

BIANCHI TYPE-III COSMOLOGICAL MODEL IN  $f(R, T)$  THEORY OF GRAVITY WITH  $\Lambda(T)$

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(Received On: 25-11-15; Revised & Accepted On: 19-12-15)

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ABSTRACT

*Spatially homogeneous Bianchi type-III cosmological models have been investigated in  $f(R, T)$  theory of gravity with  $\Lambda(T)$ , where  $R$  is the Ricci scalar and  $T$  is the trace of the energy momentum tensor, for the functional  $f(R, T) = f_1(R) + f_2(T)$ . The exact solutions of the field equations are obtained for the linearly varying deceleration parameter  $q(t)$  proposed by Akarsu and Dereli which yield the exponential and power law variation of the scale factor. It is observed that the cosmological model obtained here with power-law expansion has a point type singularity whereas the model with exponential expansion has no singularity at the initial epoch. The physical behaviors of the models are also discussed.*

**Keywords:**  $f(R, T)$  gravity, Bianchi type-III, Cosmological constant.

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1. INTRODUCTION

Recent cosmological observations such as Type Ia Supernova [1], cosmic microwave background radiation [2], large scale structure [3] indicate that the universe is currently accelerating. Quantitative analysis of all these observations suggests that there is a hitherto unknown component, dubbed dark energy which is responsible for the cosmic acceleration. The simplest candidate for dark energy is the cosmological constant ( $\Lambda$ ) which fits the observations well. But it is plagued with the fine-tuning and the cosmic coincidence problem for these reasons  $\Lambda$  with a dynamical character is preferred over a constant  $\Lambda$  especially a time dependent  $\Lambda$ . To further investigate the true nature of dark energy and the accelerated expansion of the universe, many dynamical dark energy models have been proposed, such as quintessence [4], phantom [5], tachyon [6], k-essence [7], Chaplygin gas [8], quintom[9], holographic dark energy [10], agegraphic dark energy [11] etc.

In recent years, one of the approaches to explain the late time cosmic acceleration is to modify the Einstein's General Theory of Relativity. Of late, various modified theories of gravity, such as  $f(R)$  gravity, Gauss-Bonnet gravity or  $f(G)$  gravity,  $f(T)$  gravity etc. seem attractive in the understanding of the late time cosmic acceleration. Among the various modifications,  $f(R)$  theory of gravity is treated most suitable and many authors have investigated  $f(R)$  gravity in different context. It has been suggested that cosmic acceleration can be achieved by replacing the Einstein-Hilbert action of general relativity with general function  $f(R)$  of Ricci scalar  $R$ . Nojiri and Odinstov [12] have reviewed various modified gravity theories that are considered as gravitational alternatives for dark energy. Viable  $f(R)$  gravity models have been proposed by Multamaki and Vilja [13, 14] and Shamir [15] which show the unification of early time inflation and late time acceleration.

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Very recently, Harko *et al.* [16] have developed a new modified theory of gravity known as  $f(R, T)$  gravity. In this theory, the gravitational Lagrangian is given by an arbitrary function of the scalar curvature  $R$  and the trace  $T$  of the energy tensor. They have obtained the gravitational field equations in the metric formalism as well as the equations of motion for test particles from the covariant divergence of the stress-energy tensor. The field equations corresponding to FRW metric for several particular forms of the function  $f(R, T)$  are also presented. Kumar and Singh investigated perfect fluid solutions using Bianchi type-I space-time in scalar-tensor theory [17]. Adhav [18] constructed LRS Bianchi type cosmological model in  $f(R, T)$  gravity with perfect fluid. Sharif and Zubir [19] studied the anisotropic behavior of perfect fluid and mass less scalar field for Bianchi type-I space-time in this theory. Reddy *et al.* [20] studied spatially homogeneous Bianchi type-III cosmological model in the presence of perfect fluid in  $f(R, T)$  theory with negative constant deceleration parameter. Reddy *et al.* [21] also derived Bianchi type-III dark energy model in presence of perfect fluid using special law of variation for Hubble's parameter. Yadav [22] constructed Bianchi type-V string cosmological model with power law expansion in this theory. Sahoo *et al.* [23] constructed an axially symmetric cosmological model in the presence of a perfect fluid source in  $f(R, T)$  theory. Recently, Ahmed and Pradhan [24] constructed Bianchi type-V and Sahoo and Sivakumar [25] constructed LRS Bianchi type-I cosmological models for a specific choice of  $f(R, T) = f_1(R) + f_2(T)$ . They considered the cosmological constant  $\Lambda$  as a function of the trace of the energy-momentum-tensor  $T$  and called the model as " $\Lambda(T)$  gravity".

Motivated by the above investigations we intend to study Bianchi type-III space-time with perfect fluid source in  $f(R, T)$  gravity. The paper is organized as follows: a brief introduction of the field equations in metric version of  $f(R, T)$  gravity is given in Sect. 2. Explicit field equations in  $f(R, T)$  gravity are derived using the linear form of the functions  $f_1(R) = \lambda R$  and  $f_2(T) = \lambda T$  in  $f(R, T) = f_1(R) + f_2(T)$ , where  $\lambda$  an arbitrary parameter, with the aid of Bianchi type-III metric in the presence of perfect fluid in Sect. 3. Section 4, deals with solutions of the field equations and some physical properties of the models. Finally, conclusions are summarized in Sect. 5. Here we use the natural system of units with  $G = c = 1$ , where  $G$  is the Newtonian gravitational constant and  $c$  is the speed of light in vacuum.

## 2. FIELD EQUATIONS OF $f(R, T)$ GRAVITY

The field equations of  $f(R, T)$  gravity are derived from a Hilbert-Einstein type variational principle.

The action for the modified  $f(R, T)$  gravity is

$$S = \frac{1}{16\pi} \int f(R, T) \sqrt{-g} d^4x + \int L_m \sqrt{-g} d^4x \quad (1)$$

where  $f(R, T)$  is an arbitrary function of the Ricci scalar  $R$  and the trace  $T$  of the energy momentum tensor  $T_{ij}$  of the matter, and  $L_m$  is the matter Lagrangian density. The energy momentum tensor  $T_{ij}$  is defined as

$$T_{ij} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}} \quad (2)$$

and its trace by  $T = g^{ij} T_{ij}$ .

By assuming that  $L_m$  of matter depends only on the metric tensor components  $g_{ij}$  and not on its derivatives, we obtain

$$T_{ij} = g_{ij} L_m - 2 \frac{\partial L_m}{\partial g^{ij}} \quad (3)$$

Now by varying the action (1) with respect to the metric tensor  $g^{ij}$ , we obtain the field equations of  $f(R, T)$  gravity as

$$f_R(R, T) R_{ij} - \frac{1}{2} f(R, T) g_{ij} + (g_{ij} \square - \nabla_i \nabla_j) f_R(R, T) = 8\pi T_{ij} - f_T(R, T) T_{ij} - f_T(R, T) \theta_{ij} \quad (4)$$

Where  $\theta_{ij} = -2T_{ij} + g_{ij} L_m - 2g^{lm} \frac{\partial^2 L_m}{\partial g^{ij} \partial g^{lm}}$  (5)

Here  $f_R(R, T) = \frac{\partial f(R, T)}{\partial R}$ ,  $f_T(R, T) = \frac{\partial f(R, T)}{\partial T}$ ,  $\square = \nabla^i \nabla_i$  and  $\nabla_i$  is the covariant derivative.

The trace of equation (4) gives

$$R f_R(R, T) + 3 \square f_R(R, T) - 2 f(R, T) = 8\pi T - f_T(R, T) T - f_T(R, T) \theta \quad (6)$$

where  $\theta = g^{ij} \theta_{ij}$ .

Eliminating  $\square f_R(R, T)$  from Eqs. (4) and (6), we obtain

$$f_R(R, T) \left( R_{ij} - \frac{1}{3} R g_{ij} \right) + \frac{1}{6} f(R, T) g_{ij} = (8\pi - f_T(R, T)) \left( T_{ij} - \frac{1}{3} T g_{ij} \right) - f_T(R, T) \left( \theta_{ij} - \frac{1}{3} \theta g_{ij} \right) + \nabla_i \nabla_j f_R(R, T) \quad (7)$$

Here we assume that the stress energy tensor of the matter is given by

$$T_{ij} = (p + \rho) u_i u_j - p g_{ij} \quad (8)$$

and the matter Lagrangian can be taken as  $L_m = -p$  and  $u^i = (1, 0, 0, 0)$  is the velocity vector in co-moving co-ordinate system satisfying the condition  $u^i \nabla_j u_i = 0$ ,  $u^i u_i = 0$ .

Using the variation of the stress-energy as perfect fluid in equation (5), we obtain

$$\theta_{ij} = -2T_{ij} - pg_{ij} \tag{9}$$

The field equations also depend through the tensor  $\theta_{ij}$ , on the physical nature of the matter field. Hence in the case of  $f(R, T)$  gravity depending on the nature of the matter source, we obtain several theoretical models corresponding to each choice of  $f(R, T)$ . Harko *et al.* [16] gave three classes of these models as follows

$$\begin{aligned} f(R, T) &= R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{aligned} \tag{10}$$

In this paper, we consider the case  $f(R, T) = f_1(R) + f_2(T)$  for Bianchi type-III space time.

### 3. METRIC AND FIELD EQUATIONS

We consider the spatially homogeneous and anisotropic Bianchi type-III metric given by

$$ds^2 = dt^2 - a_1^2(t)dx^2 - e^{-2mx} a_2^2(t)dy^2 - a_3^2(t)dz^2 \tag{11}$$

where  $a_1, a_2$  and  $a_3$  are cosmic scale factors and  $m$  is a positive constant.

Using the linear form of the functions  $f_1(R) = \lambda R$  and  $f_2(T) = \lambda T$  where  $\lambda$  an arbitrary parameter, the equation (4) can be written as

$$\lambda R_{ij} - \frac{1}{2}\lambda(R + T)g_{ij} + (g_{ij}\square - \nabla_i\nabla_j)\lambda = 8\pi T_{ij} - \lambda T_{ij} + \lambda(2T_{ij} + pg_{ij}) \tag{12}$$

Setting  $(g_{ij}\square - \nabla_i\nabla_j)\lambda = 0$ , we get

$$R_{ij} - \frac{1}{2}Rg_{ij} = \left(\frac{8\pi+\lambda}{\lambda}\right)T_{ij} + \left(p + \frac{1}{2}T\right)g_{ij} \tag{13}$$

This equation can be rearranged as

$$G_{ij} - \left(p + \frac{1}{2}T\right)g_{ij} = \left(\frac{8\pi+\lambda}{\lambda}\right)T_{ij} \tag{14}$$

where  $G_{ij} \equiv R_{ij} - \frac{1}{2}Rg_{ij}$  is the Einstein tensor.

The Einstein field equations with cosmological constant term are

$$G_{ij} - g_{ij} = -8\pi T_{ij} \tag{15}$$

A comparison of equations (14) and (15) provides us

$$\equiv (T) = p + \frac{1}{2}T \tag{16}$$

And 
$$\lambda = -\frac{8\pi}{8\pi+1}.$$

In a co-moving co-ordinate system the field equations (13), for the metric (11), with help of energy momentum tensor (8) can be written as

$$\frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} - \frac{m^2}{a_1^2} = -\left(\frac{8\pi+\lambda}{\lambda}\right)\rho - \tag{17}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} = \left(\frac{8\pi+\lambda}{\lambda}\right)p - \tag{18}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} = \left(\frac{8\pi+\lambda}{\lambda}\right)p - \tag{19}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} - \frac{m^2}{a_1^2} = \left(\frac{8\pi+\lambda}{\lambda}\right)p - \tag{20}$$

$$\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} = 0 \tag{21}$$

where an overhead dot denotes ordinary differentiation with respect to cosmic time 't' only.

The trace of equation (8) is given by

$$T = -3p + \rho \tag{22}$$

so that the effective cosmological constant in equation (16) reduces to

$$(T) = \frac{1}{2}(\rho - p) \tag{23}$$

Integrating (21), we obtain

$$a_1 = ka_2 \tag{24}$$

where  $k$  is any non-zero constant of integration.

The spatial volume  $V$  can be defined as

$$V = a^3 = a_1 a_2 a_3 \tag{25}$$

Now, using (24), the field equations (17) – (20) will reduce to

$$\left(\frac{\dot{a}_1}{a_1}\right)^2 + 2\frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} - \frac{m^2}{a_1^2} = -\alpha\rho - \tag{26}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} = \alpha p - \tag{27}$$

$$2\frac{\ddot{a}_1}{a_1} + \left(\frac{\dot{a}_1}{a_1}\right)^2 - \frac{m^2}{a_1^2} = \alpha p - \tag{28}$$

where  $\alpha = \frac{8\pi + \lambda}{\lambda}$ .

Equations (23), (26) and (28) give us the general formulations for the physical parameters of the  $f(R, T)$  model with respect to the Ricci scalar  $R$  and

$$R = 2\left[2\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \left(\frac{\dot{a}_1}{a_1}\right)^2 + 2\frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} - \frac{m^2}{a_1^2}\right] \tag{29}$$

Pressure, energy density and the cosmological constant for the model can be written as

$$p = \frac{(2\alpha + 1)\frac{\ddot{a}_1}{a_1} + \alpha\left(\frac{\dot{a}_1}{a_1}\right)^2 - \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} - \alpha\frac{m^2}{a_1^2}}{\alpha(1 + \alpha)} \tag{30}$$

$$\rho = \frac{\frac{\ddot{a}_1}{a_1} - \alpha\left(\frac{\dot{a}_1}{a_1}\right)^2 - (1 + 2\alpha)\frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} + \alpha\frac{m^2}{a_1^2}}{\alpha(1 + \alpha)} \tag{31}$$

$$= -\frac{\frac{\ddot{a}_1}{a_1} + \left(\frac{\dot{a}_1}{a_1}\right)^2 + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} - \frac{m^2}{a_1^2}}{1 + \alpha} \tag{32}$$

#### 4. SOLUTIONS AND THE MODELS

The field equations (26) – (28) are a system of three independent equations in four unknowns  $a_1, a_2, p, \rho$ . Hence to obtain determinate solution of the system we take the help of a generalized linearly varying deceleration parameter  $q$  [26] as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -k_1 t + s - 1 \tag{33}$$

where  $k_1 \geq 0$  and  $s \geq 0$  are constants. The universe exhibit decelerating expansion if  $q > 0$ , expand with constant rate if  $q = 0$ , accelerating power law if  $-1 < q < 0$  and exponential expansion if  $q = -1$ .

From (33), we obtain the exponential and power law variation of the scale factor as

$$a = p_1 \exp[l_1 t] \text{ for } k_1 = 0 \text{ and } s = 0 \tag{34}$$

$$a = p_2 (st + l_2)^{\frac{1}{s}} \text{ for } k_1 = 0 \text{ and } s > 0 \tag{35}$$

where  $p_1, p_2, l_1$  and  $l_2$  are integrating constants.

The average Hubble's parameter  $H$  is given by

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \tag{36}$$

where  $H_1 = \frac{\dot{a}_1}{a_1} = H_2, H_3 = \frac{\dot{a}_3}{a_3}$  are the directional Hubble parameters in the directions of  $x, y$  and  $z$  axes respectively.

The expansion scalar  $\theta$  and the shear scalar  $\sigma^2$  are defined as

$$\theta = 3H = 2\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3} \tag{37}$$

$$\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^3 H_i^2 - \frac{1}{3}\theta^2\right] = \frac{1}{3}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_3}{a_3}\right)^2 \tag{38}$$

The anisotropy parameter of the expansion  $\Delta$  is

$$\Delta = \frac{1}{3}\sum_{i=1}^3 \left(\frac{H_i - H}{H}\right)^2 = 6\left(\frac{\sigma}{\theta}\right)^2 \tag{39}$$

Since the field equations (26) – (28) are highly non-linear, we assume that expansion scalar ( $\theta$ ) is proportional to shear scalar ( $\sigma$ ) which gives

$$a_2 = a_3^n, n \neq 1 \tag{40}$$

where  $n$  is proportionality constant.

#### 4.1 Model 1

Using equations (24), (35), (40) in equation (36), we can get

$$a_2 = c_2(st + l_2)^{\frac{1}{r}} \tag{41}$$

$$a_1 = c_1(st + l_2)^{\frac{1}{r}} \tag{42}$$

$$a_3 = c_3(st + l_2)^{\frac{1}{rn}} \tag{43}$$

where  $c_1, c_2, c_3$  are integrating constants and  $c_1 = kc_2, c_3 = c_1^{\frac{1}{n}}, r = \frac{2n+1}{3n}$ .

From the above set of solutions we observe that the scale factors have constant values at  $t = 0$  and tend to infinity when  $t \rightarrow \infty$ .

The directional Hubble's parameters are

$$H_1 = \frac{s}{r(st+l_2)}, H_2 = \frac{s}{r(st+l_2)}, H_3 = \frac{s}{rn(st+l_2)} \tag{44}$$

The mean Hubble parameter is obtained as

$$H = \frac{s}{st+l_2} \tag{45}$$

The directional Hubble parameters and mean Hubble parameter become constant at the initial epoch while the values of these parameters tend to zero as  $t \rightarrow \infty$ .

The scalar of expansion  $\theta$ , shear scalar  $\sigma^2$  and the anisotropy parameter  $\Delta$  in the model are

$$\theta = \frac{3s}{st+l_2} \tag{46}$$

$$\sigma^2 = \frac{s^2(n-1)^2}{3r^2n^2(st+l_2)^2} \tag{47}$$

$$\Delta = \frac{2(n-1)^2}{9r^2n^2} \tag{48}$$

The pressure, density and cosmological constant in the model are

$$p = \frac{1}{r^2\alpha(1+\alpha)} \left[ \left\{ (1-r)(2\alpha+1) + \alpha - \frac{1}{n} \right\} s^2(st+l_2)^{-2} - \frac{\alpha r^2 m^2}{c_1^2} (st+l_2)^{\frac{-2}{r}} \right] \tag{49}$$

$$\rho = \frac{1}{r^2\alpha(1+\alpha)} \left[ \left( 1-r-\alpha - \frac{1+2\alpha}{n} \right) s^2(st+l_2)^{-2} + \frac{\alpha r^2 m^2}{c_1^2} (st+l_2)^{\frac{-2}{r}} \right] \tag{50}$$

$$= \frac{1}{r^2(1+\alpha)} \left[ \left( r-2-\frac{1}{n} \right) s^2(st+l_2)^{-2} + \frac{r^2 m^2}{c_1^2} (st+l_2)^{\frac{-2}{r}} \right] \tag{51}$$

The pressure and energy density becomes infinite at initial epoch ( $t = 0$ ) and tend to zero as  $t \rightarrow \infty$ .

The equation of state ' $\omega$ ' is

$$\omega = \left[ \left\{ (1-r)(2\alpha+1) + \alpha - \frac{1}{n} \right\} s^2(st+l_2)^{-2} - \frac{\alpha r^2 m^2}{c_1^2} (st+l_2)^{\frac{-2}{r}} \right] \times \left[ \left( 1-r-\alpha - \frac{1+2\alpha}{n} \right) s^2(st+l_2)^{-2} + \frac{\alpha r^2 m^2}{c_1^2} (st+l_2)^{\frac{-2}{r}} \right]^{-1} \tag{52}$$

Here the equation of state  $\omega$  is constant or infinity according as  $t$  is equal to zero and infinity. The equation of state  $\omega$  for this model will start from initial epoch and vary according to the time till infinite future.

In this model, the spatial volume  $V$  is zero at  $t = \frac{-l_2}{s}$  and increases with increase of  $t$ . The expansion scalar  $\theta$  is also infinite at  $t = \frac{-l_2}{s}$ . It means that the universe starts evolving with zero volume at  $t = \frac{-l_2}{s}$  with infinite rate of expansion. The anisotropy parameter is uniform throughout the whole expansion of the universe. From equations (41) – (43), we observe that the scale factors are zero at  $t = \frac{-l_2}{s}$ , and hence the model has the point-type singularity at  $t = \frac{-l_2}{s}$  [27]. As cosmic time ( $t$ ) increases the scale factors increase whereas the scalar expansion decreases. The pressure, energy density, cosmological constant, Hubble's parameter and shear scalar decreases as  $t$  increases.

The scalar curvature  $R$  for this model is given by

$$R = 2 \left[ \frac{3s^2(n-r^2n+r)}{r^2n} (st+l_2)^{-2} - \frac{m^2}{c_1^2} (st+l_2)^{\frac{-2}{r}} \right] \tag{53}$$

and it becomes constant when  $t \rightarrow \infty$ .

The trace  $T$  for this model is given by

$$T = \frac{2}{r^{2\alpha(1+\alpha)}} \left[ \left( r + 3\alpha r - 5\alpha - 1 + \frac{1-\alpha}{n} \right) s^2 (st + l_2)^{-2} + \frac{2\alpha r^2 m^2}{c_1^2} (st + l_2)^{\frac{-2}{r}} \right] \quad (54)$$

Using (53) and (54), the  $f(R, T)$  gravity from equation (10) obtained as

$$f(R, T) = 2\lambda \left[ \frac{3s^2(n-r^2n+r)}{r^{2n}} (st + l_2)^{-2} + \frac{m^2}{c_1^2} \left( \frac{1-\alpha}{1+\alpha} \right) (st + l_2)^{\frac{-2}{r}} + \frac{1}{r^{2\alpha(1+\alpha)}} \left( r + 3\alpha r - 5\alpha - 1 + \frac{1-\alpha}{n} \right) s^2 (st + l_2)^{-2} \right] \quad (55)$$

#### 4.2 Model 2

Using equations (24), (34), (40) in equation (36), we can get

$$a_1 = c_1 e^{k_2 t} \quad (56)$$

$$a_2 = c_2 e^{k_2 t} \quad (57)$$

$$a_3 = c_3 e^{\frac{k_2 t}{n}} \quad (58)$$

where  $c_1, c_2, c_3$  are integrating constants and  $c_1 = kc_2, c_3 = c_2^{\frac{1}{n}}, k_2 = \frac{3nl_1}{2n+1}$ .

We observe that the scale factors have constant values at  $t = 0$ . It indicates that the model has no singularity. The scale factors start increasing with the increase in cosmic time and finally diverge to  $\infty$  when  $t \rightarrow \infty$ .

The directional Hubble's parameters are

$$H_1 = k_2, H_2 = k_2, H_3 = \frac{k_2}{n}.$$

The mean Hubble parameter is obtained as

$$H = \frac{(2n+1)}{3n} k_2.$$

The directional Hubble's parameters and mean Hubble parameter are constant.

The scalar expansion  $\theta$ , shear scalar and the anisotropy parameter of the expansion are found as

$$\theta = \frac{2n+1}{n} k_2, \quad \sigma^2 = \frac{k_2^2(n-1)^2}{3n^2}, \quad \Delta = 2 \left( \frac{n-1}{2n+1} \right)^2 \quad (59)$$

The pressure, density and cosmological constant in the model are

$$p = \frac{1}{\alpha(1+\alpha)} \left[ \left( 3\alpha + 1 - \frac{1}{n} \right) k_2^2 - \frac{m^2\alpha}{c_1^2} e^{-2k_2 t} \right] \quad (60)$$

$$\rho = \frac{1}{\alpha(1+\alpha)} \left[ \left( 1 - \alpha - \frac{1+2\alpha}{n} \right) k_2^2 + \frac{m^2\alpha}{c_1^2} e^{-2k_2 t} \right] \quad (61)$$

$$= -\frac{1}{1+\alpha} \left[ k_2^2 \left( 2 + \frac{1}{n} \right) - \frac{m^2}{c_1^2} e^{-2k_2 t} \right] \quad (62)$$

The pressure and density are constant at  $t = 0$  and behaves monotonically in the evolving of cosmic time  $t$ .

For this model the equation of state ' $\omega$ ' is given by

$$\omega = \left[ \left( 3\alpha + 1 - \frac{1}{n} \right) k_2^2 - \frac{m^2\alpha}{c_1^2} e^{-2k_2 t} \right] \times \left[ \left( 1 - \alpha - \frac{1+2\alpha}{n} \right) k_2^2 + \frac{m^2\alpha}{c_1^2} e^{-2k_2 t} \right]^{-1} \quad (63)$$

Since  $\omega \rightarrow \text{constant}$  as  $t \rightarrow 0$  and also  $\omega \rightarrow \text{constant}$  when  $t \rightarrow \infty$ , it means that the equation of state  $\omega$  would remain ideal during the evolution.

In this model, the universe starts with a non-singular state at initial epoch. The scale factors and the spatial volume increases exponentially as  $t$  increases. The scalar expansion is constant, hence the universe exhibits uniform exponential expansion. The cosmological constant is decreasing function of time and reaches to a small value at late time which is supported by results from Supernovae observations recently obtained by High-Z supernova team and supernova cosmological project.

The Ricci scalar ' $R$ ' and the trace ' $T$ ' for this model are

$$R = 2 \left[ \left( 3 + \frac{1}{n^2} + \frac{2}{n} \right) k_2^2 - \frac{m^2}{c_1^2} e^{-2k_2 t} \right] \quad (64)$$

$$T = 2 \left[ \frac{1}{\alpha(1+\alpha)} \left( \frac{1-\alpha}{n} - 1 - 5\alpha \right) k_2^2 + \frac{2m^2}{c_1^2(1+\alpha)} e^{-2k_2 t} \right] \quad (65)$$

The function  $f(R, T) = f_1(R) + f_2(T)$  for this model is given by

$$f(R, T) = 2\lambda \left[ \left\{ \left( 3 + \frac{1}{n^2} + \frac{2}{n} \right) + \left( \frac{1-\alpha}{n} - 5\alpha - 1 \right) \frac{1}{\alpha(1+\alpha)} \right\} k_2^2 + \left\{ \frac{2m^2}{c_f^2(1+\alpha)} - \frac{m^2}{c_f^2} \right\} e^{-2k_2 t} \right] \quad (66)$$

In this model the negative deceleration parameter indicates that the universe is accelerating which is consistent with the present day observations.

## 5. CONCLUSION

In this paper, we have studied spatially homogeneous Bianchi type-III cosmological models with linearly varying deceleration parameter in the framework of a modified gravity theory known as  $f(R, T)$  gravity. Earlier several authors have discussed the cosmological consequences of exponential expansion and power law expansion for  $f(R, T) = R + 2f(T)$ . Here we consider the case  $f(R, T) = f_1(R) + f_2(T)$  as  $f_1(R) = \lambda R$  and  $f_2(T) = \lambda T$  where  $\lambda$  is an arbitrary parameter. It is observed that the model with power law expansion has a point type singularity at the initial epoch whereas the model with exponential expansion has no singularity at the initial epoch. For both the models the expressions for some important cosmological parameters have been obtained and the physical behaviors of the models are discussed in detail. The solutions obtained here are new and believed to be useful for a better understanding of the characteristic of Bianchi type-III cosmological models in the evolution of the universe within the framework of  $f(R, T)$  gravity with variable  $\Lambda(T)$ .

## ACKNOWLEDGEMENT

The financial support by UGC (NER) (Grant No. F.5-63/2013-14/MRP/NERO) is gratefully acknowledged by C. R. Mahanta for carrying out the investigations reported in this paper.

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**Source of support: UGC, India, Conflict of interest: None Declared**

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