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ECCENTRICITY PROPERTIES OF $BG_4(G)$

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ABSTRACT

Let G be a simple (p, q) graph with vertex se V(G) and edge set E(G). $B_{\overline{G},NINC,\overline{K}q}(G)$ *is a graph with vertex set* $V(G)\cup E(G)$ *and two vertices are adjacent if and only if they correspond to two non adjacent vertices of G, a vertex and an edge not incident to it in G. For simplicity, denote this graph by* $BG_4(G)$*, Boolean graph fourth kind of G. In this paper, eccentricity properties of* $BG_4(G)$ *and its complement* $\overline{BG_4}(G)$ *are studied.*

Keywords: *Eccentricity, Boolean graph* $BG_4(G)$*.*

2010 Mathematics Subject Classification: *05C69, 05C12.*

1. INTRODUCTION

Let G be a finite, simple, undirected (p, q) graph with vertex set V(G) and edge set E(G). For a graph theoretic terminology refer to Harary [4], Buckley and Harary [3].

Let G be a connected graph and u be a vertex of G. The eccentricity e(v) of v is the distance to a vertex farthest from v. Thus, $e(v) = \max\{d(u, v) : u \in V\}$. The radius r(G) is the minimum eccentricity of the vertices, whereas the diameter diam(G) is the maximum eccentricity. For any connected graph G, $r(G) \leq diam(G) \leq 2r(G)$. v is a central vertex if e(v) = r(G). The center C(G) is the set of all central vertices. The central subgraph < C(G) > of a graph G is the subgraph induced by the center. v is a peripheral vertex if e(v) = diam(G). The periphery P(G) is the set of all such vertices. For a vertex v, each vertex at distance e(v) from v is an eccentric node of v.

A subgraph of G is a graph having all of its vertices and edges in G. It is a spanning subgraph if it contains all the vertices of G. If H is a subgraph of G, then G is a super graph of H. For any set S of vertices in G, the induced subgraph < S > is the maximal subgraph with vertex set S.

A graph G is complete if every pair of its vertices is adjacent. K_n denotes the complete graph on n vertices.

The complement $\overline{G}$ of a graph G is the graph with vertex set V(G) such that two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G. A self-complementary graph is isomorphic to its complement.

A graph G is connected if there is a path joining each pair of vertices. A component of a graph is a maximal connected subgraph. If a graph has only one component, then it is connected. Otherwise it is disconnected. The diameter diam(G) of a connected graph G is the length of any, longest geodesic (diametral path).

The Line graph L(G) of a graph G is the graph whose vertices correspond to the edges of G and two vertices of L(G) are adjacent if and only if the corresponding edges of G are adjacent.

The Total graph T(G) of a graph G is the graph whose vertices correspond to the set of vertices and edges of G and two vertices of T(G) are adjacent if and only if the corresponding elements of G are adjacent or incident.

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Let G be a simple (p, q) graph with vertex set V(G) and edge set E(G). $B_{\overline{G},NINC,\overline{K}q}(G)$ is a graph with vertex set $V(G) \cup E(G)$ and two vertices are adjacent if and only if they correspond to two non adjacent vertices of G, a vertex and an edge not incident to it in G. For simplicity, denote this graph by $BG_4(G)$, Boolean graph fourth kind of G. In [1] properties of $BG_4(G)$ is studied. The vertices of $BG_4(G)$, which are in V(G) are called point vertices and vertices in E(G) are called line vertices. $V(BG_4(G)) = V(G) \cup E(G)$, $E(BG_4(G)) = E(\overline{T}(G) - E(\overline{L}(G))$, where T(G) is the total graph of G and L(G) is the line graph of G.

In [2, 5, 6, 7] Janakiraman, Bhanumathi and Muthammai have defined and studied the properties of Boolean graphs. Motivated by this, here we study the eccentric properties of Boolean graph $BG_4(G)$.

2.1 Eccentricity Properties of $BG_4(G)$

In this section, radius and diameter of $BG_4(G)$ are found out. Throughout this section, if $v \in V(G)$ and $e \in E(G)$, the corresponding vertices of $BG_4(G)$ are denoted by v' and e'.

Observation 2.1.1: If G is totally disconnected then the eccentricity of every vertex in $BG_4(G)$ is one and $BG_4(G)$ is K_p.

Proposition 2.1.1: If $BG_4(G)$ is connected, eccentricity of a point vertex is 1, 2 or 3.

Proof: Consider a point v in V(G). If G has an isolated vertex then the eccentricity of that vertex in $BG_4(G)$ is one. Assume G has no isolated vertex.

To find $d(u', v')$ in $BG_4(G)$ where $u, v \in V(G)$. G has no isolated vertex. Take $u, v \in V(G)$. If u and v are non adjacent in G then $d(u', v') = 1$ in $BG_4(G)$. Suppose u and v are adjacent in G. If u and v have a common non incident edge or non adjacent vertex then we have a shortest path $u'w'v'$ or $u'e'v'$. Hence $d(u', v') = 2$.

Suppose u and v does not have a common non incident edge or vertex in G, but u has a non adjacent vertex w and v has a non incident edge e and vice versa. Then we have a shortest path $u'w'e'v'$. since w is non incident with e in G. Hence, $d(u', v') = 3$.

Suppose a vertex is adjacent to other vertices and incident with all the edges of G, then that vertex is isolated in $BG_4(G)$.

To find $d(v', e')$ in $BG_4(G)$ where $e \in E(G)$ and $v \in V(G)$.

If e is not incident with v in G then $d(v', e') = 1$ in $BG_4(G)$.

Suppose e is incident with v in G. Let $e = vv_1 \in E(G)$. In $BG_4(G)$, e' is not incident to v'. If there exists another vertex v_2, which is not adjacent to v in G, then $v'v_2'e'$ is a shortest path in $BG_4(G)$ and hence $d(v', e') = 2$ in $BG_4(G)$.

If there exists no such vertex, then $\deg_G v = p-1$ and if there exist non incident edge e_1 in G, then $v'e_1'v_3'e'$ (where v_3 is not incident to e_1 and e in G) is a shortest path and hence $d(v', e') = 3$ in $BG_4(G)$.

Suppose this is also not possible. That is, if $\deg_G v = p-1$ and if there does not exist non incident edge $e_1$in G, then that vertex is isolated vertex in $BG_4(G)$ that means $BG_4(G)$ is disconnected.

Hence, if $BG_4(G)$ is connected, then eccentricity of point vertices is 1, 2 or 3.

Proposition 2.1.2: If $BG_4(G)$ is connected, then the eccentricity of a line vertex is 2, 3 or 4.

Proof: From the previous theorem $d(e', v') = 1, 2$ or 3 in $BG_4(G)$.

Now to find $d(e_1', e_2')$ for $e_1, e_2 \in E(G)$

Case-(i): e_1 and e_2 are non adjacent

If there exist a vertex v which is not incident with both e_1 and e_2 then $e_1'v'e_2'$ is a shortest path and hence $d(e_1', e_2') = 2$ in $BG_4(G)$. If there exists no vertex v, not incident with both e_1 and e_2 and there is no edge adjacent to both e_1 and e_2. Consider the edges $e_1 = u_1v_1$ and $e_2 = u_2v_2$ in G, $e_1'v_2'u_1'e_2'$ is a shortest path in $BG_4(G)$ then $d(e_1', e_2') = 3$ in $BG_4(G)$.

Suppose u_1 and v_1 are adjacent to all other vertices in G and there are only four vertices in G, then $e_1'w_1'e'w_2'e_2'$ is a shortest path in $BG_4(G)$. Hence, $d(e_1', e_2') = 4$ in $BG_4(G)$.

Case-(ii): e_1 and e_2 are adjacent

Let $e_1 = u_1v_1$ and $e_2 = u_2v_2$ in G. e_1' is adjacent to v_2' and e_2' is adjacent to u_1' and also u_1' and v_2' also adjacent. Hence, $e_1'v_2'u_1'e_2'$ is a shortest path in $BG_4(G)$. Hence, $d(e_1', e_2') = 3$.

Hence, distance from e' to any other vertex is 1, 2, 3 or 4 in $BG_4(G)$. This implies that the eccentricity of a line vertex is 2, 3 or 4.

Theorem 2.1.1: Let G be a (p, q) graph. Eccentricity of a point vertex is one if and only if G has an isolated vertex.

Proof: Let u be an isolated vertex in G. Then the vertex u' is adjacent to all other line vertices and point vertices of $BG_4(G)$. Hence, deg $u' = p+q-1$ and eccentricity of a vertex u' is one in $BG_4(G)$.

On the other hand, assume that eccentricity of a point vertex u is one in $BG_4(G)$. This implies, deg $u = p+q-1$. Therefore, u is not adjacent to any vertex in G. Hence, u is an isolated vertex in G.

Note 2.1: If G has an isolated vertex then $BG_4(G)$ is connected and radius of $BG_4(G)$ is one.

Theorem 2.1.2: Let G and $BG_4(G)$ be connected. Eccentricity of a vertex is one in G and has at least one non incident edge in G if and only if eccentricity of that vertex in $BG_4(G)$ is three.

Proof: Let $v \in V(G)$ and $e \in E(G)$ which is non incident with v in G.

Assume $e(v) = 1$ in G. Then the vertex v is adjacent to all vertices of G. Hence v' is not adjacent to any point vertex in $BG_4(G)$ and v' is adjacent to e' in $BG_4(G)$.

By proposition 2.1.1 we have $d(v', u') = 3$ where u is adjacent to v and incident with e in G. Hence, $e(v') = 3$.

On the other hand, assume that eccentricity of a point vertex in $BG_4(G)$ is three. By proposition 2.1.1, we have $deg_G v = p-1$, and eccentricity of v is one in G. Suppose all the edges of G are incident with v, then v will be isolated in $BG_4(G)$.

So G has at least one edge that is not incident with v. Hence, the theorem is proved.

Theorem 2.1.3: Let G be a graph. If radius of G is greater than one then eccentricity of a point vertex in $BG_4(G)$ is two.

Proof: Let G be a connected graph. Assume $u \in V(G)$, v is not adjacent to u in G. This implies that, in $BG_4(G)$, u' and v' are adjacent.

Suppose u is adjacent to v in G and $w \in G$ is not adjacent to both u and v then $v'w'u'$ is a shortest path in $BG_4(G)$.

Hence, $d(u', v') \le 2$ and $d(v', e') \le 2$ since $e(v) \ne 1$ in G. Hence, eccentricity of a point vertex is two in $BG_4(G)$.

Theorem 2.1.4: Eccentricity of a line vertex is 2 in $BG_4(G)$ if and only if G has more than four vertices and $r(G) \ne 1$.

Proof: Let G be a graph with at least five vertices and $r(G) \ne 1$. In G, any two edges e_1 and e_2 have a common non incident vertex. Hence $e_1'v'e_2'$ is a shortest path from e_1' to e_2'.

Hence $d(e_1', e_2') = 2$ in $BG_4(G)$.

Every line vertex e' corresponding to $e = uv$ is adjacent to $p-2$ point vertices in $BG_4(G)$ and since $r(G) \ne 1$, every vertex u or v in G has at least one non adjacent vertex w in G. Then there exist shortest path $e'w'u'$ or $e'w'v'$ in $BG_4(G)$.

Therefore, $d(e', v') = 2$. Hence, eccentricity of a line vertex is 2.

On the other hand, assume that in $BG_4(G)$ the eccentricity of a line vertex is 2. Let $e = uv \in E(G)$, In $BG_4(G)$, $e(e') =2$. This implies that distance between e' and other line vertices are exactly 2. Hence, for any two edges e_1 and e_2 in G, there is a common non incident vertex. This implies $p \ge 5$.

Also, e(e′) =2 implies d(u′, e′) = 2 and d(v′, e′) = 2 in BG_4(G).which implies there exist w$\in$V(G) such that w is not adjacent to both u and v in G. (That is, u′w′e′, v′w′e′ are paths in BG4(G)). Thus r(G) > 1. Hence, the theorem is proved.

Theorem 2.1.5: G = K_4 if and only if eccentricity of each line vertex is four in BG_4(G).

Proof: Assume that G = K_4. Let V(G) = {v_1, v_2, v_3, v_4} and E(G) = {e_1, e_2, e_3, e_4, e_5, e_6}. Let $e_1 = v_1v_2$, $e_2 = v_2v_3$. A line vertex e_1' is adjacent to v_3' and v_4' and we have a shortest path $e_1'v_4'e_2'$ in BG_4(G). In BG_4(G), d(e_1', e_2') = 2 for adjacent edges in G.

For non adjacent edges we have a shortest path of distance 4. Hence, eccentricity of a line vertex is 4.

Conversely, assume eccentricity of every line vertex is four. Hence d(e_1',u′) ≤ 4 in BG_4(G) and d(e_1', e_2') ≤ 4 in BG_4(G) for u $\in$ V(G) and e_1, e_2 $\in$ E(G), $|V(G)| \le 4$ since eccentricity of a line vertex is four (by theorem 2.1.4). $\overline{G}$ has anyone edge, then eccentricity of a line vertex is three, which is a contradiction to the assumption. Hence, G = K_2, K_3 or K_4.

If G = K_2 or K_3 then BG_4(G) is disconnected. Hence, G = K_4.

Note 2.2: If G=K_4, eccentricity of every point vertex is 3 and eccentricity of every line vertex is 4 in BG_4(G). Therefore, $BG_4(K_4)$ is bi-eccentric with diameter 4.

Theorem 2.1.6: G ≠ K_4, p = 4 and G has at least two edges if and only if eccentricity of each line vertex is three in BG_4(G).

Proof: Assume that G has four vertices and G ≠ K_4. Therefore $\overline{G}$ has at least one edge. Any edge in G is non incident with other two points in G. Then the line vertex e′ is adjacent to non incident vertex u′ in BG_4(G).

If e_1 and e_2 are non adjacent, by proposition 2.2.2, d(e_1', e_2') = 3 in BG_4(G) since G ≠ K_4 and p = 4.

If e_1 and e_2 are adjacent then d(e_1', e_2') = 3 in BG_4(G).

If e_1 is non incident with v_1 then d(e_1', v_1') = 1 and e_1 is incident with v_2 then d(e_1', v_2') = 3. Hence, eccentricity of a line vertex is 3.

On the other hand, assume that eccentricity of each line vertex is three in BG_4(G). Hence d(e_1', u_1') ≤ 3 and d(e_1', e_2') ≤ 3 in BG_4(G), for u $\in$ V(G) and e_1, e_2 $\in$ E(G). Hence, by theorem 2.1.4, p≤ 4 and G ≠ K_4 by theorem 2.1.5. Suppose p ≤ 3, BG_4(G) is disconnected. Hence, G ≠ K_4 and p = 4.

Theorem 2.1.7: If G = K_p, p ≥ 5, then BG_4(G) is a bi eccentric graph with radius 2.

Proof: Let G = K_p, p ≥ 5.

Case-(i): To find d(v_1', v_2') in BG_4(G) where v_1, v_2 $\in$ V(G).

Any two vertices of G are adjacent in G. So the point vertices of BG_4(G) are non adjacent. Every pair of vertices have the common non incident edge e in G. Then there exist a shortest path $v_1'e'v_2'$ in BG_4(G). Hence d(v_1', v_2') = 2 in BG_4(G).

Case-(ii): To find d(v′, e′) in BG_4(G) where v $\in$ V(G) and e $\in$ E(G).

Suppose a vertex v is incident to an edge e in G. Let e = uv, $e_1 = uv_1$, $e_2 = v_2v_3$ be the edges of G. There exist a shortest path $v'e_2'v_1'e_1'$ in BG_4(G). Hence d(v′, e′) = 3 in BG_4(G).

Case-(iii): To find d(e_1', e_2') in BG_4(G) where e_1, e_2 $\in$ E(G).

$\overline{K_q}$ is an induced subgraph of BG_4(G). Thus it follows that d(e_1', e_2') ≠ 1. Every pair of edges has a common non incident vertex v in G since G has more than four vertices. Therefore, there exist a shortest path $e_1'v'e_2'$ in BG_4(G). Hence, d(e_1', e_2') = 2.

Hence, radius of BG_4(G) is two and diameter is three.

Note 2.3:

(i) If $G = K_3$, $BG_4(G)$ is a disconnected graph.

(ii) If $G = K_4$, the diameter of $BG_4(G)$ is four by theorem 2.1.5.

Theorem 2.1.8: If G is a graph with radius 1 and diameter 2, then $BG_4(G)$ has isolated vertex or $diam(BG_4(G)) = 3$.

Proof: Let G be a graph with radius 1 and diameter 2. Assume $e(v) = 1$ in G. Suppose v is incident with all the edges of G, then v' is isolated in $BG_4(G)$. If not, by proposition 2.1.1, $e(v') = 3$ in $BG_4(G)$ and $e(e') \leq 3$ for $G \neq K_n$. Hence, $diam(BG_4(G)) = 3$.

Theorem 2.1.9: If G is a 2-self centered graph with $p \geq 5$, then $BG_4(G)$ is 2-self centered.

Proof: Let G be a 2-self centered graph with $p \geq 5$. By theorem 2.1.4, in $BG_4(G)$ eccentricity of a point vertex is 2 since $r(G) \neq 1$. By theorem 2.1.4, in $BG_4(G)$ eccentricity of a line vertex is 2 since G has more than four vertices and $r(G) \neq 1$. Therefore, $BG_4(G)$ is a 2-self centered graph.

Cor 2.1.1: If G is a 2-self centered graph with four vertices, then $BG_4(G)$ is a bi-eccentric graph with radius 2.

Proof: Let G be a 2-self centered graph with four vertices. By theorem 2.1.2, eccentricity of a point vertex is 2 in $BG_4(G)$ and by theorem 2.1.6, eccentricity of a line vertex is 3 in $BG_4(G)$. Hence, $BG_4(G)$ is a bi-eccentric graph with radius 2.

Theorem 2.1.10: If G is a graph with radius 2 and diameter 3 then $BG_4(G)$ is a graph with radius 2 and diameter 3.

Proof: Let G be a graph with radius 2 and diameter 3. Consider the vertices $u, v \in V(G)$.

Case-(i): To find $d(u', v')$ in $BG_4(G)$ where $u, v \in V(G)$.

If u and v are non adjacent in G, then they are adjacent in $BG_4(G)$.

If u and v are adjacent in G and $e(u) = 3$ or $e(v) = 3$, then u and v have the common non incident vertex in G. Therefore, $d(u', v') = 2$ in $BG_4(G)$. If $e(u) = e(v) = 3$, then u and v have the common non incident vertex in G. Hence $d(u', v') = 2$ in $BG_4(G)$. If $e(u) = e(v) = 2$ and u and v does not have a common non incident edge and not have a common non adjacent vertex, then $d(u', v') = 3$ in $BG_4(G)$. If there exist a common non incident edge or common non adjacent vertex then $d(u', v') = 2$ in $BG_4(G)$.

Case-(ii): To find $d(u', e')$ in $BG_4(G)$ where $u \in V(G)$ and $e \in E(G)$.

If a vertex u is incident with an edge $e = uv$ in G, then $d(u', e') = 2$ in $BG_4(G)$ since $e(u) = 2$ or $e(v) = 3$ (that is the eccentric vertex of u in G is adjacent to u and e in $BG_4(G)$).

If u is not incident with e in G, then $d(u', e') = 1$ in $BG_4(G)$.

Case-(iii): To find $d(e_1', e_2')$ in $BG_4(G)$ where $e_1, e_2 \in E(G)$.

By proposition 2.1.2, $d(e_1', e_2') = 2$ or 3.

Hence, $BG_4(G)$ is a graph with radius 2 and diameter 3.

Theorem 2.1.11: If G is a graph with radius 2 and diameter 4, then $BG_4(G)$ is a 2-self centered graph.

Proof: Let G be a graph with radius 2 and diameter 4.

Case-(i): To find $d(u', v')$ in $BG_4(G)$ where $u, v \in V(G)$.

Let $uv = e \in E(G)$. The non adjacent vertices of G are adjacent in $BG_4(G)$. If the eccentricity of a vertex $v \in V(G)$ is 3 or 4, then u and v have the common non adjacent vertex w in G. Therefore, there exists a shortest path $u'w'v'$ in $BG_4(G)$. Hence $d(u', v') = 2$ in $BG_4(G)$.

If u and v have eccentricity 2, then $d(u', v') = 2$ or 3 in $BG_4(G)$. (The proof is similar as in theorem 2.1.10 case (i)). By theorem 2.1.9, $d(u', v') = 2$ in $BG_4(G)$.

Case-(ii): To find d(u′, e′) in $BG_4(G)$, where u $\in$ V(G), e$\in$E(G).

If u is not incident with e in G, then d(u′, e′) = 1 in $BG_4(G)$. If u is incident with e in G then d(u′, e′) = 2. (the proof is similar to theorem 2.1.10 case(ii)).

Case-(iii): To find d(e_1′, e_2′) in $BG_4(G)$, where e_1, $e_2$$\in$E(G).

The graph G has more than four vertices since diameter of G is four. By the theorem 2.1.4, eccentricity of a line vertex is 2.

Thus, the eccentricity of every point vertex and line vertex is two in $BG_4(G)$. Hence, $BG_4(G)$ is 2-self centered.

Theorem 2.1.12: If G is a graph with $r(G) \geq 3$, then $BG_4(G)$ is a 2-self centered graph.

Proof: Let G be a graph with $r(G) \geq 3$.

Case-(i): To find d(u′, v′) in $BG_4(G)$, where u, v$\in$V(G).

If u and v are non adjacent in G, then d(u′, v′) = 1 in $BG_4(G)$.

If u and v are adjacent in G, then u and v have a common non adjacent vertex in G.

By the definition, d(u′, v′) = 2 in $BG_4(G)$.

Case-(ii): To find d(u′, e′) in $BG_4(G)$, where u $\in$ V(G), e$\in$E(G).

If u is incident with an edge e in G, then u and e have the common non incident (adjacent) vertex in G. By proposition 2.1.2, we have d(u′, e′) = 2 in $BG_4(G)$.

If u is non incident with an edge e in G, then e′ and v′ are adjacent in $BG_4(G)$.

Case-(iii): To find d(e_1′, e_2′) in $BG_4(G)$, where $e_1 = u_1v_1$, $e_2 = u_2v_2$$\in$E(G).

G has more than five vertices since r(G) = 3. By theorem 2.1.4, d(e_1′, e_2′) = 2.

Therefore, e(u′) = 2 and e(e′) = 2 in $BG_4(G)$. Hence, $BG_4(G)$ is a 2-self centered graph.

2.2 Eccentricity properties of $\overline{BG_4}(G)$

In this section, radius and diameter of $\overline{BG_4}(G)$ are found out. Throughout this section, if v$\in$V(G) and e$\in$E(G), the corresponding vertices of $\overline{BG_4}(G)$ are denoted by v′ and e′.

Proposition 2.2.1: If $\overline{BG_4}(G)$ is connected, eccentricity of a point vertex is 1, 2 or 3.

Proof: Consider a point vertex v in V(G).

To find d(u′, v′) in $\overline{BG_4}(G)$ where u, v $\in$ V(G).

Case-(i): If a vertex u has degree p-1 and incident with all the edges of G, then in $\overline{BG_4}(G)$, u′ is adjacent to all the line vertices and point vertices. Hence, eccentricity of point vertex v is one.

Case-(ii): If a vertex u is adjacent to v in G then d(u′, v′)=1 in $\overline{BG_4}(G)$. If a vertex u is non adjacent to v in G, then u′e_1′e_2′v′ is a shortest path and d(u′, v′) = 3 in $\overline{BG_4}(G)$, where edge e_1 is incident with u and edge e_2 is incident with v in G.

To find $d(u', e')$ in $\overline{BG}_4(G)$ where $e \in E(G)$.

Case-(i): If a vertex u is incident with an edge e in G, then $d(u', e') = 1$ in $\overline{BG}_4(G)$.

Case-(ii): If a vertex u is not incident with an edge e in G, then there exist a shortest path $u'e_1'e'$ and $d(u', e') = 2$ in $\overline{BG}_4(G)$, where e_1 is incident with u in G.

Hence, distance from u' to any other vertex is 1, 2 or 3 in $\overline{BG}_4(G)$. Hence, the eccentricity of a point vertex is 1, 2 or 3.

Proposition 2.2.2: If $\overline{BG}_4(G)$ is connected, then the eccentricity of a line vertex is 1 or 2.

Proof: From the proposition 2.2.1 $d(e_1', v') = 1$ or 2.

Now to find $d(e_1', e_2')$ for $e_1, e_2 \in E(G)$.

K_q is an induced sub graph of $\overline{BG}_4(G)$. Hence, $d(e_1', e_2') = 1$. Hence the eccentricity of a line vertex is 1 or 2.

Observation 2.2.1: If a graph G has a vertex of degree $p-1$ and incident with all the edges of G, that is $G = K_{1, p-1}$, then radius of $\overline{BG}_4(G)$ is one.

Theorem 2.2.1: If $G = K_p$, then $\overline{BG}_4(G)$ is a two self centered graph.

Proof: Let $G = K_p$.

Case-(i): To find $d(v_1', v_2')$ in $\overline{BG}_4(G)$, where $v_1, v_2 \in V(G)$.

All the vertices of K_p are adjacent to each other and G is an induced sub graph of $\overline{BG}_4(G)$. Hence, $d(v_1', v_2') = 1$ in $\overline{BG}_4(G)$.

Case-(ii): To find $d(v', e')$ in $\overline{BG}_4(G)$, where $v \in V(G)$ and $e \in E(G)$.

If a vertex v is incident with an edge e, then $d(v', e') = 1$ in $\overline{BG}_4(G)$.

Suppose a vertex v is not incident with an edge e, then there exists a shortest path $v'e_1'e'$ in $\overline{BG}_4(G)$, where e_1 is incident with v in G. Therefore, $d(v', e') = 2$ in $\overline{BG}_4(G)$.

Case-(iii): To find $d(e_1', e_2')$ in $\overline{BG}_4(G)$ where $e_1, e_2 \in E(G)$.

Since K_q is an induced sub graph of $\overline{BG}_4(G)$. $d(e_1', e_2') = 1$ in $\overline{BG}_4(G)$.

Therefore, radius of $\overline{BG}_4(G)$ is a 2-self centered graph.

Theorem 2.2.2: If $G \neq K_{1, n}$ is a graph with radius 1 and diameter 2, then $\overline{BG}_4(G)$ is a 2-self centered graph.

Proof: Let G be a graph with radius 1 and diameter 2. Consider $v_1, v_2 \in V(G)$, $d(v_1', v_2') = 1$ in $\overline{BG}_4(G)$ if v_1 and v_2 are adjacent in G. $d(v_1', v_2') = 2$ in $\overline{BG}_4(G)$ if v_1 and v_2 are non adjacent in G since diameter of G is 2. $d(v_1', e') = 1$ in $\overline{BG}_4(G)$ if v_1 is non incident with an edge e in G since there exist a shortest path $v_1'e_1'e'$ in $\overline{BG}_4(G)$, where $e_1 = v_1v_2$ in G.

$d(e_1', e_2') = 1$ in $\overline{BG_4}(G)$ since K_q is an induced sub graph of G.

Therefore, $\overline{BG_4}(G)$ is a 2-self centered graph.

Theorem 2.2.3: If G is a 2-self centered graph then $\overline{BG_4}(G)$ is a 2-self centered graph.

Proof: Assume G is a 2-self centered graph.

Case-(i): To find $d(v_1', v_2')$ in $\overline{BG_4}(G)$ where $v_1, v_2 \in V(G)$.

Suppose v_1 and v_2 are adjacent in G, then $d(v_1', v_2') = 1$ in $\overline{BG_4}(G)$. Suppose v_1 and v_2 are non adjacent in G, $d(v_1', v_2') = 2$ in $\overline{BG_4}(G)$, since G is an induced sub graph of $\overline{BG_4}(G)$.

Case-(ii): To find $d(v', e')$ in $\overline{BG_4}(G)$, where $v \in V(G)$ and $e \in E(G)$.

If a vertex v is incident with e in G, then $d(v', e') = 1$ in $\overline{BG_4}(G)$.

If a vertex v is not incident with e in G, then there exists a shortest path $v'e_1'e'$ in $\overline{BG_4}(G)$ where e_1 is incident with v in G. Therefore, $d(v', e') = 2$ in $\overline{BG_4}(G)$.

Case-(iii): To find $d(e_1', e_2')$ in $\overline{BG_4}(G)$, where $e_1, e_2 \in E(G)$.

In $\overline{BG_4}(G)$, K_q is an induced sub graph. Hence, $d(e_1', e_2') = 1$ in $\overline{BG_4}(G)$.

Therefore, $\overline{BG_4}(G)$ is a 2-self centered graph.

Theorem 2.2.4: If G is a graph with radius 2 and diameter 3, then $\overline{BG_4}(G)$ is a graph with radius 2 and diameter 3.

Proof: Assume a graph G with radius 2 and diameter 3.

Case-(i): To find $d(v_1', v_2')$ in $\overline{BG_4}(G)$ where $v_1, v_2 \in V(G)$.

The adjacent vertices of G are adjacent in $\overline{BG_4}(G)$. Consider a vertex v such that $e(v) = 2$ and w is an eccentric vertex of v. Then, $d(v', w') = 2$ in $\overline{BG_4}(G)$, otherwise $d(u', v') = 3$ in $\overline{BG_4}(G)$ since u and v are at distance 3 in G.

Case-(ii): To find $d(v', e')$ in $\overline{BG_4}(G)$ where $v \in V(G)$ and $e \in E(G)$.

By proposition 2.2.1, $d(v', e') = 1$ or 2 in $\overline{BG_4}(G)$.

Case-(iii): To find $d(e_1', e_2')$ in $\overline{BG_4}(G)$ where $e_1, e_2 \in E(G)$.

In $\overline{BG_4}(G)$ $d(e_1', e_2') = 1$ since K_q is an induced sub graph of $\overline{BG_4}(G)$.

Hence, $\overline{BG_4}(G)$ is a graph with radius 2 and diameter 3.

Theorem 2.2.5: If G is a graph with radius 2 and diameter 4, then $\overline{BG_4}(G)$ is a bi-eccentric graph with radius two.

Proof: Let G be a graph with radius 2 and diameter 4.

Case-(i): To find $d(v_1', v_2')$ in $\overline{BG_4(G)}$ where $v_1, v_2 \in V(G)$.

If v_1 and v_2 are adjacent in G, then $d(v_1', v_2') = 1$ in $\overline{BG_4(G)}$.

If v_1 and v_2 are non-adjacent in G. Assume $e(v'_1) = 2$ in G, then $e(v_1') = 2$ in $\overline{BG_4(G)}$. If $e(v'_1) = 3$ in G, then $e(v'_1) = 3$ in $\overline{BG_4(G)}$ by the proposition 2.2.1.

Case-(ii): To find $d(v', e')$ in $\overline{BG_4(G)}$ where $v \in V(G)$ and $e \in E(G)$.

By proposition 2.2.1, $d(v', e') = 1$ or 2 in $\overline{BG_4(G)}$.

Case-(iii): To find $d(e_1', e_2')$ in $\overline{BG_4(G)}$ where $e_1, e_2 \in E(G)$.

K_q is an induced sub graph of $\overline{BG_4(G)}$. Thus $e(e') = 1$ in $\overline{BG_4(G)}$.

Therefore, $\overline{BG_4(G)}$ is a bi-eccentric graph with radius two.

Theorem: 2.2.6 If G is a connected graph $p \geq 3$ with radius greater than 2, then $\overline{BG_4(G)}$ is a bi-eccentric graph with radius 2.

Proof: Let G be a graph with $r(G) \geq 3$.

Case-(i): To find $d(v_1', v_2')$ in $\overline{BG_4(G)}$, where $v_1, v_2 \in V(G)$.

Consider $v_1, v_2 \in V(G)$ and $e_1, e_2 \in E(G)$. If v_1 and v_2 are adjacent, then $d(v_1', v_2') = 1$ in $\overline{BG_4(G)}$.

If v_1 is an eccentric vertex of v_2, then $d(v_1', v_2') = 3$ in $\overline{BG_4(G)}$ since there exist a shortest path $v_1'e_1'e_2'v_2'$ in $\overline{BG_4(G)}$, where $e_1 = v_1w_1$ and $e_2 = v_2w_2$ in G. Hence $d(v_1', v_2') = 3$ in $\overline{BG_4(G)}$.

Case-(ii): To find $d(v', e')$ in $\overline{BG_4(G)}$, where $v \in V(G)$ and $e \in E(G)$.

If a vertex v is incident with an edge e in G, then $d(v', e') = 1$ in $\overline{BG_4(G)}$.

If a vertex v is non-incident with an edge e in G, then there exist a shortest path $v'e_1'e'$ in $\overline{BG_4(G)}$, where $e_1 = vv_1$ in G. Hence, $d(v', e') = 2$ in $\overline{BG_4(G)}$.

Case-(iii): To find $d(e_1', e_2')$ in $\overline{BG_4(G)}$, where $e_1 = uv$, $e_2 = u_1v_1 \in E(G)$.

$d(e_1', e_2') = 1$ in $\overline{BG_4(G)}$ since K_q is an induced sub graph of $\overline{BG_4(G)}$. Hence, eccentricity of a line vertex is 2.

Therefore, $\overline{BG_4(G)}$ is a bi-eccentric graph with radius 2.

CONCLUSION

It is proved that either $BG_4(G)$ is disconnected or it is a graph of diameter at most 4. Also, we have characterized graphs G for which $BG_4(G)$, $\overline{BG_4(G)}$ are 2-self centered, bi-eccentric, etc.

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