International Journal of Mathematical Archive-2(7), July - 2011, Page: 1094-1098
Available online through www.ijma.info ISSN 2229 – 5046

SPECIAL PYTHAGOREAN TRIANGLES WITH PERIMETER TO BE SUM TWO SQUARES AND A QUARTIC

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(Received on: 20-06-11; Accepted on: 10-07-11)

ABSTRACT

Special Pythagorean Triangle, with perimeter to be sum two squares and a quartic are obtained various 3D graphs of corresponding Pythagorean Triplets are plotted. A few interesting results are observed.

Key words: *Pythagorean Triangles, sum of two legs is a quartic.*

Subject Classification Code*: 11D25, 11-04 and 11Y50.*

INTRODUCTION:

In the Pythagorean Mathematics, the determination of classifying all Pythagorean triangles wherein each of which the perimeter can always be represented as the sum of three distinct squares by Gopalan and Devibala **[1]**, motivated me to examine whether there exists any Pythagorean Triangle, with sum of legs to be a quartic. The existence of such triangles with two sides consecutive is also investigated. It is towards this end, an attempt has been made to find patterns of Special Pythagorean Triangles with perimeter as sum of two squares and a quartic.

METHOD OF ANALYSIS:

The primitive solutions of the Pythagorean Equation

$$X^2 + Y^2 = Z^2 \quad (1)$$

is given by [2]

$$X = m^2 - n^2,\ Y = 2mn,\ Z = m^2 + n^2 \quad (2)$$

for some integers ***m, n*** of opposite parity such that $m > n > 0$ and $(m, n) = 1$.

(I) SUM OF TWO LEGS IS A QUARTIC:

$$X + Y = \beta^{10} \quad (3)$$

$$\text{i.e., } m^2 + 2\,m\,n - n^2 - \beta^4 == 0 \quad (4)$$

The solution of (4) and subsequently (3) is given by,

$\beta == - (m^2 + 2\,m\,n - n^2)^{¼}$

$\beta == - i (m^2 + 2\,m\,n - n^2)^{¼}$

$\beta == (m^2 + 2\,m\,n - n^2)^{¼}$

$\beta == i (m^2 + 2\,m\,n - n^2)^{¼}$ (5)

From these values only those values are taken for which β is a positive integer.

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(II) Perimeter is sum of two squares and a Quartic:

From (2) and (3),

$$X + Y + Z = \beta^4 + m^2 + n^2 \quad (6)$$

(2) and (4) generates X and Y which satisfy (1) and (3) in correspondence with **(6)**. Solving the Diophantine equation using software *Mathematica*,

$m^2 + 2\,m\,n - n^2 - \beta^4 == 0$ for $n < m$, $0 < n < 10^6$, $0 < m < 10^6$, $0 < \beta < 10^6$, we get the 1698 values for X,Y, Z which will consume around 90 pages to write down! Following are the few examples (precisely 74) with $0 < \beta < 10^2$ given in Table -1:

Table-1

S.N.	m	n	β	$X = m^2 - n^2$	$Y = 2mn$	$Z = m^2 + n^2$
1	35	28	7	441	1960	2009
2	41	10	7	1581	820	1781
3	140	112	14	7056	31360	32144
4	164	40	14	25296	13120	28496
5	205	182	17	8901	74620	75149
6	221	102	17	38437	45084	59245
7	315	252	21	35721	158760	162729
8	369	90	21	128061	66420	144261
9	389	238	23	94677	185164	207965
10	391	230	23	99981	179860	205781
11	560	448	28	112896	501760	514304
12	656	160	28	404736	209920	455936
13	689	528	31	195937	727584	753505
14	775	248	31	539121	384400	662129
15	820	728	34	142416	1193920	1202384
16	884	408	34	614992	721344	947920
17	875	700	35	275625	1225000	1255625
18	1025	250	35	988125	512500	1113125
19	1189	1148	41	95817	2729944	2731625
20	1625	58	41	2637261	188500	2643989
21	1260	1008	42	571536	2540160	2603664
22	1476	360	42	2048976	1062720	2308176
23	1556	952	46	1514832	2962624	3327440
24	1564	920	46	1599696	2877760	3292496
25	1565	1428	47	410041	4469640	4488409
26	1739	658	47	2591157	2288524	3457085
27	1715	1372	49	1058841	4705960	4823609
28	1781	1020	49	2131561	3633240	4212361
29	1855	798	49	2804221	2960580	4077829
30	2009	490	49	3795981	1968820	4276181
31	1845	1638	51	720981	6044220	6087069
32	1989	918	51	3113397	3651804	4798845
33	2240	1792	56	1806336	8028160	8228864
34	2624	640	56	6475776	3358720	7294976
35	2756	2112	62	3134992	11641344	12056080
36	3100	992	62	8625936	6150400	10594064
37	2835	2268	63	2893401	12859560	13181049
38	3321	810	63	10372941	5380020	11685141
39	3280	2912	68	2278656	19102720	19238144
40	3536	1632	68	9839872	11541504	15166720
41	3501	2142	69	7668837	14998284	16845165
42	3519	2070	69	8098461	14568660	16668261
43	3500	2800	70	4410000	19600000	20090000
44	4100	1000	70	15810000	8200000	17810000
45	3865	1752	71	11868721	13542960	18007729
46	4331	852	71	18031657	7380024	19483465

47	3869	2628	73	8062777	20335464	21875545
48	4105	1802	73	13603821	14794420	20098229
49	4235	3388	77	6456681	28696360	29413769
50	4961	1210	77	23147421	12005620	26075621
51	4549	2988	79	11765257	27184824	29621545
52	5135	1422	79	24346141	14603940	28390309
53	4756	4592	82	1533072	43679104	43706000
54	6500	232	82	42196176	3016000	42303824
55	5040	4032	84	9144576	40642560	41658624
56	5904	1440	84	32783616	17003520	36930816
57	5125	4550	85	5563125	46637500	46968125
58	5525	2550	85	24023125	28177500	37028125
59	5785	3738	89	19493581	43248660	47438869
60	5989	2990	89	26928021	35814220	44808221
61	5915	4732	91	12595401	55979560	57379049
62	6929	1690	91	45154941	23420020	50867141
63	6224	3808	92	24237312	47401984	53239040
64	6256	3680	92	25595136	46044160	52679936
65	6201	4752	93	15870897	58934304	61033905
66	6975	2232	93	43668801	31136400	53632449
67	6260	5712	94	6560656	71514240	71814544
68	6956	2632	94	41458512	36616384	55313360
69	7421	2772	97	47387257	41142024	62755225
70	8245	1358	97	66135861	22393420	69824189
71	6860	5488	98	16941456	75295360	77177744
72	7124	4080	98	34104976	58131840	67397776
73	7420	3192	98	44867536	47369280	65245264
74	8036	1960	98	60735696	31501120	68418896

Following Table- 2 verifies the result for first ten special Pythagorean triplets:-

Table- 2

S.N.	X^2	Y^2	$X^2 + Y^2$	Z^2	$X+Y=\beta^4$	$X + Y + Z = \beta^4 + m^2 + n^2$
1	194481	3841600	4036081	4036081	$2401=7^4$	$4410=7^4+35^2+28^2$
2	2499561	672400	3171961	3171961	$2401=7^4$	$4182=7^4+41^2+10^2$
3	49787136	983449600	1033236736	1033236736	$38416=14^4$	$70560=14^4+140^2+112^2$
4	639887616	172134400	812022016	812022016	$38416=14^4$	$66912=14^4+164^2+40^2$
5	79227801	5568144400	5647372201	5647372201	$83521=17^4$	$158670=17^4+205^2+182^2$
6	1477402969	2032567056	3509970025	3509970025	$83521=17^4$	$142766=17^4+221^2+102^2$
7	1275989841	25204737600	26480727441	26480727441	$194481=21^4$	$357210=21^4+315^2+252^2$
8	16399619721	4411616400	20811236121	20811236121	$194481=21^4$	$338742=21^4+369^2+90^2$
9	8963734329	34285706896	43249441225	43249441225	$279841=23^4$	$487806=23^4+389^2+238^2$
10	9996200361	32349619600	42345819961	42345819961	$279841=23^4$	$485622=23^4+391^2+230^2$

(III) One leg and hypotenuse are consecutive:

In such cases, $m = n + 1$

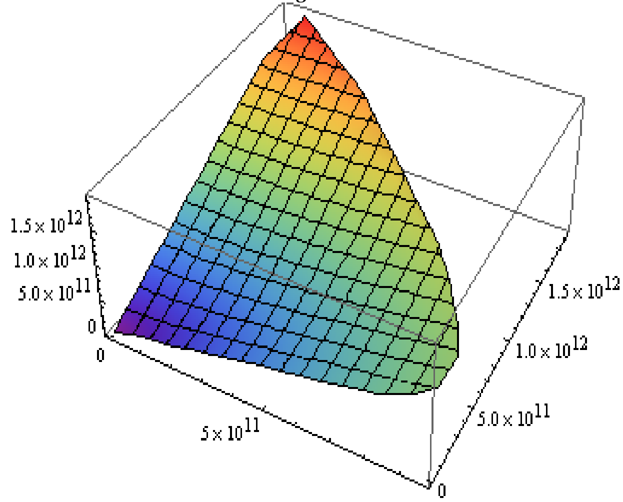
Substituting the value of m in equations (2) and (4), we get the Diophantine equation

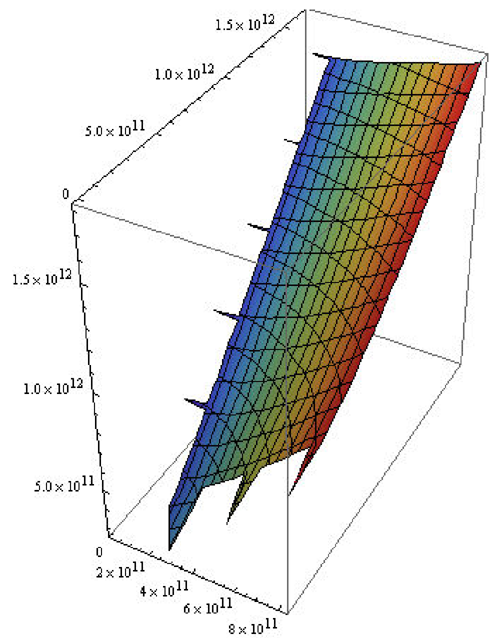
$$2n^2 + 4n+1- \beta^4 = 0 \quad (7)$$

Using the software Mathematica equation (7) was solved for $0 < n < 10^7$ and $0 < \beta < 10^7$ and it was found that it has no solution. It needs further investigations to find whether the solution of (7) exists for $n \geq 10^7$.

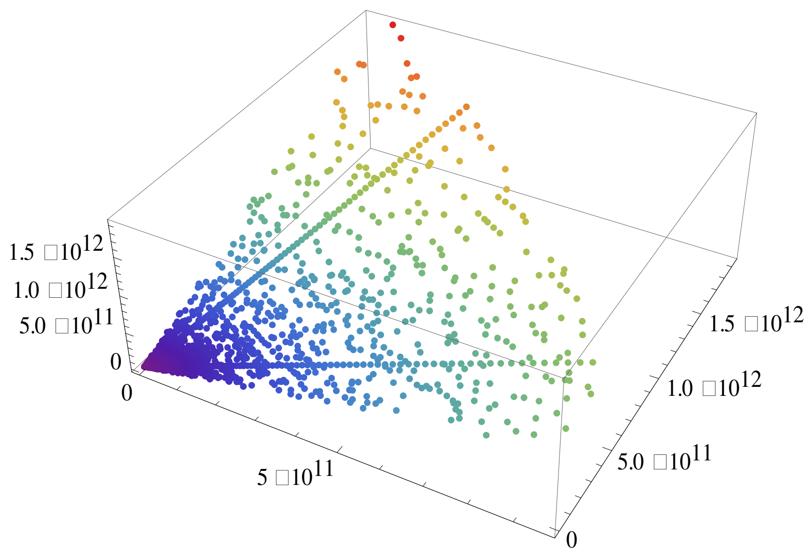
For $n < m$, $0 < n < 10^6$, $0 < m < 10^6$, $0 < \beta < 10^6$, we get the 1698 values for X, Y, Z. Plotting these values as

1. ListPlot3D: Figure 1
2. ListSurfacePlot3D: Figure2





3. ListPointPlot3D: Figure3



It is also observed that

1. $X + Y + Z = 0 \pmod 2$
2. $(Y + Z - X)^2 = 2(Y + Z)(Z - X)$
3. $(X + 2Y + Z)^2 = (Z - X)^2 + 4(X + Y)(Y + Z)$
4. $X + 2Y + Z \pm 2\{(X + Y)(Z - Y)\}^{1/2} = 0(\text{mod}16)$ or $= 0(\text{mod}4)$

In conclusion one may find other patterns of Pythagorean Triangles given by above stated problem.

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[2] Ivan Niven, Herbert S. Zuckerman; *An Introduction to the Theory of Numbers;* Wiley Eastern Limited; New Delhi; 1976; Page No. 106.

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