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ON PRE GENERALIZED REGULAR WEAKLY-OPEN SETS IN A TOPOLOGICAL SPACE

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ABSTRACT

This paper introduces the pgrw-open set in a topological space and studies some of its properties. Also in this paper we introduce pgrw-interior, pgrw-neighbourhood, pgrw-limit points in topological spaces. Using pgrw-closed sets we introduce pgrw-closure and discuss some of its basic properties.

Keywords: *Topological spaces, pgrw-closed sets, pgrw-open sets.*

1. INTRODUCTION

Regular open sets and rw-open sets have been introduced and investigated by Stone [9] and Benchalli and Wali [1] respectively. Levine [4] introduced and investigated semi open sets. Maki *et.al* [5] introduced and studied generalized α-closed sets and α-generalized closed sets. R.S.Wali and P S. Mandalgeri [11] introduced and studied αrω-closed sets. R.S. Wali and V.T. Chilakwad [10] introduced and studied pgrw-closed sets.

2. PRELIMINARIES

For a subset A of a space X, cl (A), Int(A) and A^c denote the Closure of A, Interior of A and Complement of A in X respectively.

Definition 2.1: Let (X, τ) be a topological space and A $\subseteq$ X.

The intersection of all semi closed (pre-closed, α-closed, and semi-pre-closed) subsets containing A is called the Semi closure (pre-closure, α-closure and Semi-pre-closure) of A and is denoted by scl(A) [pcl(A),αcl(A), spcl(A)].

Definition 2.2: A subset A of a topological space (X, τ) is called

a) a #regular generalized closed (briefly #rg-closed) set [9] if cl(A)⊆U whenever A ⊆ U and U is rw-open.
b) a generalized semi-pre closed set(briefly gsp-closed) [7] if spcl(A) ⊆ U whenever A⊆U and U is open in X.
c) a generalized semi pre regular closed (briefly gspr-closed) set [6] if spcl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
d) generalized pre regular closed set (briefly gpr-closed) [2] if pcl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
e) a generalized pre closed (briefly gp-closed) set [4] if pcl(A)⊆U whenever A⊆U and U is open in X.
f) a α-regular w- closed set[11] if αcl(A) ⊆ U whenever A⊆U and U is rw -open in X.

The complements of the above mentioned closed sets are their open sets respectively.

3. PRE GENERALISED REGULAR WEAKLY-CLOSED SETS [10] IN A TOPOLOGICAL SPACE

Definition 3.1: A subset A of a topological space X is called a Pre generalized regular weakly-closed [pgrw-closed set] set if pcl(A)⊆U whenever A ⊆ U and U is rω-open in X.

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Results 3.2[10]:
- i) Every closed set is a pgrw-closed set in X
- ii) Every α-closed set in X is pgrw-closed.
- iii) Every regular closed set is pgrw-closed in X.
- iv) Every pgrw-closed set is gspr-closed [gsp-closed, gp-closed, gpr-closed] in X.

4. pgrw-OPEN SETS

In this section, we define pgrw-open set in a topological space and obtain some of its properties.

Definition 4.1: A subset A of a topological space X is called a pre generalised regular-weakly open (briefly pgrw-open) set in X if the complement A^c of A is pgrw-closed in X.

Example 1: X = {a. b. c}, T={X, ϕ, {a}, {b}, {a, b}} Open sets are X, ϕ, {a}, [b}, {a, b}.
Closed sets are X, ϕ, {b, c}, {a, c},{c}.
Rw-open sets are X, ϕ, {a},{[b}, {c},{a, b}.
Pgrw closed sets are X, ϕ, {b, c}, {a, c},{c}.
Pgrw –open sets are X, ϕ, {a},{b},{a, b}

Example 2: X={a. b. c}, T={X, ϕ,{a}}.
Rw open sets are X, ϕ, {b}, {a}, {b}, {c},{a, b}, {b, c},{a, c} .
Pgrw –closed sets are X, ϕ, {b},{c},{b, c}.
Pgrw-open sets are X, ϕ, {a, c},{a, b},{a}.

Theorem 4.2: For any topological space X
- i) Every open (α-open, regular-open, αrω-open, #rg-open, pgpr-open) set is pgrw-open.
- ii) Every pgrw-open set is gspr–open (gsp-open, gp-open and gpr-open).

Remark 4.3: The union and intersection of pgrw-open sets in X are generally not pgrw-open.

Example: X={a, b, c}, T={X, ϕ, {a}, {b}, {a, b}}. {a, b}, {a} are pgrw – open. But {a, b}∩{a} is not pgrw-open. X = {a, b, c, d}, T = {X, ϕ, {{a}}, {c, d}, {a, c, d}}{a} & {b} are pgrw-open, but {a}U{b} = {a, b} is not pgrw-open.

Theorem 4.4: A subset A of a topological space X is pgrw-open iff U⊆ p-int(A), whenever U is rω-closed and U ⊆ A.

Proof: A and U are subsets of a topological space X such that A is pgrw-open, U is rw-closed and U⊆A.
=> X-A is pgrw-closed, X-U is rw-open and X-A⊆X-U.
=> pcl(X-A) ⊆X-U by the defn of pgrw-closed set.
=> U⊆X-pcl(X-A)
=> U⊆p-intA.

Conversely, Suppose U⊆ p-int (A) whenever U is rω-closed and U ⊆ A.
=> X- p-int (A) ⊆X-U whenever X-U is rw-open and X-A⊆X-U.
=> pcl(X-A) ⊆X-U whenever X-U is rw-open and X-A⊆ X-U ’.’ pcl(X-A)= X-p-int(A)
=> X-A is pgrw-closed.
=> A is pgrw-open.

Theorem 4.5: If p-int(A) ⊆B⊆A and A is a pgrw-open set, then B is pgrw-open.

Proof: p-int(A)⊆B⊆A & A is pgrw open.
=> X-A ⊆X-B⊆X-p-int(A) &X-A is pgrw-closed.
=> X-A ⊆X-B⊆pcl(X-A) & X-A is pgrw-closed. ‘.’ Pcl(X-A) = X-p-intA
=> X-B is pgrw-closed [th 3.21[10]].
=> B is pgrw-open**.**

Theorem 4.6: If A is a pgrw-closed set, then pcl(A)-A is pgrw-open.

Proof: A is pgrw-closed. F is rw-closed and F⊆pcl(A)-A
=> F=φ [th3.22 [10]
=> F⊆ p-int (pcl(A)-A)
=> pcl(A)-A is pgrw-open[by th 4.4].
The converse is not true.

Example 4.7: Let X={a, b, c, d}, T = {X, ϕ, {a} {b}, {a, b}, {a, b, c}}. Let A = {a, d}
pcl(A)-A = {a, c, d}-{a, d} = {c} which is pgrw-open. But A is not pgrw-closed.

Theorem 4.8: For a pgrw-open set A and rw-open set U in a topological space X if p-int(A) $\cup$ A^c $\subseteq$U, then U=X.

Proof: In a topological space X, for any two sets A & U, (p-int(A))$\cup A^c \subseteq$U
=> $U^c \subseteq$[(p-int(A))$\cup A^c]^c$
=> $U^c \subseteq$[p-int(A)]$^c \cap (A^c)^c$
=> $U^c \subseteq$ pcl(A^c)$\cap$A
=> $U^c \subseteq$pcl(A^c)-A^c
=> U^c= ϕ [Using th. 3.22[10]]
=> U=X.

Theorem 4.9: If A and B are two subsets of a space X such that A is pgrw-open and p-int(A)$\subseteq$B, then A$\cap$B is also pgrw-open.

Proof: A is pgrw-open and p-int(A)$\subseteq$B . ……..(i)

For any set A, p-int(A)$\subseteq$A…………………....(ii)
∴. From (i) and (ii), p-int(A)$\subseteq$A$\cap$B. Also A$\cap$B$\subseteq$A.
∴ p-int(A)$\subseteq$A$\cap$B$\subseteq$A. => A$\cap$B is also a pgrw-open set in X. [Th.4.5]

5. pgrw-CLOSURE

In this section the pgrw-closure and pgrw-Interior are defined and some of their basic properties are studied.

Definition 5.1: For a subset A of a topological space X, pgrw-closure of A is defined as intersection of all pgrw-closed sets containing A.

Notation: pgrwcl(A) .

Example: X={a, b, c, d}, T={X, ϕ,{a},{b},{a, b} {a, b, c}}. Let A={a, b},then pgrwcl(A)={a, b, d}

Theorem 5.2: A and B are subsets of a space X
i) pgrwcl(X) = X, pgrwcl(ϕ)=ϕ
ii) A $\subseteq$ pgrwcl(A)
iii) If B is any pgrw-closed set containing A, then pgrwcl(A) $\subseteq$ B
iv) If A$\subseteq$ B then pgrwcl(A)$\subseteq$ pgrwcl(B)
v) pgrwcl(A)= pgrwcl(pgrwcl (A))
vi) pgrwcl(A) $\cup$ pgrwcl(B) $\subseteq$ pgrwcl(A$\cup$B)

Proof:
i) X is the only pgrw-closed set containing X. Therefore pgrwcl(X) = Intersection of all the pgrw-closed sets containing X=$\cap${X} =X. Therefore pgrwcl (X) = X and again by definition of pgrw-closure, pgrwcl(ϕ) = Intersection of all pgrw-closed sets containing ϕ = ϕ $\cap$ any pgrw-closed set containing ϕ =ϕ. Therefore pgrwcl(ϕ)=ϕ.
ii) By definition of pgrw-closure of A, it is obvious that A $\subseteq$ pgrwcl(A).
iii) Let B be any pgrw-closed set containing A. Since pgrwcl(A) is the intersection of all pgrw-closed sets containing A, pgrwcl(A) is contained in every pgrw-closed set containing A. Hence in particular pgrwcl(A)$\subseteq$B.
iv) Let A and B be subsets of X such that A$\subseteq$B.
pgrwcl(B)=$\cap${F: B $\subseteq$F and F is a pgrw-closed set}. If B $\subseteq$F ϵ PGRWC(X),
then pgrwcl(B) $\subseteq$ F. since A $\subseteq$B, A$\subseteq$ B $\subseteq$F ϵ PGRWC(X), we have pgrwcl(A) $\subseteq$F by (iii),
pgrwcl(A) $\subseteq\cap${F : B $\subseteq$F ϵ PGRWC(X)} = pgrwcl(B).Therefore pgrwcl(A) $\subseteq$ pgrwcl(B).
v) A is a subset of X. pgrwcl(A)= $\cap${F : A $\subseteq$F ϵ PGRWC(X).
If A$\subseteq$F ϵ PGRWC(X), then pgrwcl(A) $\subseteq$ F, since F is pgrw-closed set containing pgrwcl(A) by (iii) pgrwcl(pgrwcl(A)) $\subseteq$ F, for every pgrw-closed set containing A.
Hence pgrwcl(pgrwcl(A)) $\cap${F : A $\subseteq$F ϵ PGRWC(X)}.
Then pgrwcl(pgrwcl(A)) $\subseteq$ pgrwcl(A)……………………………………………………....(I)
Next, A$\subseteq$ pgrwcl(A) (from (ii)). ∴ pgrwcl(A) $\subseteq$ pgrwcl(pgrwcl(A)) .[using (iv) above]….(II)

Therefore from I and II pgrwcl(pgrwcl(A)) = pgrwcl(A).

vi) Let A and B be subsets of X. Clearly $A \subseteq A \cup B$, $B \subseteq A \cup B$.
From (iv) pgrwcl(A) $\subseteq$ pgrwcl($A \cup B$), pgrwcl(B) $\subseteq$ pgrwcl($A \cup B$)
Hence pgrwcl(A) $\cup$pgrwcl(B)$\subseteq$ pgrwcl($A \cup B$).

Theorem 5.3: If $A \subseteq X$ is a pgrw-closed set, then pgrwcl(A) =A

Proof: Let A be a pgrw-closed subset of X. then by theorem 5.2(ii), A $\subseteq$ pgrwcl(A) .-----(i)

Also $A \subseteq A$ and A is pgrw-closed set. $\therefore$ by theorem5.2 (iii) pgrwcl(A) $\subseteq$A. -------------(ii)
(i) and (ii) pgrwcl(A) =A.

The Converse of the above need not be true as seen from the following example.

Example 5.4: Let X={a, b, c, d},T={X, ϕ ,{a},{c, d},{a, c, d}}A={a} which is pgrw-open. But pgrwcl(A)=A.

Theorem 5.5: If A and B are subsets of a space X, then pgrwcl($A \cap B$)$\subseteq$ pgrwcl(A)$\cap$pgrwcl(B)

Proof: Let A and B be subsets of X, Clearly $A \cap B \subseteq A$, $A \cap B \subseteq B$. By theorem 5.2(iv)
pgrwcl($A \cap B$) $\subseteq$ pgrwcl(A), pgrwcl($A \cap B$) $\subseteq$ pgrwcl(B).
Hence pgrwcl($A \cap B$) $\subseteq$ pgrwcl(A)$\cap$pgrwcl(B).

Remark 5.6: In general pgrwcl(A)$\cap$pgrwcl(B) $\nsubseteq$ pgrwcl($A \cap B$) as seen from the following example.

Example 5.7: Consider X={a, b, c, d}, T={X, ϕ,{a},{b},{a, b},{a, b, c}}, A={b, c, d}, B={a, b, c}
Pgrwcl(A)={b, c, d}. Pgrwcl(B)=X. Pgrwcl(A)$\cap$pgwcl(B)={b, c, d}
Pgrwcl($A \cap B$)=pgrwcl({b, c})={b, c}. .'. Pgrwcl(A)$\cap$pgwcl(B) $\nsubseteq$ Pgrwcl($A \cap B$).

Theorem 5.8: For an $x \in X$, $x \in$ pgrwcl(A) if and if $A \cap V \neq \phi$ for every pgrw-open set V containing x.

Proof: Let $x \in$ pgrwcl(A) To prove $A \cap V \neq \phi$ for every pgrw-open set V containing x by contradiction.

Suppose there exists a pgrw-open set V containing x s.t $A \cap V = \phi$. Then $A \subseteq X-V$, X-V is a pgrw-closed set, pgrwcl(A) $\subseteq$ X-V. This shows that $x \notin V$ which is a contradiction.

Hence $A \cap V \neq \phi$ for every pgrw-open set V containing x.

Conversely, Let $x \in X$ and $A \cap V \neq \phi$ for every pgrw-open set V containing x. To prove $x \in$pgrwcl(A). We prove the result by contradiction. Suppose $x \notin$ pgrwcl(A) then there exists a pgrw-closed set F containing A such that $x \notin F$. Then $x \in$ X-F is pgrw-open. Also (X-F) $\cap$ A=ϕ which is a contradiction. Hence $x \in$ pgrwcl(A).

Theorem 5.9: For every subset A of X, (i) pgrwcl(A) $\subseteq$ cl(A) (ii) pgrwcl(A) $\subseteq$ pcl(A)

Proof:

(i) Let A be a subset of a topological space X. Then by definition,
cl(A)= $\cap${F: $A \subseteq F$, F is closed}. If $A \subseteq F \in C(X)$, then $A \subseteq F \in$PGRWC(X) because every closed set is pgrw-closed that is pgrwcl(A)$\subseteq$F.
Therefore pgrw cl(A)$\subseteq\cap${F : $A \subseteq F \in C(X)$} = cl(A). Hence pgrωcl(A)$\subseteq$cl(A).

(ii) Let A be a subset of topological space X. By definition, pcl(A)= $\cap${F : A $\subseteq$F, a pre closed set}.
If $A \subseteq F$, a pre closed set, then $A \subseteq F \in$ PGRWC(X) because every pre-closed set is pgrw-closed that is pgrwcl(A)$\subseteq$F. Therefore pgrwcl(A) $\subseteq\cap${F : $A \subseteq F \in$ pC(X)} = pcl(A).
Hence pgrw cl(A) $\subseteq$ pcl(A).

Theorem 5.10: If A is a subset of a space X, then gprcl(A)$\subseteq$ pgrwcl(A) .

Proof: Let A be a subset of X. pgrw-cl(A)=$\cap${F : $A \subseteq F \in$ PGRWC(X)}.

If $A \subseteq F \in$ PGRWC(X), then $A \subseteq F \in$ GPRC(X), because every pgrw-closed set is gpr-closed i.e. gprcl(A) $\subseteq$F therefore gprcl(A) $\subseteq\cap${F : $A \subseteq F \in$ PGRWC(X)}= pgrwcl(A). Hence gprcl(A) $\subseteq$ pgrwcl(A).

pgrw-interior:

Definition 5.11: For a subset A of a topological space X, pgrw-interior of A is defined as pgrwint(A) = ∪{G: G⊆A and G is pgrw-open in X} or ∪{G: G⊆A and G ϵ PGRWO(X)}. i.e pgrwint(A) is the union of all pgrw -open sets contained in A. Every point of pgrw-interior of A is called pgrw-interior point of A.

Example X={a, b, c, d}, T={X, ϕ, {b, c},{b, c, d},{a, b, c}}. Let A={a, b}. pgrwint(A)={a, b}.

Theorem 5.12: Let A and B be subsets of a space X. Then

i) pgrwint(X) = X, pgrwint(ϕ)=ϕ
ii) pgrwint(A) ⊆ A
iii) If B is any pgrw-open set contained in A, then B ⊆ pgrwint(A)
iv) If A ⊆ B then pgrwint(A) ⊆ pgrwint(B)
v) pgrwint(A) ⊆ pgrwint(pgrwint(A))
vi) pgrwint(A∩B) ⊆ pgrwint(A)∩pgrwint(B)

Proof: i) and ii) are obvious by definition of pgrw-interior of A.
iii) Let B be any pgrw-open set s.t B ⊆ A. Let x ϵ B, B is an pgrw -open set contained in A, x is an pgrw-interior point of A i.e. x ϵ pgrwint(A). Hence B ⊆ pgrwint(A).
iv), v), vi) have similar proofs as in theorem5.2 and using definition of pgrw-interior.

Theorem 5.13: If a subset A of a topological space X is pgrw-open, then pgrwint(A)=A

Proof: Let A be a pgrw-open subset of X. From 5.12 (ii) pgrwint(A)⊆A …..(1) A⊆A and A is pgrw-open =>A⊆pgrwint(A) from 5.12(iii)…..(2) is pgrw-open set contained in A from Theorem 5.12(iii) A ⊆ pgrw int(A) --(2).

Hence from (1) and (2) pgrwint(A)=A.

Corollary: If a subset A of a topological space X is open then int(A)= pgrwint(A).

Proof: A⊆X and A is open.
∴ int(A) =A and as every open is pgrw-open A= pgrwint(A) .
∴ int(A)=pgrwint(A).

Converse is not true.

Example: X={a, b, c, d}, T={X, ϕ, {a},{b},{a, b}, {a, b, c}}.
A={b, d}, int(A)={b}. pgrwint(A)={b}. Here intA=pgrwintA, but A is not open.

Theorem 5.14 If A and B are subsets of a space X. Then, pgrωint(A) ∪ pgrωint(B) ⊆ pgrωint(A∪B).

Proof: For any two subsets of X, A⊆A∪B and B⊆A∪B and so in space X. pgrωint(A)⊆pgrωint(A∪B) and pgrωint(B)⊆pgrωint(A∪B). This implies that pgrωint(A)∪pgrωint(B) ⊆pgrwint(A∪B)**.**

Remark 5.15: The converse of the above theorem need not be true as seen from the following example.

Example 5.16: Let X={a, b, c, d**},** T ={X, ϕ,{a},{b},{a, b},{a, b, c}}, A={b, c}, B={a, d},
A∪B ={a, b, c, d}. pgrwint(A)={b, c} pgrwint(B)={a}, pgrwint(A∪B)=X,
pgrωint(A) ∪ pgrωint(B)={a, b, c}.Therefore pgrωint(A∪B) ⊈ pgrωint(A) ∪ pgrωint(B).

Theorem 5.17: For any subset A of a topological space of X, int(A) ⊆ pgrωint(A)

Proof: Let A be a subset of a space X. Then x ϵ int(A) => x ϵ ∪ {G : G is open, G ⊆ A}
=> ∃ an open set G s.t. x ϵ G ⊆ A
=> There exists a pgrω-open set G s.t. x ϵ G⊆A, as every open set is a pgrω-open set in X and so x ϵ ∪{G :G is pgrω-open and G⊆ A}.
=> x ϵ pgrωint(A). Thus x ϵ int(A).
=> x ϵ pgrωint(A). Hence int(A) ⊆ pgrωint(A).

Remark: converse is not true.

Example: X{a, b, c}, T={X, ϕ, {a}}, int({a, c}) ={a}, pgrwint({a, c}) = {a, c}.

Theorem 5.18: If A is a subset of a topological space X, then pgrωint(A)⊆ gpr-int(A),

Proof: A is a subset of a space X. x ϵ pgrwint(A).
=> x ϵ ∪{G : G is pgrw-open , G ⊆ A}
=> There exists an pgrω-open set G such that x ϵ G ⊆ A.
=> there exists a gpr-open set G such that x ϵ G and G⊆ A '.' Every pgrw open set is gpr-open.
=> x ϵ ∪{G : G is gpr-open, G ⊆ A}
=> x ϵ gpr-int(A). Thus x ϵ pgrω int(A).
=> x ϵ gpr-int(A). Hence pgrωint(A) ⊆ gprint(A).

Remark: Converce is not true.

Theorem 5.19: For any subset A of X
i) X - pgrwint(A) = pgrwcl(X - A)
ii) pgrωint(A) = X - pgrωcl(X - A)
iii) X - pgrωint(X - A)= pgrωcl(A)
iv) X - pgrωcl(A) = pgrωint(X - A)

Proof:
(i) If x ϵ X-pgrw-int(A) then x is not in pgrw-int(A) i.e. every pgrw-open set G containing x is such that G⊈A. This implies every pgrw-open set G containing x intersects (X - A) i.e. $G \cap (X - A) \neq \phi$. Then by theorem 5.8 x ϵ pgrwcl(X-A).
.'. X-pgrw-int(A) ⊆ pgrwcl(X-A) -----------------------------------(1)

Let x ϵ pgrwcl(X-A), then by theorem 5.8 every pgrw-open set G containing x intersects X–A i.e. $G \cap (X - A) \neq \phi$, i.e. every pgrw-open set G containing x s.t.G⊈A. Then x is not in pgrw-int(A), i.e. x ϵ X-pgrw-int(A) and so Pgrwcl(X-A) ⊆X-pgrw-int(A)--- (2)

From (1) and (2) X-pgrwint(A)=pgrwcl(X-A).

i) Follows by taking complements in i).
ii) Follows by replacing A by X-A in i)
iii) Follows by taking complements in iii).

6. pgrω-NEIGHBOURHOOD AND pgrω-LIMIT POINTS

In this section we define the notion of pgrω-neighbourhood, pgrω-limit points and pgrω- derived set.

Definition 6.1: Let X be a topological space and $x \in X$. A subset N of X is said to be a pgrω-neighbourhood of x if there exists a pgrw open set G such that $x \in G \subseteq N$.

Definition 6.2:
i) Let X be a topological space and A be a subset of X. A subset N of X is said to be a pgrω-neighbourhood of A if there exists a pgrw open set G such that $A \subseteq G \subseteq N$.
ii) The collection of all pgrω-neighbourhoods of $x \in X$ is called pgrω-neighbourhood system of x and shall be denoted by pgrw-N(x)

Definition 6.3: Let X be a topological space and A be a subset of X. Then a point $x \in X$ is called a pgrw limit point of A iff every pgrω-neighbourhood of x contains a point of A distinct from x i.e. $(N - \{x\}) \cap A \neq \phi$ for each pgrω-neighbourhood N of x. The set of all pgrw-limit points of a set A is called the derived set of A and is denoted by pgrwd(A).

Theorem 6.4: Every neighbourhood N of a point x of a topological space X is a pgrω-neighbourhood of x.

Proof: X is a topological space and x ϵ X. Let N be a neighbourhood of x. By definition, ∃ an open set G such that $x \in G \subset N$. Then ∃ a pgrw open set G such that $x \in G \subset N$. ‘.’an open set is pgrw-open. Hence N is a pgrω-neighbourhood of x.

Remark 6.5: A pgrω- neighbourhood N of x∈X need not be a neighbourhood of x in X, as seen from the following example.

Example 6.6: Let X = {a, b, c, d} with topology
T={X, ϕ, {a},{b},{a, b},{a, b, c}}.

Here {a, c, d} is pgrw-neighbourhood of c. But it is not a neighbourhood of c.

Theorem 6.7: A pgrw-open set is a pgrw-neighbourhood of each of its points.

Proof: Suppose N is pgrω-open. Let x ∈ N. We claim that N is pgrω- neighbourhood of x. For N is a pgrω-open set such that x ∈ N ⊆ N. Since x is an arbitrary point of N, it follows that N is a pgrω- neighbourhood of each of its points.

Remark 6.8: The converse of the above theorem is not true as seen from the following example.

Example 6.9: Let X = {a, b, c, d} with T={X, ϕ,{a},{b},{a, b},{a, b, c}}.

Here {a, c, d} is pgrw-neighbourhood of each of its points but it is not pgrw-open.

Theorem 6.10: If F is a pgrω-closed subset of a topological space X, and $x \in F^{c}$, then there exists a pgrω-open set N containing x such that N∩F =ϕ.

Proof: F is a pgrω-closed subset of X and $x \in F^c$. F^c is a pgrω-open set of X. So by theorem 6.7, F^c is a pgrω- neighbourhood of each of its points. Hence there exists a pgrω-open set N containing x such that $N \subset F^c$. That is N ∩ F = ϕ.

Theorem 6.11: Let X be a topological space and for each x∈X, let pgrω-N (x) be the collection of all pgrω-neighbourhood of x. Then we have the following results.

(i) ∀x ∈ X, pgrω-N (x) ≠ ϕ.
(ii) N ∈ pgrω-N (x) ⇒ x ∈ N.
(iii) N∈pgrω-N (x), M ⊃N ⇒ M∈ pgrω-N (x).
(iv) N ∈ pgrω-N (x) ⇒ there exists M ∈ pgrω-N (x) such that M ⊂ N and M ∈ pgrω-N (y) for every y ∈ M.

Proof:

(i) Since X is a pgrω-open set, it is a pgrω- neighbourhood of every x ∈ X. Hence there exists at least one pgrω-neighbourhood (namely - X) for each x ∈ X. Hence pgrω-N (x) ≠ ϕ for every x ∈ X.
(ii) If N ∈ pgrω-N (x), then N is a pgrω- neighbourhood of x. So by definition of pgrω- neighbourhood, x ∈ N.
(iii) Let N ∈ pgrω-N (x) and M ⊃ N. Then there is a pgrω-open set G such that x ∈ G ⊂ N.
Since, N ⊂ M, x ∈ G ⊂ M and so M is pgrω- neighbourhood of x. Hence M ∈ pgrω-N (x).
(iv) If N ∈ pgrω-N (x), then there exists a pgrω-open set M such that x ∈ M ⊂N. Since M is a pgrω-open set, it is pgrω- neighbourhood of each of its points. M ∈ pgrω-N (y) for every y ∈ M.

REFERENCES

1. Benchalli, S. S and R.S Wali., On rω- Closed sets is Topological Spaces, *Bull, Malays, Math, sci, soc* 30 (2007), 99-110.
2. Gnanambal, Y. On generalized pre regular closed sets in topological spaces, Indian J. Pure. Appl. Math., 28(3), (1997), 351-360.
3. Jiling Cao, Maximilian Ganster, Chariklia Konstadilaki and Ivan L. Reilly, On Preclosed Sets and Their Generalizations, www.google.co.in.
4. Levine, N. Semi- open sets and semi continuity in topological space, Amer.math.monthly, 70(1963), 36-41.
5. Maki, H., Devi, R. and Balachandran, K. Associated topologies of generalized α-closed sets and α-generalized closed sets, Mem. Fac. Sci. Kochi Univ. Ser.A. Math., 15(1994), 51-63.
6. Navalagi, G, Chandrashekarappa A.S and S.V. Gurushantanavar. On Gspr-Closed Sets Intopological Spaces. International Journal of Mathematics and Computing Applications Vol. 2, (1-2), January-December, 2010.

7. Sarsak M.S. and N. Rajesh (2010), genrealized semi pre closed sets, Int Math Forum 5(12), 573-578.
8. Stone, M. Applications of the theory of Boolean rings to general topology. http://www.ams.org/journals/tran/1937-041-03/S0002-9947-1937-1501905-7/S0002-9947-1937-1501905-7.pdf.
9. Syed Ali Fathima and M. Mariasingam, On #Regular Generalized Closed Sets In Topological Spaces, International Journal of Computer Applications 42(7), March, 2012.
10. R. S. Wali and Chilakwad, V. T., On Pre Generalized Regular Weakly [pgrw]-Closed sets in a Topological Space, Int. J. of Math Archive, 6(1), Jan.-2015.
11. Wali, R.S and Mandalgeri, P. S., On α regular ω-closed sets in Topological spaces, Int. J. of Math Archive 5(10), 2014, 68-76.

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