

ON INFRA TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduced and investigate infra-topological spaces which deduced from topological spaces and studies the properties of subsets of infra topological spaces such as infra topological space, infra derived set, infra-interior set, infra-closure set, infra-exterior set and infra-boundary set.

Keywords: infra - topological space, infra -derived set, infra- interior point, infra- closure, infra-exterior point and infra-boundary set.

1. INTRODUCTION

In 1983, A.S.Mashhour *et al.* [1] introduced the supra topological space and studied s -ontinuous functions and s^* -continuous functions. In this paper we introduced Infra -Topological Space (ITS) and analogue concepts associated with infra- topological space. Such as, infra- derived set (resp.infra-closure, infra-interior, infra-exterior and infra-boundary) of subset A of infra-space X . we will be denoted by $ids(A)$ (resp. $icp(A)$, $iip(A)$, $iep(A)$ and $ibp(A)$). Many results of topologic space remain valid in infra- topological space, Whereas some become invalid in infra-topological space.

2. INFRA -TOPOLOGICAL SPACES

Definition 2.1: Let X be any arbitrary set. An *Infra-topological space* on X is a collection τ_{iX} of subsets of X such that the following axioms are satisfying:

Ax-1 $\emptyset, X \in \tau_{iX}$.

Ax-2 The intersection of the elements of any subcollection of τ_{iX} in X .

i.e, If $O_i \in \tau_{iX}$, $1 \leq i \leq n \rightarrow \bigcap O_i \in \tau_{iX}$.

Terminology, the ordered pair (X, τ_{iX}) is called *Infra-Topological Space*. we simply say X is a *Infra -space*.

Definition 2.2: Let (X, τ_{iX}) be an (ITS) and $A \subset X$. A is called *infra -open set* (IOS) if $A \in \tau_{iX}$.

Definition 2.3: Let X be any arbitrary set and $\tau = \{\emptyset, X\}$, then (X, τ_{iX}) is called *indiscrete infra-topology space* or is called *trivial infra- topological space*

Definition 2.4: Let X be any countable arbitrary set and $\tau = P(X)$ the set of all subsets of X , then (X, τ) is called *discrete infra-topology space* or is called *maximal infra- topological space*.

Theorem 2.1: Let (X, τ) be a *topological -space* (TS), then (X, τ) is an *infra-topological space* (IITS).

Proof: Suppose that (X, τ) is a topological space, then by axioms it is clear that (X, τ) is infra topological space. The converse of above theorem is not true.

Example 2.1: If $X = \{a, b, c\}$ and $\tau_{iX} = \{\emptyset, X, \{a\}, \{b\}\}$, then (X, τ_{iX}) is *infra- topological space*, but not topological space.

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Theorem 2.2: Let (X, τ_{iX}) be *infra-topological space*. Then:

1. \emptyset, X are *infra-open set*.
2. Any arbitrary intersections of *infra-open sets* are *infra-open sets*.
3. Finite union of *infra-open sets* may not be *infra-open sets*.

Proof

1. It is clear that \emptyset, X are *infra open set* by Ax-1 and definition 2.1.
2. Let $C_{i \in J} \in \tau_{iX}$, by Ax-2 and definition 2.1. $\cap O_i \in \tau_{iX}$ are *infra open set*.
3. By counter example 2.1: $\{a\}, \{b\} \in \tau_{iX}$, but $\{a\} \cup \{b\} = \{a, b\} \notin \tau_{iX}$.

Theorem 2.3: let (X, τ_{iX}) and (X, τ_{iX}^*) be two *infra topological Spaces* on set X . Then the intersection τ_{iX} and τ_{iX}^* is an *infra topological space*.

Proof: let (X, τ_{iX}) and (X, τ_{iX}^*) be two *infra topological Spaces* on set X .

By Ax-1 $\emptyset, X \in \tau_{iX}$ and $\emptyset, X \in \tau_{iX}^*$, so $\emptyset, X \in \tau_{iX} \cap \tau_{iX}^*$. Suppose that $O_i \in \tau_{iX} \cap \tau_{iX}^*$.

$1 \leq i \leq n$, implies that $O_i \in \tau_{iX}$ and $O_i \in \tau_{iX}^*$ Consequently, $\cap_{i=1}^n O_i \in \tau_{iX}$ and $\cap_{i=1}^n O_i \in \tau_{iX}^*$ and hence $\cap_{i=1}^n O_i \in \tau_{iX} \cap \tau_{iX}^*$.

Theorem 2.4: Let (X, τ_{iX}) and (X, τ_{iX}^*) be two *infra topological Spaces* on set X . then the union τ_{iX} and τ_{iX}^* is an *infra topological space*.

Proof: Let (X, τ_{iX}) and (X, τ_{iX}^*) be two *infra topological infra spaces* on set X . By Ax-1, $\emptyset, X \in \tau_{iX}$ and $\emptyset, X \in \tau_{iX}^*$, so $\emptyset, X \in \tau_{iX} \cup \tau_{iX}^*$.

Suppose that $O_i \in \tau_{iX} \cup \tau_{iX}^*$; $1 \leq i \leq n$, implies that $O_i \in \tau_{iX}$ or $O_i \in \tau_{iX}^*$ for some $i, 1 \leq i \leq n$. So $\cap_{i=1}^n O_i \in \tau_{iX}$ or $\cap_{i=1}^n O_i \in \tau_{iX}^*$ and hence $\cap_{i=1}^n O_i \in \tau_{iX} \cup \tau_{iX}^*$.

Remark: The union of *infra-topological spaces* may not be *infra-topological space*, in general, by the following example.

Example 2.2: Let X be a set $X = \{a, b, c, d\}$, $\tau_{iX} = \{\emptyset, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$ and $\tau_{iX}^* = \{\emptyset, X, \{c\}, \{d\}, \{b, c\}, \{c, d\}\}$. Now $\tau_{iX} \cup \tau_{iX}^* = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{c, d\}\}$. is not *infra-topological space*.

3. PROPERTIES OF SUBSETS ON INFRA TOPOLOGICAL SPACES

Definition 3.1: Let (X, τ_{iX}) be an (ITS) and $A \subset X$. A point $x \in X$ is called *Infra-Cluster Point* (ICP) of A , if for all *Infra-open set* O containing x , then $A \cap (O \setminus \{x\}) \neq \emptyset$.

Definition 3.2: Let (X, τ_{iX}) be an (ITS) and $A \subset X$. The set of all *Infra-Cluster Point* (ICP) of A is called the *Infra-Derived Set* (IDS) of A and is denoted by $ids(A)$.

Definition 3.3: Let (X, τ_{iX}) be *infra-topological space*. A subset $C \subset X$ is called *infra-closed set* in X if $X - C$ is *infra-open set* in X . That is C is *infra-closed set* (ICS) iff $X - C \in \tau_{iX}$.

Theorem 3.1: Let (X, τ_{iX}) be *infra-topological space*. Then:

- i. $\emptyset, X \in \tau_{iX}$ are *infra-closed set*.
- ii. Any arbitrary finite intersections of *infra-closed sets* is an *infra-closed sets*.

Proof:

- i. Since $X - \emptyset = X \in \tau_{iX}$ and $X - X = \emptyset \in \tau_{iX}$ are *infra-closed sets*.
- ii. Let $\{C_i : i \in I\}$ be an arbitrary family of *infra closed sets* such that $C_i \in \tau_{iX}$ for all $i \in I$. Now, $X - C_i \in \tau_{iX}$ is *infra-open set* for all $i \in I$.
But $X - C_i = C_i^c \in \tau_{iX}$, then $\cap C_i^c = \cap (X - C_i) = X - \cap C_i \in \tau_{iX}, \forall i \in I$. Hence $\cap C_i \in \tau_{iX}, \forall i \in I$ is *infra-closed set*.

Remark: Finite union of *infra-closed sets* may not be *infra-closed sets*, in general.

Definition 3.4: Let (X, τ_{iX}) be an (ITS) and $A \subset X$. The *Infra Closure Point* (ICP) of A is a set denoted by $icp(A)$ and given by: $icp(A) = \cap \{C_i : A \subset C_i, X - C_i \in \tau_{iX}\}$. That is, $icp(A)$ is the intersection of all *infra closed set* contained the set A .

Remark: Since $icp(A)$ is the intersection of all infra closed sets containing in A , then $A \subset icp(A)$ and $icp(A)$ is the smallest infra closed sets .

Definition 3.5: Let (X, τ_{iX}) be an (ITS) and $A \subset X$. The *Infra-Interior Points* (IIP) of A is a set denoted by $iip(A)$ and given by: $iip(A) = \cup \{O_i : O_i \subset A, O_i \in \tau_{iX}\}$. That is, $iip(A)$ is the union of all infra-open set contained in the set A .

Remark: Since $iip(A)$ is the union of all infra-open sets contained in A , then $iip(A) \subset A$ and $icp(A)$ is the smallest infra-open sets. Also if O is infra-open set contained in A , then $O \subset iip(A)$.

Definition 3.6 Let (X, τ_{iX}) be an (ITS) and $A \subset X$. The *Infra-Exterior Points* (IEP) of A is a set denoted by $iep(A)$ and given by: $iep(A) = iip(A^c)$. That is, Set of all infra-interior point of complement of A .

Definition 3.7: Let (X, τ_{iX}) be an (ITS) and $A \subset X$. The *Infra-Boundary Points* (IBP) of A is a set denoted by $ibp(A)$ and given by: $ibp(A) = X \setminus iip(A) \cup iep(A)$

Theorem 3.2: Let (X, τ_{iX}) be an (ITS) and $A, B \subset X$. The *Infra-Derived Set Axioms* (IDSA) satisfies the followings:

- (IDSA)₁ : $ids(\emptyset) = \emptyset$.
- (IDSA)₂ : If $A \subset B$, then $ids(A) \subset ids(B)$.
- (IDSA)₃ : if $x \in ids(A)$, then $x \in ids(A \setminus \{x\})$.
- (IDSA)₄ : $ids(A \cap B) \subset ids(A) \cap ids(B)$.
- (IDSA)₅ : $ids(A \cup B) = ids(A) \cup ids(B)$.

Proof:

(IDSA)₁ : Suppose that $ids(\emptyset) \neq \emptyset \rightarrow \exists x \in ids(\emptyset) \rightarrow \emptyset \cap (O \setminus \{x\}) \neq \emptyset$
 $\rightarrow x \in \emptyset$ and $x \notin \emptyset$. That is contradiction.
 $\rightarrow ids(\emptyset) = \emptyset$.

(IDSA)₂ : Suppose that $A \subset B$. Let $x \in ids(A) \rightarrow \forall O \ni x, A \cap (O \setminus \{x\}) \neq \emptyset$.
 $\rightarrow \forall O \ni x, B \cap (O \setminus \{x\}) \neq \emptyset$.
 $\rightarrow x \in ids(B)$
 $\rightarrow ids(A) \subset ids(B)$.

(IDSA)₃ : Assume that $x \in ids(A) \rightarrow \forall O \ni x, A \cap (O \setminus \{x\}) \neq \emptyset$.
 $\rightarrow \forall O \ni x, A \cap (O \cap \{x\}^c) \neq \emptyset$.
 $\rightarrow \forall O \ni x, A \cap (O \cap \{x\}^c \cap \{x\}^c) \neq \emptyset$.
 $\rightarrow \forall O \ni x, A \cap (\{x\}^c \cap O \cap \{x\}^c) \neq \emptyset$.
 $\rightarrow \forall O \ni x, (A \cap \{x\}^c) \cap (O \cap \{x\}^c) \neq \emptyset$.
 $\rightarrow \forall O \ni x, (A \setminus \{x\}) \cap (O \setminus \{x\}) \neq \emptyset$.
 $\rightarrow x \in ids(A \setminus \{x\})$.

(IDSA)₄: Since $A \cap B \subset A \wedge A \cap B \subset B$
 $\rightarrow ids(A \cap B) \subset ids(A) \wedge ids(A \cap B) \subset ids(B)$.
 $\rightarrow ids(A \cap B) \subset ids(A) \cap ids(B)$.

It can be easily shown by example the equality is not hold like topological space.

(IDSA)₅: Since $A \subset A \cup B$ and $B \subset A \cup B$, then $ids(A) \subset ids(A \cup B)$ and $ids(B) \subset ids(A \cup B)$, hence $ids(A) \cup ids(B) \subset ids(A \cup B)$. Conversely,

Suppose that $x \in ids(A \cup B) \rightarrow \forall O \ni x, (A \cup B) \cap (O \setminus \{x\}) \neq \emptyset$.
 $\rightarrow \forall O \ni x, A \cap (O \setminus \{x\}) \neq \emptyset \cup B \cap (O \setminus \{x\}) \neq \emptyset$.

$\rightarrow x \in ids(A) \cup ids(B)$. Hence,
 $ids(A \cup B) = ids(A) \cup ids(B)$.

Theorem 3.3: Let (X, τ_{iX}) be an (ITS) and $A, B \subset X$. The *Infra Closure Point Axioms* (ICPA) satisfying the following conditions:

- (ICPA)₁ : A is infra-closed iff $A = icp(A)$.
- (ICPA)₂ : $icp(\emptyset) = \emptyset$ and $icp(X) = X$.
- (ICPA)₃ : $icp(icp(A)) = icp(A)$.
- (ICPA)₄ : If $A \subset B$, then $icp(A) \subset icp(B)$.
- (ICPA)₅ : $icp(A \cap B) \subset icp(A) \cap icp(B)$.

Proof:

(ICPA)₁ : Suppose that A is infra-closed set. Since $A \subset A$ and $A \cap A = A \rightarrow icp(A) \subset A$, Also $A \subset icp(A) \rightarrow A = icp(A)$. Conversely, Let $A = icp(A)$, obviously, $icp(A)$ is the smallest infra-closed set. Hence A is infra-closed set.

(ICPA)₂ : Since X and \emptyset are infra- closed sets, so by (ICPA)₁ $icp(\emptyset) = \emptyset$ and $icp(X) = X$. (ICPA)₃: Since $icp(A)$ is the intersection of all infra-closed sets are closed sets, then $icp(icp(A)) = icp(A)$.

(ICPA)₄: Consider $A \subset B$. Since $A \subset icp(A)$ and $B \subset icp(B)$, so $icp(A) \subset icp(B)$.

(ICPA)₅: Since $(A \cap B \subset A \wedge A \cap B \subset B)$, then $icp(A \cap B) \subset icp(A)$ and $icp(A \cap B) \subset icp(B) \rightarrow icp(A \cap B) \subset icp(A) \cap icp(B)$.

Remark: In (ICPA)₅ the equality does not work like topological space.

Theorem 3.4: Let (X, τ_{iX}) be an (ITS) and $A, B \subset X$. The *Infra-Interior Points Axioms* (IIPA) given by:

$$(IIPA)_1: A \text{ is infra-open set iff } A = iip(A).$$

$$(IIPA)_2: iip(X) = X \text{ and } iip(\emptyset) = \emptyset$$

$$(IIPA)_3: iip(iip(A)) = iip(A).$$

$$(IIPA)_4: \text{ If } A \subset B, \text{ then } iip(A) \subset iip(B).$$

$$(IIPA)_5: iip(A \cap B) = iip(A) \cap iip(B).$$

Proof:

(IIPA)₁ : Suppose that A is infra-open set. Since $A \subset A$, then A is infra-open set containing itself, so $A \subset iip(A)$ and $iip(A) \subset A$, that implies $A = iip(A)$. Conversely, Let $A = iip(A)$, suppose that $A = iip(A)$. Since $iip(A)$ is infra-open set, then A is infra- open set.

(IIPA)₂: Since X, \emptyset are infra-open sets, by (IIPA)₁, we have $iip(X) = X$ and $iip(\emptyset) = \emptyset$.

(IIPA)₃: Since $iip(A)$ is infra-open set. so by (IIPA)₁ $iip(iip(A)) = iip(A)$.

(IIPA)₄ : Suppose that If $A \subset B$. Let $O_i \in iip(A) \rightarrow O_i \subset A \rightarrow O_i \subset B \rightarrow O_i \in iip(B)$.

Therefore $iip(A) \subset iip(B)$.

(IIPA)₅: Let $O_i \in iip(A) \cap iip(B) \leftrightarrow O_i \in iip(A) \wedge O_i \in iip(B)$.

$$\leftrightarrow \cup O_i, O_i \subset A, \forall i \wedge \cup O_i, O_i \subset B, \forall i.$$

$$\leftrightarrow \cup O_i, O_i \subset A \cap B, \forall i.$$

$$\leftrightarrow O_i \in iip(A \cap B), \forall i.$$

Theorem 3.5: Let (X, τ_{iX}) be an (ITS) and $A, B \subset X$. The *Infra -Exterior points Axioms* (IEPA) given by:

$$(IEPA)_1: iep(X) = \emptyset \text{ and } iep(\emptyset) = X.$$

$$(IEPA)_2: iep(A) \subset A^c.$$

$$(IEPA)_3: iep(A \cup B) = iep(A) \cap iep(B).$$

$$(IEPA)_4: \text{ If } A \subset B, \text{ then } iep(B) \subset iep(A).$$

$$(IEPA)_5: iep(A \cap B) \subset iep(A) \cup iep(B).$$

Proof:

(IEPA)₁: $iep(X) = iip(X^c) = iip(\emptyset) = \emptyset$ and $iep(\emptyset) = iip(\emptyset^c) = iip(X) = X$.

(IEPA)₂: $iep(A) = iip(A^c) \subset A^c$.

(IEPA)₃: $iep(A \cup B) = iip(A \cup B)^c = iip(A^c \cap B^c) = iip(A^c) \cap iip(B^c) = iep(A) \cap iep(B)$.

(IEPA)₄: let $A \subset B \rightarrow B^c \subset A^c \rightarrow iip(B^c) \subset iip(A^c) \rightarrow iep(B) \subset iep(A)$.

(IEPA)₅: $iep(A \cap B) = iip(A \cap B)^c = iip(A^c \cup B^c) \subset iip(A^c) \cup iip(B^c) = iep(A) \cup iep(B)$.

Theorem 3.6: Let (X, τ_{iX}) be an (ITS) and $A \subset X$. The *Infra-Boundary Points Axioms* (IBPA) given by:

$$\begin{aligned} \text{(IBPA)}_1 &: \text{ibp}(X) = \text{ibp}(\emptyset) = \emptyset. \\ \text{(IBPA)}_2 &: \text{ibp}(A \cap B) = \text{ibp}(A) \cup \text{ibp}(B). \end{aligned}$$

Proof:

$$\text{(IBPA)}_1: \text{ibp}(X) = X \setminus \text{iip}(X) \cup \text{iep}(X) = X \setminus X \cup \emptyset = X \setminus X = \emptyset.$$

$$\text{ibp}(\emptyset) = X \setminus \text{iip}(\emptyset) \cup \text{iep}(\emptyset) = X \setminus \emptyset \cup X = X \setminus X = \emptyset$$

$$\begin{aligned} \text{(IBPA)}_2: \text{ibp}(A \cap B) &= X \setminus \text{iip}(A \cap B) \cup \text{iep}(A \cap B) \\ &= X \setminus (\text{iip}(A) \cap \text{iip}(B)) \cup \text{iep}(A \cap B) \\ &= X \setminus \text{iip}(A) \cup X \setminus \text{iip}(B) \cup \text{iep}(A \cap B) \\ &= X \setminus \text{iip}(A) \cup X \setminus \text{iip}(B) \cup \text{iep}(A) \cup \text{iep}(B) \\ &= \text{ibp}(A) \cup \text{ibp}(B). \end{aligned}$$

The following theorem illustrate the relations between $\text{ids}(A)$, $\text{icp}(A)$, $\text{iip}(A)$, $\text{iep}(A)$ and $\text{ibp}(A)$.

Theorem 3.7: Let (X, τ_{iX}) be an (ITS) and $A \subset X$. then:

1. $A \subset \text{icp}(A) \rightarrow \text{ids}(A) \subset \text{ids}(\text{icp}(A))$.
2. $\text{iip}(A) \subset A \rightarrow \text{ids}(\text{iip}(A)) \subset \text{ids}(A)$.
3. If A is infra-closed, then $\text{ids}(A) \subset A$.
4. $\text{icp}(A) = A \cup \text{ids}(A)$.
5. $\text{ibp}(A) = \text{icp}(A) \setminus \text{iip}(A)$.
6. $\text{icp}(A) = \text{ibp}(A) \cup \text{iip}(A)$.
7. $\text{ibp}(A) \subset \text{icp}(A)$.
8. $\text{iip}(A) \cap \text{ibp}(A) = \emptyset$.

Proof:

- (1) Let $A \subset \text{icp}(A)$. By (IDSA)₂ $\text{ids}(A) \subset \text{ids}(\text{icp}(A))$.
- (2) Let $\text{iip}(A) \subset A$. By (IDSA)₂ $\text{ids}(\text{iip}(A)) \subset \text{ids}(A)$.
- (3) Let A be a infra closed set and $x \in \text{ids}(A)$, then $\forall O \ni x, A \cap (O - \{x\}) \neq \emptyset$ Hence $x \in A$ and $\text{ids}(A) \subset A$.
- (4) Since $A \subset \text{icp}(A)$ and $\text{ids}(A) \subset \text{ids}(\text{icp}(A)) \subset \text{icp}(A)$. we have $A \cup \text{ids}(A) \subset \text{icp}(A)$. Another direction, To show that $\text{icp}(A) \subset A \cup \text{ids}(A)$.

Let $x \in \text{icp}(A)$, but $A \subset \text{icp}(A)$, then $x \in A$ or $x \notin A$.

Probability 1) If $x \in A$, then $x \in A \cup \text{ids}(A)$.

Probability 2) if $x \notin A$, Let $x \notin \text{ids}(A) \rightarrow \exists O \ni x, A \cap (O \setminus \{x\}) = \emptyset$, but $x \notin A$, that is contradiction, therefore $x \in \text{ids}(A)$ and $x \in A \cup \text{ids}(A)$.

So $\text{icp}(A) = A \cup \text{ids}(A)$.

- (5) By defⁿ: $\text{ibp}(A) = X \setminus \text{iip}(A) \cup \text{iep}(A)$
 $= X \setminus \text{iip}(A) \cap X \setminus \text{iep}(A)$
 $= X \setminus \text{iip}(A) \cap \text{icp}(A)$

Since $\text{iip}(A) \subset \text{icp}(A) \subset X \rightarrow \text{icp}(A) \cap (X \setminus \text{iip}(A)) = \text{icp}(A) \setminus \text{iip}(A)$. Then we have $\text{ibp}(A) = \text{icp}(A) \setminus \text{iip}(A)$.

- (6) By (1) $\text{ibp}(A) = \text{icp}(A) \setminus \text{iip}(A)$
 $\rightarrow \text{ibp}(A) \cup \text{iip}(A) = \text{icp}(A) \setminus \text{iip}(A) \cup \text{iip}(A) = \text{icp}(A)$.
- (7) By (2) it is clear that $\text{ibp}(A) \subset \text{icp}(A)$.
- (8) $\text{iip}(A) \cap \text{ibp}(A) = \text{iip}(A) \cap \text{icp}(A) \setminus \text{iip}(A) = \emptyset$.

Theorem 3.8: Let X be any finite set and $A \subset X$ and order $(o(A) = 1)$. The collection $\tau_{iX} = \{\emptyset, X\} \cup \{A \subset X \text{ such that } o(A)=1\}$ is infra topological space.

Proof: Since $\emptyset, X \in \{\emptyset, X\}$, so $\emptyset, X \in \tau_{iX}$ and A_{x-1} is hold. Now, Assume that $A_i \in \tau_{iX}, 1 \leq i \leq n$. And $o(A) = 1$ Then $A_i \cap \emptyset = \emptyset, \forall i$ and $A_i \cap X = A_i, \forall i$, so that A_{x-2} is hold.

The pair (X, τ_{iX}) is called the Particular singleton set of infra-topological space on X .

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