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E AND V –MAGIC STRENGTH OF H-MULTIPLE LABELING OF GRAPHS

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ABSTRACT

A one-to-one map f from $V \cup E$ of a graph G onto the integers $\{h, 2h, 3h, ..., (m+n)h\}$ is a E-Magic h-multiple labeling if there exist a constant k such that $\varepsilon k(f) = \sum_{v\in V} d(v) f(v) + \sum_{e\in E} f(e)$. A one-to-one map f from $V \cup E$ of a graph G onto the integers $\{h, 2h, 3h, ..., (m+n)h\}$ is a V-Magic h-multiple labeling if there exist a constant k such that $f(u) + \sum_{v\in N(u)} f(u)v = k$ where h is any positive integer. In this Paper, we obtain the E-magic strength of h-multiple labeling of the path P_n on n-vertices, the n-bi star $B_{n,n}$ obtained from two disjoint copies of $K_{1,n}$ by joining the center vertices by an edge and the tree $< K_{1,n} : 2 >$ obtained from $B_{n,n}$ by subdividing the middle edge with a new vertex.Also we obtain the V-magic strength of h-multiple labeling of the path P_n on n-vertices and 4-connected graph $H_{4,n}$ on n-vertices

Keywords: *Magic labeling, E-Magic Strength, E-Magic h-multiple labelling, graph nP_2, V-Magic h-multiple labeling and structure of $H_{m,n}$*

1. INTRODUCTION

A labeling of a graph G(V,E) is a mapping from the set of vertices ,edges or both vertices and edges to the set of labels. Based on the domain we distinguish vertex labelings, edge labelings and total labelings. In most applications the labels are positive (or nonnegative) integers, though in general real numbers could be used. Various labelings are obtained based on the requirements put on the mapping. Magic labelings were introduced by sedlacek in 1963[6]. In general for a magic type labeling we require the sum of labels related to a vertex (a vertex magic labeling) or to an edge (an edge magic labeling) to be constant all over the graph.

Definition 1.1: For any magic labeling f of G, there is a constant k such that $f(x) + f(y) + f(xy) = k$, $xy \in E(G)$.

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2. E-MAGIC STRENGTH OF H-MULTIPLE LABELING OF GRAPHS

Definition 2.1 [3]: The E-magic strength of G, Em(G),is defined as the minimum of all k where the minimum is taken over all magic labelings f of G. That is Em(G)=min{k:f is a magic labeling of G}.Let f be a magic labeling of G with the constant k. Then adding all constants obtained at each edge we get $\varepsilon k(f)=\sum_{v\in V} d(v)f(v)+\sum_{e\in E} f(e)$

Definition 2.2: Let h be any positive integer.A one-to-one map f from $V\cup E$ of a graph G onto the integers $\{h,2h,3h,...,(m+n)h\}$ is a E-Magic h-multiple labeling if there exist a constant k such that

$$\varepsilon k(f)=\sum_{v\in V} d(v)f(v)+\sum_{e\in E} f(e)$$

A graph which admits E-Magic h-multiple labeling then it is called E-Magic h-multiple graph.

Theorem 2.1: $Em(P_{2n})=h(5n+1)$ and $Em(P_{2n+1})=h(5n+3)$

Proof: We prove this theorem by assigning a magic labeling to P_{2n} and P_{2n+1}

Let $v_1,v_2,...,v_{2n}$ be the consecutive vertices and $e_1,e_2,...,e_{2n-1}$ be the consecutive edges of P_{2n}

That is , $e_i=v_iv_{i+1}\quad for\quad 1\le i\le 2n-1$

Define $f:V\cup E\to\{h,2h,...,(2n-1)h\}$ by

$$f(v_{2i-1})=ih\quad for\quad 1\le i\le n$$

$$f(v_{2i})=h(n+i)\quad for\quad 1\le i\le n$$

$$f(e_i)=h(4n-i)\quad for\quad 1\le i\le 2n-1$$

Thus $Em(P_{2n})\le h(5n+1)$

Since $\varepsilon(P_{2n})=2n-1$, and hence $\varepsilon+v=4n-1$

If f is a magic labeling of P_{2n} with constant $k(f)$ then $\varepsilon k(f)=\sum_{v\in V} d(v)f(v)+\sum_{e\in E} f(e)$

$$\begin{aligned}\text{That is , }(2n-1)k(f)&=\sum_{i=2}^{2n-1}2h(v_i)+h(v_1)+h(v_{2n})+h\sum_{i=1}^{2n-1}f(e_i)\\
&= h\sum_{i=1}^{2n}f(v_i)+h\sum_{i=1}^{2n-1}f(e_i)+h\sum_{i=2}^{2n-1}f(v_i)\\
&= (h+2h+...+(4n-1)h)+h\sum_{i=2}^{2n-1}f(v_i)\\
&= h(4n-1)(2n)+h\sum_{i=2}^{2n-1}f(v_i)\end{aligned}$$

$$\begin{aligned}\therefore k(f)&=\frac{h2n(4n-1)}{2n-1}+h\sum_{i=2}^{2n-1}\frac{f(v_i)}{2n-1}\\
&>(4n-1)h+2h+(n-1)h=5n\end{aligned}$$

Hence $k(f) \geq (5n+1)h$ which implies that $Em(P_{2n}) \geq h(5n+1)$

$\therefore\ Em(P_{2n}) = h(5n+1)$

Similarly, we can prove that $Em(P_{2n+1}) = h(5n+3)$

Example 2.1: Fig. 2.1 Shows that $Em(P_9) = 46$

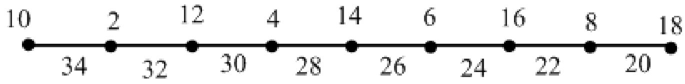

Fig. 2.1

Theorem 2.2: $Em(B_{n,n}) = h(5n+6)$

Proof: Let $V(B_{n,n}) = \{u, v; u_1, u_2, ..., u_n; v_1, v_2, ..., v_n\}$ and $E(B_{n,n}) = \{uu_i, vv_i, uv : 1 \leq i \leq n\}$

Define a function $f : V \cup E \to \{h, 2h, ..., (2n+1)h\}$ by

$f(u_i) = ih$ for $1 \leq i \leq n$, $f(u) = h(n+2)$, $f(v) = h(n+1)$, $f(u)v = h(3n+3)$, $f(u_i)u = h(4n+4-i)$ for $1 \leq i \leq n$ and $f(vv_i) = h(2n+3-i)$ for $1 \leq i \leq n$

Clearly f is a E-magic h-multiple labeling of $B_{n,n}$ with $k(f) = h(5n+6)$ and hence $Em(B_{n,n}) \leq h(5n+6)$

Let f be a E-magic h-multiple labeling of $B_{n,n}$ with $k(f)$

Then $$\varepsilon k(f) = \sum_{v \in V} d(v) f(v) + \sum_{e \in E} f(e)$$
$$= h\sum_{i=1}^{n} f(v_i) + h\sum_{i=1}^{n} f(u_i) + h(n+1) f(u) + h(n+1) f(v) + h\sum_{e \in E} f(e)$$

$$(2n+1)k(f) = h\sum_{v \in V} f(v) + h\sum_{e \in E} f(e\ ) + nhf(u) + nhf(v)$$
$$= h + 2h + ... + (4n+3)h + nhf(u) + nhf(v)$$
$$= \frac{h(4n+3)(4n+4)}{2} + nh(f(u) + f(v))$$

$$\therefore k(f) = h(4n+5) + \frac{h}{2n+1}(n(f(u) + f(v)) + 1)$$

Since $k(f)$ is an integer, $h\left(\frac{n(f(u) + f(v)) + 1}{2n+1}\right)$ is also an integer.

That is $n(f(u) + f(v)) + 1 \equiv 0 \bmod (2n+1)$

And hence $n(f(u) + f(v)) \equiv 2n \bmod (2n+1)$ which implies that $f(u) + f(v) \equiv 2 \bmod (2n+1)$

But $f(u) + f(v) \geq 3$ and thus $f(u) + f(v) \geq 2n+3$

$$\therefore k(f) \geq h(4n+5) + [n(2n+3)+1]h / 2n+1$$
$$= h(5n+6)$$

This shows that $Em(B_{n,n}) \geq h(5n+6)$

Hence $Em(B_{n,n}) = h(5n+6)$

Example 2.2: Fig. 2.2 Shows that $Em(B_{5,5}) = 155$

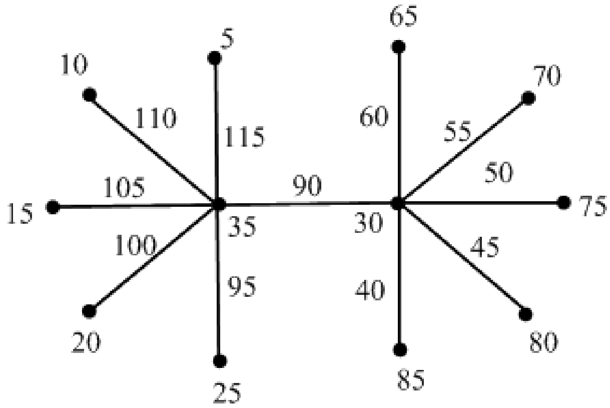

Fig. 2.2

Theorem 2.3: $Em(< K_{1,n} : 2 >) = h(4n+9)$

Proof: Define the labeling f on $< K_{1,n} : 2 >$ as follows

$f(u) = h,\ f(v) = 2h,\ f(w) = h(n+3)$

$f(u_i) = h(i+2)$ for $1 \le i \le n$,

$f(v_i) = h(n+3+i)$ for $1 \le i \le n$,

$f(uw) = h(3n+5),\ f(vw) = h(3n+4),\ f(uu_i) = h(4n+6-i)$ for $1 \le i \le n$

$f(vv_i) = h(3n+4-i)$ for $1 \le i \le n$

One can check that the above labeling f is a E-magic h-multiple labeling of $< K_{1,n} : 2 >$ with $k(f) = h(4n+9)$

Thus $Em(< K_{1,n} : 2 >) \le h(4n+9)$

Now we show that $Em(< K_{1,n} : 2 >) \ge h(4n+9)$

Let f be a magic labeling of $< K_{1,n} : 2 >$ with constant $k(f)$

Then $\varepsilon k(f) = \sum_{v \in V} d(v) f(v) + \sum_{e \in E} f(e)$

That is, $(2n+2)k(f) = \sum_{i=1}^{n} h(u_i) + h\sum_{i=1}^{n} f(v_i) + h(n+1) f(u) + h(n+1) f(v) + 2h(w) + h\sum_{e \in E} f(e)$

$$(2n+2\ ()f) = h\sum_{v \in V} f(v) + h\sum_{e \in E} f(e\ ) + nhf(u) + nhf(v) + hf(w)$$

$$= h + 2h + \ldots + (4n+5)h + nhf(u) + nhf(v) + hf(w)$$

$$= \frac{h(4n+5)(4n+6)}{2} + nh(f(u) + f(v)) + hf(w)$$

$$= \frac{h(4n+5)(4n+6)}{2(2n+2)} + h(n(f(u) + f(v)) + f(w)) / (2n+2)$$

$$\therefore k(f) = h(4n+7) + \frac{h}{2n+2}(n(f(u) + f(v)) + f(w) + 1)$$

Since $k(f)$ is an integer and $f(u)+f(v)\geq 3$, $Em(<K_{1,n}:2>)\geq h(4n+9)$

Hence $Em(<K_{1,n}:2>)=h(4n+9)$

Example 2.3: Fig. 2.3 Shows that $Em(<K_{1,4}:2>)=50$

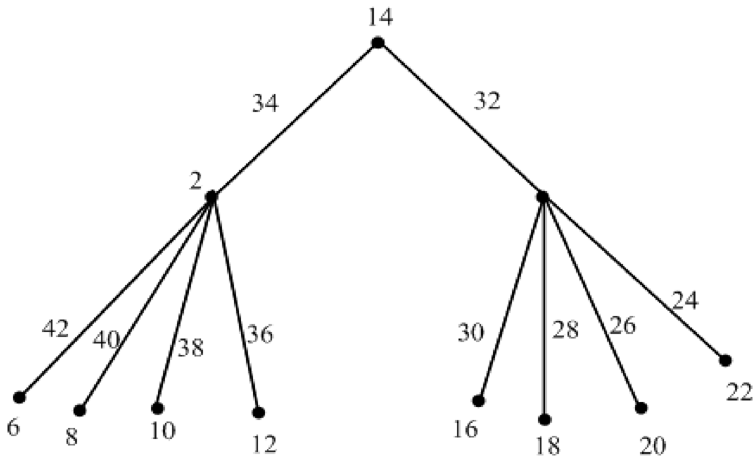

Fig. 2.3

Definition 2.2: The graph nP_2 is defined as the disjoint union of n copies of P_2.

Theorem 2.4: $Em((2n+1)P_2)=h(9n+6)$ for any $n\geq 0, n$ is odd

Proof: Let the vertices of $(2n+1)P_2$ be $u_1,u_2,...,u_{2n+1};v_1,v_2,...,v_{2n+1}$ and let the edge set be $\{u_iv_i:1\leq i\leq 2n+1\}$

Define $f:V\cup E\to\{h,2h,3h,...,h(6n+3)\}$ in such a way that $f(u_i)=ih$ for $1\leq i\leq 2n+1$

$f(v_i)=h(6n+4-2i)$ for $1\leq i\leq n$,

$f(v_{n+i})=h(6n+5-2i)$ for $1\leq i\leq n+1$,

$f(u_iv_i)=h(3n+2+i)$ for $1\leq i\leq n$,

$f(u_{n+i}v_{n+i})=h(2n+1+i)$ for $1\leq i\leq n+1$

It is easy to check that f is a E- magic h-multible labeling of $(2n+1)P_2$ with $k(f)=h(9n+6)$

Let f be a E-magic h-multiple labeling of $(2n+1)P_2$ with constant $k(f)$

Then $\varepsilon k(f)=\sum_{v\in V}d(v)f(v)+\sum_{e\in E}f(e)$

$$(2n+1)k(f)=h\sum_{v\in V}f(v)+h\sum_{e\in E}f(e)$$
$$=h+2h+...+(6n+3)h$$
$$=\frac{h(6n+3)(6n+4)}{2}$$

$$\therefore k(f)=\frac{h(6n+3)(6n+4)}{(2n+1)2}$$
$$=h(9n+6)$$

This is true for any magic labeling f of $(2n+1)P_2$

Therefore, $Em((2n+1)P_2)=h(9n+6)$

Example 2.4: Fig. 2.4 Shows that $Em(11P_2) = 255$

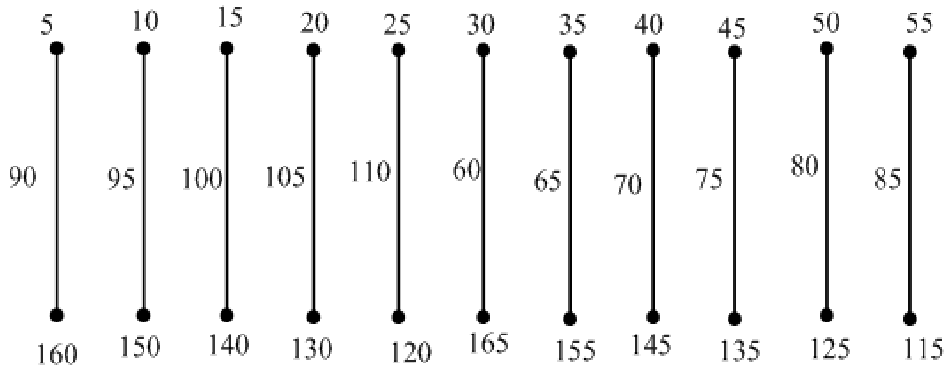

Fig. 2.4

Theorem 2.5: $Em(C_{2n+1}) = h(5n+4)$

Proof: Let the vertices of C_{2n+1} be $v_1v_2...v_{2n+1}v_1$

Define $f : V \cup E \to \{h, 2h, 3h, ..., h(5n+4)\}$ in such a way that $f(v_{2i+1}) = h(i+1)$ for $0 \le i \le n$
$f(v_{2i}) = h(n+1+i)$ for $1 \le i \le n$,
$f(v_iv_{i+1}) = h(4n+2-i)$ for $1 \le i \le 2n$,
$f(v_{2n+1}v_1) = h(4n+2)$

It is easy to check that f is a E- magic h-multiple labeling of C_{2n+1} with $k(f) = h(5n+4)$

Thus, $Em(C_{2n+1}) \le h(5n+4)$

Let f be a E-magic h-multiple labeling of C_{2n+1} with constant $k(f)$

Then $\varepsilon k(f) = \sum_{v \in V} d(v) f(v) + \sum_{e \in E} f(e)$

$$(2n+1)k(f) = h\sum_{v \in V} 2 f(v) + h\sum_{e \in E} f(e\ )$$

$$= h\sum_{v \in V} f(v) + h\sum_{e \in E} f(e\ ) + h\sum_{v \in V} f(v)$$

$$= h + 2h + ... + (4n+2)h + h\sum_{v \in V} f(v)$$

$$\ge \frac{h(4n+2)(4n+3)}{2} + h + 2h + ... + (2n+1)h$$

$\therefore k(f) \ge h(5n+4)$

$Em(C_{2n+1}) \ge h(5n+4)$

Hence $Em(C_{2n+1}) = h(5n+4)$

Example 2.5: Fig. 2.5 Shows that $Em(C_9) = 48$

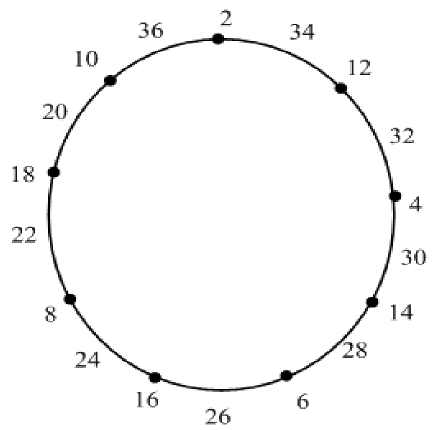

Fig 2.5

Theorem 2.6: $Em(P_n^2) = 3nh$

Proof: Let the vertices of $P_n{}^2$ be $v_1, v_2, ..., v_n$

Define $f : V \cup E \to \{h, 2h, 3h, ..., 3nh\}$ in such a way that
$f(v_i) = ih$ for $1 \le i \le n$,
$f(v_i v_{i+1}) = h(3n - (2i+1))$ for $1 \le i \le n-1$,
$f(v_i v_{i+2}) = h(3n - (2i+2))$ $1 \le i \le n-2$
Thus, $Em(P_n{}^2) \le 3nh$
But since $\varepsilon(P_n{}^2) = (3n-3)h,$ we have $Em(P_n{}^2) \ge 3nh$
Hence $Em(P_n^2) = 3nh$

Example 2.6: Fig. 2.6 Shows that $Em(P_6{}^2) = 90$

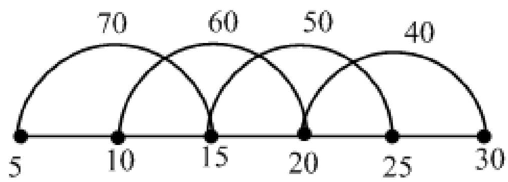

Fig. 2.6

3. V- MAGIC STRENGTH OF H-MULTIPLE LABELING OF GRAPHS

Definition 3.1 [4]: A one-to-one map $f : V \cup E \to \{1, 2, 3, ..., m+n\}$ is a vertex –magic total labeling of G if there is a constant k so that for every vertex u, $w_k(u) = f(u) + \sum_{v \in N(u)} f(u)v = k$. So the magic requirement is the associated weight $w_k(u) = k$ for all u. The fixed integer k is called the magic constant of f.

Definition 3.2 [4]: Let h be any positive integer. A one-to-one map f from $V \cup E$ of a graph G onto the integers $\{h, 2h, 3h, ..., (m+n)h\}$ is a V-Magic h-multiple labeling if there exist a constant k such that $f(u) + \sum_{v \in N(u)} f(u)v = k$. A graph which admits V-Magic h-multiple labeling then it is called V-Magic h-multiple graph.

Lemma 3.1: [1] If a non-trivial graph G is vertex magic, then the magic constant k is given by $k = q + \frac{p+1}{2} + \frac{q(q+1)}{p}$

Theorem 3.1: $k(P_n) = \frac{h(5n-3)}{2} \quad if \quad n \quad is \quad odd \quad and \quad n \geq 3$

Proof: Let $\{v_1, v_2, ..., v_n\}$ be the consecutive vertices and $\{e_1, e_2, ..., e_{n-1}\}$ be the consecutive edges of P_n.
That is $E(P_n) = \{e_i = v_i v_{i+1} / 1 \leq i \leq n-1\}$

Define $f: V \cup E \to \{h, 2h, 3h, ..., h(2n-1)\}$ as follows

$$f(v_1) = h(2n-1)$$

$$f(v_i) = h(n-2+i) \ \ for \ \ 2 \leq i \leq n$$

$$and \ \ f(e_i) = \begin{cases} \frac{h(n-i)}{2} & if \quad i \quad is \quad odd \\ h(n - \frac{i}{2}) & if \quad i \quad is \quad even \end{cases}$$

Let f be an V –Magic h-multiple labeling of a graph G with the magic number k.

Then $f(E) = \{h, 2h, 3h, ..., hm\}$ and $f(u) + \sum_{v \in N(u)} f(uv) = k \ \ for \quad all \quad u \in V$

That is $nk(P_n) = \sum_{u \in V} f(u) + \sum_{u \in V} \sum_{v \in N(u)} f(uv) \ \ for \quad all \quad u \in V$

$$= \sum_{u \in V} f(u) + 2\sum_{e \in E} f(e)$$

$$= hn(n-1) + \frac{hn(n+1)}{2} + hn(n-1)$$

$$\therefore k(P_n) = h(n-1) + \frac{h(n+1)}{2} + h(n-1)$$

$$= 2h(n-1) + \frac{h(n+1)}{2}$$

$$= \frac{h(5n-3)}{2} \quad if \quad n \quad is\, odd \quad and \quad n \geq 3$$

Example 3.1: Fig. 3.1 Shows that $k(P_5) = 55$

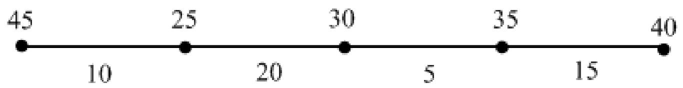

Fig 3.1

Remark 3.1: In [5], Bondy and Murty constructed an m-connected graph $H_{m,n}$ on n-vertices that has exactly {mn/2} edges. The structure of $H_{m,n}$ depends on the parities of m and n; there are three cases.

Case-1: m even. Let m=2r.Then $H_{2r,n}$ is constructed as follows. It has vertices 0,1,...,n-1 and two vertices i and j are joined if $i - r \leq j \leq i + r$ (where addition is taken modulo n)

Case-2: m odd, n even. Let m=2r+1.Then $H_{2r+1,n}$ is constructed by first drawing $H_{2r,n}$ and then adding edges joining vertex i to vertex i+(n/2) for $1 \le i \le n/2$

Case-3: m odd, n odd. Let m=2r+1.Then $H_{2r+1,n}$ is constructed by first drawing $H_{2r,n}$ and then adding edges joining vertex 0 to vertices(n-1)/2 and (n+1)/2 and vertex i to vertex i+(n+1)/2 for $1 \le i \le (n-1)/2$

Theorem 3.2: $k(H_{4,n}) = \dfrac{h(13n+5)}{2} \quad if \quad n \quad is \quad odd$

Proof: Let the vertex set of $H_{4,n}$ be $V = \{v_1, v_2, ..., v_n\}$ and the edge set of $H_{4,n}$ be

$$E = \{v_i v_{i+1} / 1 \le i \le n-1\} \cup \{v_n v_1\} \cup \{v_i v_{i+2} / 1 \le i \le n-2\} \cup \{v_{n-1} v_1\} \cup \{v_n v_2\}$$

Define $f : V \cup E \to \{h, 2h, 3h, ..., 3nh\}$ as follows

$$f(v_i) = \begin{cases} \dfrac{h}{2}(6n+1-i) & for \quad i \equiv 1 (\bmod 2) \\ \dfrac{h}{2}(5n+1-i) & for \quad i \equiv 0 (\bmod 2) \end{cases}$$

$$f(v_i v_{i+1}) = \begin{cases} \dfrac{h}{2}(n+2+i) & for \quad i \equiv 1 (\bmod 2), \quad i \ne n \\ \dfrac{h}{2}(i+2) & for \quad i \equiv 0 (\bmod 2) \end{cases}$$

$$f(v_n v_1) = h$$

$$f(v_i v_{i+2}) = \begin{cases} 2h(n-i) & for \quad i = 1, 2, ..., \dfrac{n-1}{2} \\ h(3n-2i) & for \quad i = \dfrac{n+1}{2}, \dfrac{n+3}{2}, ..., n-2 \end{cases}$$

$$f(v_{n-1} v_1) = h(n+2)$$

$$f(v_n v_2) = 2hn$$

Suppose there exists an vertex –magic h-multiple total labeling f of a graph G with the magic number k.

Then by lemma [3.1],

$$k = h(2n) + \frac{h(n+1)}{2} + \frac{h(2n)(2n+1)}{n}$$
$$= h[2n + \frac{n+1}{2} + 4n + 2]$$
$$= \frac{h}{2}(13n+5)$$

Example 3.2: Fig. 3.2 Shows that $k(H_{4,7}) = 96$

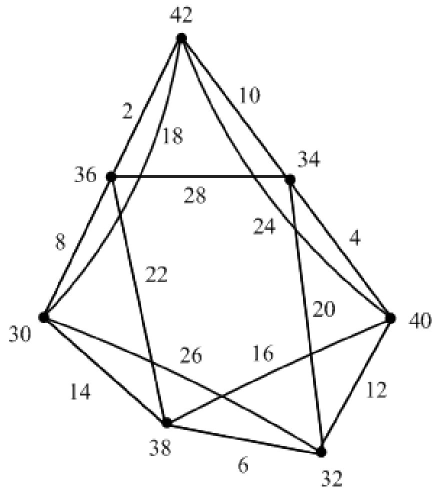

Fig. 3.2

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