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γ-OPEN SETS AND DECOMPOSITION OF CONTINUITY VIA GRILLS

C. Janaki*

Asst. Professor, Dept. of Mathematics, L. R. G Govt. Arts College for Women, Tirupur-4

E-mail: janakicsekar@yahoo.com

I. Arockiarani**

*****Associate Professor, Dept. of Mathematics, Nirmala College for Women, Coimbatore-18***

*****E-mail: stell11960@yahoo.co.in***

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ABSTRACT

In this paper, we apply the notion of γ-open sets to define a new class of sets via grills and investigate their properties with the existing sets. This concept is further extended to define Gγ-sets to obtain a new decomposition of continuity.*

Keywords: *G-γ-open sets, G-γ-continuous functions, Gγ*-sets, Gγ*-continuous function, G-γ-open function.*

AMS Subject classification*: 54C10, 54C08, 54C05.*

1. INTRODUCTION:

The idea of grill on a topological space was first introduced by Choquet [1] in 1947.The concept of grills has shown to be a powerful supporting and useful tool like nets and filters for many topological investigations. In [4] Roy and Mukerjee defined a new topology associated naturally to the existing topology and a grill on a given topological space. In [3], Ravi and Ganesan have defined and studied G-α-open sets and G-α-continuous functions in grill topological spaces. In this paper, we introduce G-γ-open sets, G-γ-continuous functions, Gγ*-sets, Gγ*-continuous functions and investigate the relation between such sets with other grill sets (functions) and obtain a decomposition of continuity.

2. PRELIMINARIES:

Throughout this paper , (X,τ) or X represent a topological space on which no separation axioms are assumed unless otherwise mentioned .For a subset A of a space X, Cl(A) and Int(A) denote the closure and the interior of A respectively. The power set of X will be denoted by $\wp(X)$.A collection G of a non-empty subsets of a space X is called a grill [1] on X if

(1) $A \in G$ and $A \subset B \Rightarrow B \in G$

(2) $A, B \subset X$ and $A \cup B \in G \Rightarrow A \in G$ or $B \in G$.

For any point x of a topological space(X,τ), τ(x) denote the collection of all open neighborhoods of x.

Definition 2.1: [4] Let (X,τ) be a topological space and G be a grill on X. The mapping $\Phi: \wp(X)\to \wp(X)$,denoted by $\Phi_G(A, \tau)$ for $A\in \wp(X)$ or simply Φ (A) called the operator with the grill G and the topology τ and is defined by

$\Phi_G(A, \tau) = \{x\in X | A\cap U \in G, \forall U\in \tau(x)\}$

Result 2.2[4]: Let G be a grill on a space X. Then a map $\Psi: \wp(X)\to \wp(X)$ is defined by Ψ(A) = A ∪ Φ (A), for all $A\in \wp(X)$.The map Ψ satisfies Kuratowski closure axioms. Corresponding to a grill G on a topological space (X,τ), there exists a unique topology τ_G on X given by $\tau_G = \{U\subset X | \Psi(X-U) = X-U\}$,

***Corresponding author: C. Janaki*, *E-mail: janakicsekar@yahoo.com**

where for any $A \subset X$, $\Psi(A) = A \cup \Phi(A) = \tau_G\text{-}Cl(A)$.

For any grill G on a topological space (X,τ), $\tau \subset \tau_G$. If (X,τ) is a topological space and G is a grill on X, then (X,τ, G) denote a grill topological space.

Theorem 2.3: [4]

(1)If G_1 and G_2 are two grills on a space X with $G_1 \subset G_2$, then $\tau_{G_1} \subset \tau_{G_2}$.

(2)If G is a grill on a space X and $B \notin G$, then B is closed in (X,τ, G).

(3)For any subset A of a space X and any grill G on X, $\Phi(A)$ is τ_G-closed.

Theorem2.4: [4] Let (X,τ) be a topological space and G be any grill on X. Then

(1) $A \subseteq B\ (\subseteq X) \Rightarrow \Phi(A) \subseteq \Phi(B)$;

(2) $A \subseteq X$ and $A \notin G \Rightarrow \Phi(A) = \phi$;

(3) $\Phi(\Phi(A)) \subseteq \Phi(A) = Cl(\Phi(A)) \subseteq Cl(A)$, for any $A \subseteq X$;

(4) $\Phi(A \cup B) = \Phi(A) \cup \Phi(B)$;

(5) $A \subseteq \Phi(A) \Rightarrow Cl(A) = \tau_G\text{-}Cl(A) = Cl(\Phi(A)) = \Phi(A)$;

(6) $U \in \tau$ and $\tau \setminus \{\phi\} \subseteq G \Rightarrow U \subseteq \Phi(U)$;

(7) If $U \in \tau$ then $U \cap \Phi(A) = U \cap \Phi(U \cap A)$, for any $A \subseteq X$.

Theorem2.5: [4] Let (X,τ) be a topological space and G be any grill on X. Then for any $A, B \subseteq X$,

(1) $A \subseteq \Psi(A)$;

(2) $\Psi(\phi) = \phi$;

(3) $\Psi(A \cup B) = \Psi(A) \cup \Psi(B)$;

(4) $Int(A) \subseteq Int(\Psi(A))$;

(5) $Int(\Psi(A \cap B)) \subseteq Int(\Psi(A))$;

(6) $Int(\Psi(A \cap B)) \subseteq Int(\Psi(B))$;

(7) $Int(\Psi(A)) \subseteq \Psi(A)$;

(8) $A \subseteq B \Rightarrow \Psi(A) \subseteq \Psi(B)$.

Theorem 2.6:[4] Let (X,τ) be a topological space and G be any grill on X. Then for any $A, B \subseteq X$,

(1) $\Phi(A) \subseteq \Psi(A) = \tau_G\text{-}Cl(A) \subseteq Cl(A)$;

(2) $A \cup \Psi(Int(A)) \subseteq Cl(A)$;

(3) $A \subseteq \Phi(A)$ and $B \subseteq \Phi(B) \Rightarrow \Psi(A \cap B) \subseteq \Psi(A) \cap \Psi(B)$.

Definition 2.7: Let (X,τ) be a topological space and G be any grill on X. A subset A in X is said to be

(1) Φ-open set [2] if $A \subseteq Int(\Phi(A))$;

(2) g-set [2] if $Int(\Psi(A)) = Int(A)$;

(3) G-preopen set [2] if $A \subseteq Int(\Psi(A))$;

(4) G-α-open set [3] if $A \subseteq Int(\Psi(Int(A)))$;

(5) G-semiopen set if $A \subseteq \Psi(IntA)$.

Definition 2.8:[2] Let (X,τ,G) be a grill topological space and $A\subseteq X$ is called G-set if $A = U\cap V$ where $U\in\tau$ and V is a g-set in (X, τ, G).

Definition 2.9:[2] A function $f: (X,\tau,G)\rightarrow(Y,\sigma)$ is said to be G-continuous function(resp. G-precontinuous, G- α-continuous function, G-semicontinuous function) if for each open set V in Y, $f^{-1}(V)$ is G-open set(resp. G-preopen, G-α-open, G-semiopen)

3. G-γ-OPEN SETS:

Definition 3.1: Let G be a grill on a topological space (X,τ).A set $A \subseteq X$ is called G-γ-open set if $A\subseteq Int(\Psi(A)) \cup \Psi(Int(A))$. The complement of such a set is called G-γ-closed set.

Proposition 3.2: Let G be a grill on a topological space (X,τ). If $\tau \setminus \{\phi\}\subseteq G$ then
(1) Every G-open set is G-γ-open.

(2) Every G-preopen set is G-γ-open

(3) Every G-α-open set is G-γ-open.

(4) Every G-semiopen set is G-γ-open.

(5) Every G-γ-open set is γ-open.

Remark 3.3: Converse of the above need not be true as seen in the following examples.

Example3.4: Let $X = \{a, b, c\}$ and $\tau = \{\phi,X,\{a\},\{b, c\}\}$.If $G =\{X,\{a\},\{b\},\{a, b\},\{b, c\}\}$,then G is a grill on X such that $\tau \setminus \{\phi\}\subset G$. Let $A = \{a, b\}$.Then A is G-γ-open but not G-semiopen set, G-α-open set.

Example 3.5: Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\},\{b\},\{a, b\}\}$. If $G =\{X,\{a\},\{b\},\{a, b\},\{b, c\}\}$,then G is a grill on X such that $\tau \setminus \{\phi\}\subset G$. Let $A = \{a, c\}$.Then A is G-γ-open but not G-preopen set, G-open set.

Example3.6: Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, X, \{d\},\{a, c\},\{a, c, d\}\}$. If $G = \{X,\{a\},\{d\},\{a, c\},\{a, c, d\}\}$,then G is a grill on X such that $\tau \setminus \{\phi\}\subset G$. Let $A = \{a, b, c\}$.Then A is γ-open but not G-γ-open set.

Proposition 3.7: Let A be G-γ-open set such that $Int(A) = \phi$.Then A is G-preopen set.

Proof: Since $A\subseteq Int(\Psi(A)) \cup \Psi(IntA) \subseteq Int(\Psi(A))$ it follows A is G-preopen set.

Proposition3.8: Let G be a grill on a topological space (X,τ). If $\tau \setminus \{\phi\}\subseteq G$, then every open set is G-γ-open set.

Proof: $U\in\tau$ and $\tau \setminus \{\phi\}\subseteq G \Rightarrow U\subseteq \Phi(U)$. Hence $U\subseteq Int(\Phi(U)) \subseteq Int(\Psi(U)) \subseteq Int(\Psi(U)) \cup \Psi(Int(U)$ implies U is G-γ-open set.

Proposition 3.9: Each G-γ-open set which is τ_G-closed is G-semiopen set.

Proof: Let A be G-γ-open set and τ_G-closed. Then $A\subseteq Int(\Psi(A)) \cup \Psi(IntA)$.Since A is τ_G-closed $\Psi(A) = A$. Hence $A\subseteq Int((A)) \cup \Psi(Int(A)) \subseteq \Psi(Int(A))$ implies A is G-semiopen set.

Proposition 3.10: Let (X,τ, G) be a grill topological space and A,B be subsets of X.

(i) If $A_i\in$ G-γ-open set for each $i\in\Delta$ then $\cup\{A_i: i\in\Delta\}$ is G-γ-open set.

(ii) If A is G-γ-open set and $U\in\tau$ then $A\cap U$ is G-γ-open set.

Proof:
(i) Let A_i be G-γ-open set. then $A_i\subseteq Int(\Psi(A_i)) \cup \Psi(Int(A_i))\ \forall\ i\in\Delta$

$$\begin{aligned}\text{Thus } \cup_{i\in\Delta}A_i &\subseteq \cup_{i\in\Delta}\left(Int(\Psi(A_i)) \cup \Psi(Int(A_i))\right)\\ &\subseteq Int((\cup_{i\in\Delta}A_i)\cup \Phi(\cup_{i\in\Delta}(A_i))) \cup Int((\cup_{i\in\Delta} A_i) \cup\Phi(Int(\cup_{i\in\Delta}A_i))\\ &\subseteq Int(\Psi(\cup_{i\in\Delta}A_i)) \cup\Psi(Int(\cup_{i\in\Delta}A_i)).\end{aligned}$$

(ii) Let A be G-γ-open set and $U\in \tau$. Then $A \subseteq \text{Int}(\Psi (A)) \cup \Psi (\text{Int}(A))$.So

$A \cap U \subset (\Psi (\text{Int}(A)) \cup \text{Int}(\Psi (A)))\cap U \subset (U\cap(\text{Int}(A) \cup\Phi(\text{Int}(A))) \cup (U\cap\text{Int}(A\cup\Phi(A)))$

$\subset ((U\cap\text{Int}(A)) \cup (U\cap\Phi(\text{Int}(A))) \cup(\text{Int}(U\cap (A\cup\Phi(A))))$

$\subset (\text{Int}(U\cap A)) \cup(U\cap\Phi(\text{Int}(A) \cap U)) \cup (\text{Int}((U\cap A) \cup (U\cap\Phi(A)))$

$\subset (\text{Int}(U\cap A) \cup (U\cap\Phi(\text{Int}(A\cap U))) \cup \text{Int}((U\cap A) \cup (U\cap\Phi(A\cap U)))$

$\subset (\text{Int}(U\cap A) \cup\Phi (\text{Int}(A\cap U))) \cup \text{Int}((U\cap A) \cup (\Phi(A\cap U)))$

$\subset \Psi(\text{Int}(U\cap A)) \cup\text{Int}(\Psi(U\cap A))$.Hence $A\cap U$ is G-γ-open set.

Remark 3.11: Intersection of two G-γ-open sets is not G-γ-open set.

Example 3.12: Let X = {a, b, c, d}} and τ = {ϕ, X,{ a, b, d}}. If G = {X,{a, b, d},{a},{a, d} }, then G is a grill on X such that $\tau \setminus \{\phi\}\subset G$. Then {a, b, d} and {a, b, c} are G-γ-open sets but {a, b} is not G-γ-open set.

Definition 3.13: Let (X,τ,G) be a grill topological space and $A\subset X$ is called Gγ*-set if $A = U\cap V$ where $U\in\tau$ and V is a g-set in (X,τ,G) and $\text{Int}(\Psi(V)) = \Psi(\text{Int}(V))$.

Remark 3.14: Every open set is Gγ*-set.

Converse need be true as seen in the following example

Example 3.15: Let X = {a, b, c} and τ = {ϕ, X,{a},{b, c}}. If G ={X,{a},{b},{a, b},{a, c},{b, c}},then G is a grill on X such that $\tau \setminus \{\phi\}\subset G$. Take A = {a,c}, then A is a Gγ*-set but not an open set.

Remark 3.16: G-γ-open set and Gγ*-set are independent of each other.

Example 3.17: In example 3.15, A = {a, c} is a Gγ*-set but not a G-γ-open set and in example 3.4 A = {a, b} is G-γ-open set but not an Gγ*-set.

Proposition 3.18: For a subset $A\subset (X,\tau, G)$ the following conditions are equivalent:

(a) A is open

(b) A is G-γ-open set and Gγ*-set .

Proof: a ⇒b Obvious

b⇒a: Let A be G-γ-open set and Gγ*-set . Since A is Gγ*-set and G-γ-open set we have

$A = U\cap V\subset \Psi(\text{Int}(U\cap V)) \cup \text{Int}(\Psi(U\cap V))$ where U is open and V is a g-set and $\text{Int}(\Psi(V)) = \Psi(\text{Int}(V))$.

Now $A = (U\cap V) \cap U \subset (\Psi(\text{Int}(U\cap V))\cup\text{Int}(\Psi(U\cap V))) \cap U\subset(\Psi(\text{Int}(U\cap V)) \cap U) \cup(\text{Int}(\Psi(U\cap V)) \cap U) \subset (\Psi(\text{Int}(V)) \cap U) \cup (\text{Int}(\Psi(U\cap V) \cap\Psi(U\cap V)) \cap U) \subset (\text{Int}(\Psi (V)) \cap U) \cup (\text{Int}(\Psi(U) \cap\Psi(V)) \cap U) \subset (\text{Int}(\Psi (V)) \cap U) \cup ((\text{Int}(\Psi(U)) \cap\text{Int}(\Psi(V))) \cap U)\subset (\text{Int}(\Psi (V)) \cap U) \cup ((\text{Int}(\Psi(U)) \cap\text{Int}(\Psi(V))) \cap U)\subset (\text{Int}(V) \cap U) \cup (\text{Int}(\Psi(U)) \cap\text{Int}(V) \cap U) \subset (\text{Int}(V) \cap U) \cup (\text{Int}((U\cup\Phi(U))) \cap\text{Int}(V) \cap U)$
$\subset \text{Int}(V \cap U) \cup (\text{Int}((\text{Cl}(U)\cap U)) \cap\text{Int}(V) \} \subset \text{Int}(V \cap U) \subset \text{Int} (A)$.

Hence A is open.

Proposition 3.19: If A is both regular open and G-γ-open set then it is closed.

Proof: $A \subseteq \text{Int}(\Psi (A)) \cup \Psi (\text{Int}A) \subseteq \text{Int}(A\cup\Phi (A)) \cup \Psi (A) \subseteq \text{Int}(A\cup \text{Cl} (A)) \cup \text{Cl} (A) \subseteq \text{Int}(\text{Cl} (A)) \cup \text{Cl} (A) \subseteq A\cup \text{Cl} (A) \subseteq \text{Cl} (A)$.Hence A is closed.

Proposition 3.20: Every Gγ*-set is G-set.

4. DECOMPOSITION OF CONTINUITY:

Definition 4.1: A function $f: (X,\tau,G)\rightarrow(Y,\sigma)$ is said to be G-γ-continuous function if for each open set V in Y, $f^{-1}(V)$ is G-γ-open set

Definition 4.2: A function f: (X,τ, G)→(Y,σ) is said to be Gγ*-continuous function if for each open set V in Y, $f^{-1}(V)$ is Gγ* set.

Proposition 4.3:

(a) Every G-continuous function is G-γ-continuous function.

(b) Every G-precontinuous function is G-γ-continuous function.

(c) Every G-α-continuous function is G-γ-continuous function.

(d) Every G-semicontinuous function is G-γ-continuous function.

(e) Every G-γ-continuous function is γ-continuous function

Proof: Obvious. Converse need not be true can be seen in the following examples.

Example 4.4: Let X = {a, b, c} and τ = {ϕ,X,{a},{b, c}}.If G ={X,{a},{b},{a,b},{b,c}},then G is a grill on X such that τ \ {ϕ}⊂ G. Let Y = {a,b} with topology σ = { ϕ,Y,{a}}.

Let f: (X,τ, G)→(Y,σ) be defined by f(a) = a ,f(b) = a and f(c) = b .Then f is G-γ-continuous but it is neither G-semicontinuous nor G-α-continuous.

Example 4.5: Let X ={a, b, c} and τ = {ϕ, X,{a},{b},{a, b}}. If G ={X,{a},{b},{a, b},{b, c}},then G is a grill on X such that τ \ {ϕ}⊂ G. Let Y = {a, b} with topology σ = { ϕ,Y,{a}}.

Let f: (X,τ,G)→(Y,σ) be defined by f(a) = a ,f(b) = b and f(c) = a .Then f is G-γ-continuous but it is neither G-precontinuous nor G-continuous.

Example 4.6: Let X = {a, b, c, d} and τ = {ϕ, X,{d},{a, c},{a, c, d}}. If G ={X, {a}, {d}, {a, c}, {a, c, d}}, then G is a grill on X such that τ \ {ϕ}⊂ G. Let Y = {a, b, c} with topology σ = { ϕ, Y,{a, b, c}} Let f: (X,τ,G)→(Y,σ) be the identity function . Then f is γ-continuous but not G-γ-continuous.

Proposition 4.7: Every continuous function is Gγ*-continuous function.

Remark 4.8: Converse of the above need not be true as seen in the following example.

Example 4.9: Let X = {a, b, c} and τ = {ϕ,X,{a},{b, c}}.If G ={X,{a},{b},{a, b},{a, c},{b, c}},then G is a grill on X such that τ \ {ϕ}⊂ G.

Let Y = {a, b} with topology σ = { ϕ,Y,{a}}, Let f: (X,τ,G)→(Y,σ) be defined by f(a) = a ,f(b) = b and f(c) = a. Then f is Gγ*-continuous function but not continuous.

Proposition 4.10: A function f: (X,τ, G)→(Y,σ) is continuous if and only if it is G-γ-continuous and Gγ*-continuous.

Proof: It follows from proposition 3.18.

Proposition 4.11: The following statements are equivalent for a function f: (X,τ, G)→(Y,σ):

(a) f is G-γ-continuous;

(b) for each x∈ X and each open set V in Y with f(x) ∈ V, there exists a G-γ-open set U containing x such that f(U)⊂V;

(c) for each x∈ X and each open set V in Y with f(x) ∈ V, $f^{-1}(V)$ is a G-γ-neighborhood of x.

(d) the inverse image of each closed set in (Y,σ) is G-γ-closed.

Proof: a⇒b: Let x ∈ X and V be an open set in Y such that f(x) ∈ V. Since f is G-γ-continuous, $f^{-1}(V)$ is G-γ-open.

Put U = $f^{-1}(V)$, then U is a G-γ-open set containing x such that f(U)⊂V.

b⇒c: Let V be an open set in Y such that $f(x) \in V$. Then by assumption, there exists a G-γ-open set U containing x such that $f(U) \subset V$. So $x \in U \subset f^{-1}(V)$. Hence $f^{-1}(V)$) is a G-γ-neighborhood of x

c⇒a: Let V be an open set in Y such that $f(x) \in V$. Then by c) $f^{-1}(V)$ is a G-γ-neighborhood of x. Thus for each $x \in f^{-1}(V)$, there exists a G-γ-open set U_x containing x such that $x \in U_x \subset f^{-1}(V)$.Hence $f^{-1}(V) \subset \cup_{x \in f-1(V)} U_x$.

So $f^{-1}(V)$) is G-γ-open set.
a⇔d:Obvious.

Proposition 4.12: Let f: $(X,\tau,G_1) \to (Y,\sigma,G_2)$ and g: $(Y,\sigma,G_2) \to (Z,\eta)$ be two functions where G_1, G_2 are grills on X and Y respectively. Then $g \circ f$ is G-γ-continuous if f is G-γ-continuous and g is continuous.

Proof: Straight forward

Definition 4.13: A grill topological space (X,τ, G) is said to G-γ-connected if X is not the union of two disjoint G-γ-open subsets of X.

Definition 4.14: A grill topological space (X,τ, G) is said to G-γ-normal if for each pair of non-empty disjoint closed sets of X ,it can be separated by disjoint G-γ-open sets.

Theorem 4.15: A G-γ-continuous image of a G-γ-connected space is connected.

Proof: Let f: $(X,\tau,G) \to (Y,\sigma)$ be G-γ-continuous function of a G-γ-connected space X on to a topological space Y. Suppose Y is connected .Then A and B are clopen and $Y = A \cup B$, where $A \cap B = \phi$.Since f is G-γ- continuous,
$X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$,$f^{-1}(B)$ are non-empty G-γ-open sets in X. Also $f^{-1}(A) \cap f^{-1}(B) = \phi$.

Hence X is not G-γ-connected which is a contradiction. Therefore, Y is connected.

Theorem 4.16: If f: $(X,\tau, G) \to (Y,\sigma)$ is G-γ-continuous, closed injective and Y is normal, then X is G-γ-normal.

Proof: Let F_1 and F_2 be disjoint closed subsets of X. Since f is closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint closed subsets of Y. Since Y is normal, $f(F_1)$ and $f(F_2)$are separated by disjoint open sets V_1 and V_2 respectively.$f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint G-γ-open sets containing F_1 and F_2.Hence X is G-γ-normal.

Definition 4.17: A function f: $(X, \tau, G_1) \to (Y, \sigma, G_2)$ is called G-γ-open(resp. G-γ-closed) if for each $U \in \tau$ (resp.closed set F), f(U) is G-γ-open(resp. G-γ-closed).

Theorem 4.18: A function f: $(X, \tau, G_1) \to (Y, \sigma, G_2)$ is G-γ-open if and only if for for each $x \in X$ and each neighborhood U of x, there exists G-γ-open set V in Y containing f(x) such that $V \subset f(U)$.

Proof: Suppose f is G-γ-open function. For each $x \in X$ and each neighborhood U of x, there exists $U_x \in \tau$ such that $x \in U_x \subset U$. Since f is G-γ-open, $V = f(U_x)$ is G-γ-open set in Y and hence $f(x) \in V \subset f(U)$.Conversely,

Let U be an open set of (X,τ).For $x \in U$, there exists G-γ-open set V_x in Y containing f(x) such that $f(x) \in V_x \subset f(U)$.

Therefore, we obtain $f(U) = \cup\{ V_x : x \in U\}$ and hence f(U) is G-γ-open and hence f is G-γ-open.

Theorem 4.19: Let f: $(X, \tau, G_1) \to (Y, \sigma, G_2)$ be G-γ-open. If W is any subset of Y and F is closed subset of X containing $f^{-1}(W)$,then there exist a G-γ-closed subset H of Y containing W such that $f^{-1}(H) \subset F$.

Proof: Suppose f is G-γ-open function. Let W be any subset of Y and F a closed subset of X containing $f^{-1}(W)$.Then X–F is open. Since f is G-γ-open, f(X–F) is G-γ-open.

Hence H = Y–f(X–F) is G-γ-closed. It follows from $f^{-1}(W) \subset F$ that $f^{-1}(H) \subset F$.

Theorem4.20: For any bijective function f: $(X, \tau) \to (Y, \sigma, G)$, the following are equivalent:

(I) f^{-1} :$(Y, \sigma, G) \to (X, \tau)$ is G-γ-continuous.

(ii) f is G-γ-open.

(iii) f is G-γ-closed.

Proof: Straight forward.

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