\ X_4\ X_6 \ldots X_{p-1}$ which is a path with $\left\lfloor \frac{p}{2} \right\rfloor$ vertices. These two paths are disjoint. Hence

$$N_{oe}\left(P_p\right) = 2P_{\left\lfloor \frac{p}{2} \right\rfloor}, \quad \text{if } p \text{ is odd.}$$

Theorem 4: If C_p is a cycle with $p \geq 3$ vertices, then

$$N_{oe}(C_p) = C_p, \text{ if } p \text{ is odd.}$$
$$= 2C_{\frac{p}{2}}, \text{ if } p \text{ is even and } p \geq 6,$$
$$= 2P_2, \text{ if } p = 4.$$

Theorem 5: For any star $K_{1,p}$ with $p \geq 3$ vertices, $N_{oe}(K_{1.p}) = K_p$.

Proof: Let $E(K_{1,p}) = \{e_1, e_2, \ldots, e_p\}$. Then $N(e_i) = E(K_{1,p}) - \{e_i\}$, for $1 \leq i \leq p$. Thus $N(e_i) \cap N(e_j) \neq \phi$, $i \neq j$. Hence every pair of corresponding vertices of $N(e_1)$, $N(e_2)$, …, $N(e_p)$ are adjacent in $N_{oe}(K_{1,p})$. Since $N_{oe}(K_{1,p})$ has p vertices, $N_{oe}(K_{1,p}) = K_p$.

The double star $S_{m,n}$ is the graph obtained from joining the centres of two stars $K_{1,m}$ and $K_{1,n}$ with an edge. The centres of $K_{1,m}$ and $K_{1,n}$ are called central vertices of $S_{m,n}$. Thus $S_{m,n}$ has $m+n+2$ vertices and $m+n+1$ edges.

Theorem 6: If $S_{m,n}$, $2 \le m \le n$ is a double star, then $N_{oe}(S_{m,n}) = K_{m+n+1}$.

Proof: Let $S_{m,n}$ be a double star, $2 \le m \le n$. By the definition, $E(S_{m,n}) = E(K_{1,m}) \cup E(K_{1,n}) \cup \{e\}$, where e is the edge joining the centres of two stars $K_{1,m}$ and $K_{1,n}$. Let $E(K_{1,m}) = \{e_1, e_2, ..., e_m\}$, $E(K_{1,n}) = \{e'_1, e'_2, ..., e'_n\}$. Then $N(e_i) = E(K_{1,m}) \cup \{e\} - e_i$, $N(e'_i) = E(K_{1,n}) \cup \{e\} - e'_i$ and $N(e) = E(K_{1,m}) \cup E(K_{1,n})$, $2 \le m \le n$. Let $S_1 = \{N(e_1), N(e_2), ..., N(e_m)\}$, $S_2 = N(e'_i) = \{N(e'_i), N(e'_i), ..., N(e'_n)\}$. Then $V(N_{oe}((S_{m,n})) = S_1 \cup S_2 \cup N(e)$ and any two open edge neighborhood sets of $S_{m,n}$ are not disjoint. Therefore every pair of vertices of $S_1 \cup S_2 \cup N(e)$ are adjacent in $N_{oe}(S_{m,n})$. Since $N_{oe}(S_{m,n})$ has $m+n+1$ vertices, it implies that $N_{oe}(S_{m,n}) = K_{m+n+1}$.

Theorem 7: If G is a graph with an endvertex v which is adjacent with a cutvertex u of degree 2, then $N_{oe}(G)$ is not complete.

Proof: Let G be a graph with an end vertex v and let v be adjacent with a cut vertex u of degree 2. Then $e_1 = vu$ and $e = uw$ are edges of G. Clearly deg $w \ge 1$. Then e is adjacent with $e_1, e_2, \ldots, e_n$, $n \ge 1$. We have $N(e_1) = \{e\}$ and $N(e) = \{e_1, e_2, ..., e_n\}$. Then $N(e_1) \cap N(e) = \phi$. Therefore the corresponding vertices of $N(e_1)$ and $N(e)$ are not adjacent in $N_{oe}(G)$. Thus $N_{oe}(G)$ is not complete.

Corollary 8: If $S_{1,n}$, $1 \le n$, is double star, then $N_{oe}(S_{1,n})$ is not complete.

Remark 9: Let G_1 be a graph as in Figure 2. Let $G_2 = K_{1,4}$. Then $N_{oe}(G_1) = N_{oe}(G_2) = K_4$. Thus $N_{oe}(G_1) = N_{oe}(G_2)$ does not imply $G_1 = G_2$.

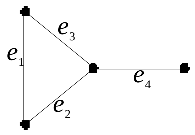


Figure-2

Problem 10: Characterize graphs whose open edge neighborhood graphs are complete.

Proposition 11: $N_{oe}(G) = 2nK_1$ if and only $G = nP_3$, $n \ge 1$.

Proposition 11 can be stated as given below.

Proposition 12: The open edge neighborhood graph $N_{oe}(G)$ is totally disconnected if and only if every component of G is P_3.

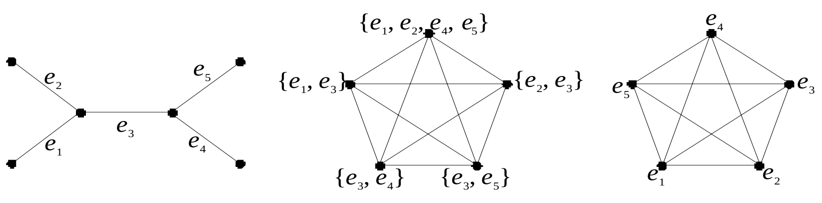
3. COMMON EDGE NEIGHBORHOOD GRAPHS

Recently the concept of the common minimal total dominating graph $CD_t(G)$ of a graph G was introduced by Kulli in [22] as the graph having the same vertex set as G with two vertices in $CD_t(G)$ adjacent if there is a minimal total dominating set in G containing them.

The definition of the common minimal total dominating graph inspired us to introduce the common edge neighborhood graph in graph theory.

Definition 13: Let $G = (V, E)$ be a graph without isolated vertices and without isolated edges. The common edge neighborhood graph $N_{ce}(G)$ of G is the graph having the vertex set E and with two vertices are adjacent in $N_{ce}(G)$ if there exists an open edge neighborhood set in G containing them.

Example 14: In Figure 3, a graph G and its open edge neighborhood graph $N_{oe}(G)$ and common edge neighborhood graph $N_{ce}(G)$ are shown. For the graph G in Figure 3, the open edge neighborhood sets are $N(e_1) = \{e_2, e_3\}$, $N(e_2) = \{e_1, e_3\}$, $N(e_3) = \{e_1, e_2, e_4, e_5\}$, $N(e_4) = \{e_3, e_5\}$, $N(e_5) = \{e_3, e_4\}$.



G $N_{oe}(G)$ $N_{ce}(G)$

Figure-3

From Figure 3, we have

Remark 15: If G is a graph as shown in Figure 3, then $N_{oe}(G) = N_{ce}(G)$.

Conjecture 16: For any graph G without isolated vertices and without isolated edges, $N_{oe}(G) = N_{ce}(G)$.

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