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MHD FLOW PAST AN IMPULSIVELY STARTED VERTICAL PLATE WITH VARIABLE TEMPERATURE IN THE PRESENCE OF HALL CURRENT

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ABSTRACT

In the present paper, MHD flow past an impulsively started vertical plate with variable temperature in the presence of Hall current is studied. The fluid considered is electrically conducting, absorbing-emitting radiation in a non-scattering medium. The Laplace transform technique has been used to find the solutions for the velocity profiles and Skin friction. The velocity profile and Skin friction have been studied for different parameters like Prandtl number, Hall parameter, magnetic field parameter, Thermal Grashof number, and time. The effect of parameters is shown graphically and the value of the skin-friction for different parameters has been tabulated.

Keywords: *MHD, Heat transfer, Hall current, Skin friction.*

INTRODUCTION

The study of MHD flow was first initiated by a Swedish electrical engineer Hannes Alfven [5]. MHD flow and Hall effect are encountered in power genertors, refrigeration coils, and electric transfers etc. The researchers have studied the effect of Hall current in various flow models. Ram *et al.* [10] have studied the Hall effects on heat and mass transfer flow through porous medium. Sulochana[13] have analyzed Hall effects on unsteady MHD three dimensional flow through a porous medium in a rotating parallel plate channel with effect of inclined magnetic field. Attia [6] studied the effect of variable properties on the unsteady Hartmann flow with heat transfer considering the Hall effect. Attia along with Ahmed [7] have further Studied the Hall effect on unsteady MHD Couette flow and heat transfer of a Bingham fluid with suction and injection. Ahmed *et al.* [9] have studied unsteady MHD free convective flow past a vertical porous plate immersed in a porous medium with Hall current, thermal diffusion and heat source. Rajput and Kumar [16] have studied MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion. We are extending their [16] work to study the effect of hall current in the flow.

The effects of Hall current on the velocity have been observed with the help of graphs, and the skin friction has been tabulated.

MATHEMATICAL FORMULATION

We consider an unsteady viscous incompressible electrically conducting fluid past an impulsively started vertical plate. The plate is electrically non conducting. A uniform magnetic field B is assumed to be applied on the flow. Initially the fluid and plate are at the same temperature T_∞. At time t>0, temperature of the plate is raised to T_w. Using the relation $\nabla \bullet B = 0$, for the magnetic field $\overline{B} = (B_x, B_y, B_z)$, we obtain B_y (say B_0) = constant, for $B = (0, B_0, 0)$, where B_0 is the externally applied transverse magnetic field. Due to the Hall effects there will be two components of the momentum equation, which are as under. The usual assumptions have been taken in to consideration.

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma\mu^2 B_0^{\ 2}}{\rho(1+m^2)}(u + mw), \tag{1}$$

$$\frac{\partial w}{\partial t} = \upsilon \frac{\partial^2 w}{\partial y^2} - \frac{\sigma\mu B_0^{\ 2}}{\rho(1+m^2)}(w - mu), \tag{2}$$

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$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

The following boundary conditions have been assumed:

$$\left.\begin{aligned} & t \le 0 : u = 0, w = 0, T = T_\infty \text{ for all the values of } y, \\ & t > 0 : u = u_0, w = 0, T = T_\infty + (T_w - T_\infty)\frac{u_0^{\,2} t}{\upsilon} \text{ at } y = 0, \\ & u \to 0, w \to 0, T \to T_\infty \text{ as } y \to 0, \end{aligned}\right\} \tag{4}$$

where u is the velocity of the fluid in x- direction, w - the velocity of the fluid in z- direction, m is the Hall parameter, g–acceleration due to gravity, β -volumetric coefficient of thermal expansion, t- time, T_∞ - the temperature of the fluid near the plate, T_w - temperature of the plate, T - the temperature of the fluid far away from the plate, k - the thermal conductivity, υ is the kinematic viscosity, ρ -the fluid density, σ -electrical conductivity, μ -the magnetic permeability, and C_P the specific heat at constant pressure. Here $m = \omega_e \tau_e$ with ω_e - cyclotron frequency of electrons and τ_e - electron collision time.

To write the equations (1) - (3) in dimensionless from, we introduce the following non - dimensional quantities:

$$\left.\begin{aligned} & \bar{u} = \frac{u}{u_0}, \bar{w} = \frac{w}{u_0}, \bar{y} = \frac{y u_0}{\upsilon}, \Pr = \frac{\mu C_P}{k}, \bar{t} = \frac{t u_0^{\,2}}{\upsilon}, \\ & M = \frac{\sigma B_0^2 \upsilon}{\rho u_0^2}, Gr = \frac{g\beta\upsilon(T_w - T_\infty)}{u_0^{\,3}}, \theta = \frac{T - T_\infty}{T_w - T_\infty}. \end{aligned}\right\} \tag{5}$$

Here the symbols used are:
$\bar{u}$ -dimensionless velocity, $\bar{w}$ -dimensionless velocity, Pr-Prandtl number, M-magnet field parameter, $\bar{y}$ -dimensionless coordinate axis normal to the plate, θ - dimensionless temperature, Gr - thermal Grashof number.

The dimensionless forms of Equation (1), (2), and (3) are as follows:

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + Gr\theta - \frac{M}{1+m^2}(\bar{u} + m\bar{w}), \tag{6}$$

$$\frac{\partial \bar{w}}{\partial \bar{t}} = \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} - \frac{M}{1+m^2}(\bar{w} - u\bar{m}), \tag{7}$$

$$\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial \bar{y}^2}. \tag{8}$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} & \bar{t} \le 0, \bar{u} = 0, \theta = 0, \bar{w} = 0, \text{ for all values of } y, \\ & \bar{t} > 0, \bar{u} = 1, \bar{w} = 0, \theta = t \text{ at } \bar{y} = 0, \\ & \bar{u} \to 0, \theta \to 0, \bar{w} \to 0 \text{ as } \bar{y} \to \infty. \end{aligned}\right\} \tag{9}$$

Dropping the bars and combining the Equations (6) and (7), we get

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} - \frac{M}{1+m^2} q(1 - mi) + Gr\theta, \tag{10}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}, \tag{11}$$

where $q = u + iw$, with corresponding boundary conditions

$$\left.\begin{array}{l} t \le 0, q = 0, \theta = 0, \text{for all values of y}, \\ t > 0, q = 1, \theta = t, \quad \text{at y} = 0, \\ q \to 0, \theta \to 0, w \to 0 \text{ as } \quad y \to \infty. \end{array}\right\} \tag{12}$$

Solving the above equations analytically, we obtain

$$q = \frac{1}{2} e^{-\sqrt{a}\, y}\left(1 + Erf\left[\frac{2\sqrt{at} - y}{2\sqrt{t}}\right] + e^{2\sqrt{a}\, y} Erfc\left[\frac{2\sqrt{at} + y}{2\sqrt{t}}\right]\right)$$
$$+\frac{1}{4a^2} y\, Gr\left\{A(1-a) + \sqrt{a}\, e^{-\sqrt{a}y}\left(1 - e^{2\sqrt{a}y} + Erf\left[\frac{2\sqrt{at} - y}{2\sqrt{t}}\right] + e^{2\sqrt{a}y} Erf\left[\frac{2\sqrt{at} + y}{2\sqrt{t}}\right]\right) + B(1-\Pr) - A\Pr\right\}$$
$$+\frac{1}{2a^2\sqrt{\pi}} yGr\left\{a\left(2e^{\frac{-y^2 \Pr}{4t}}\sqrt{t} + \sqrt{\pi}y\right)\left(-1 + Erf\left[\frac{y\sqrt{\Pr}}{2\sqrt{t}}\right]\right)\right\}\sqrt{\Pr}$$
$$+\left\{\left(\frac{C}{ye^{\frac{at}{-1+\Pr} - y\sqrt{\frac{a}{-1+\Pr}}\sqrt{\Pr}}\sqrt{\Pr}} - \frac{C}{ye^{\frac{at}{-1+\Pr} - y\sqrt{\frac{a}{-1+\Pr}}\sqrt{\Pr}}}\right) + (-1 + at + \Pr)D\right\}\sqrt{\Pr} \tag{13}$$

$$\theta = t\left[\left(1 + 2\eta^2 \Pr\right) erfc\left(\eta\sqrt{\Pr}\right) - \frac{2\eta\sqrt{\Pr}}{\sqrt{\pi}} e^{-\eta^2 \Pr}\right] \tag{14}$$

The expressions for the constants involved in the above equations are given in the appendix.

Skin friction

The dimensionless skin friction at the plate y=0

$$\left(\frac{dq}{dt}\right)_{y=0}$$

Separating $\left(\frac{dq}{dt}\right)_{y=0}$ into real and imaginary parts, the dimensionless skin friction component $\tau_x = \left(\frac{du}{dy}\right)_{y=0}$ and $\tau_z = \left(\frac{dw}{dy}\right)_{y=0}$ can be computed.

DISCUSSION AND RESULTS

The numerical values of velocity and skin friction are computed for different parameters like, Thermal Grashof number Gr, Magnetic field parameter M, Hall parameter m, Prandtl number Pr, and time t. The values of the main parameters considered are Gr=10, 20, 30, magnetic parameter M= 1, 3, 5, Hall parameter 0.5, 1.0, 1.5, Prandtl number Pr = 2, 4, 7 and t=0.4, 0.6, 0.8.

From figure - 1, 2 and 5 it is observed that the primary velocity (u) increases when Hall parameter, Grashof number and time are increased. However, it decreases when Magnetic field parameter and Prandtl number are increased (figure - 3, 4). And from figure - 6, 7, 8, 9 and 10 it is observed that the secondary velocity (w) increases when Hall parameter, Grashof number, Magnetic field parameter, Prandtl number, and time are increased. The values of the skin frictions τ_x and τ_z are tabulated in table –1. τ_x decreases when Grashof number, magnetic field parameter and time are increased.

But, it increases when Prandtl number and Hall parameter are increased. The skin friction τ_z increases with increase in Grashof number, time and magnetic field parameter. However τ_z decreases when Prandtl number is increased.

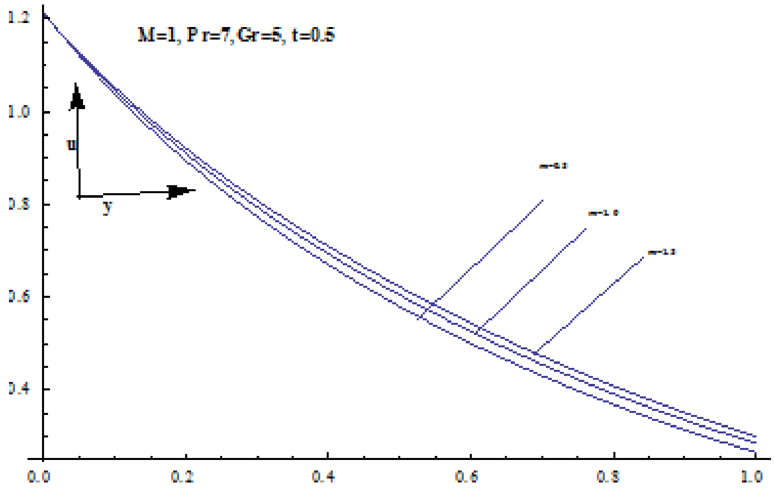

Figure-1: Velocity profiles u for different values of m.

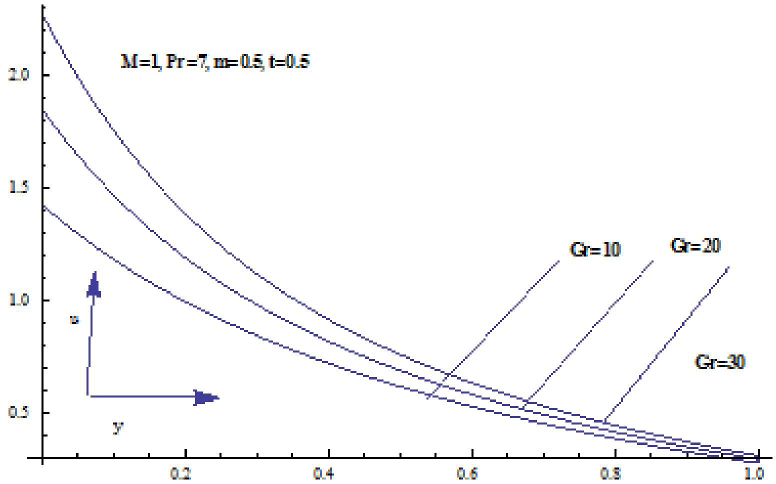

Figure-2: Velocity profiles for different values of Gr.

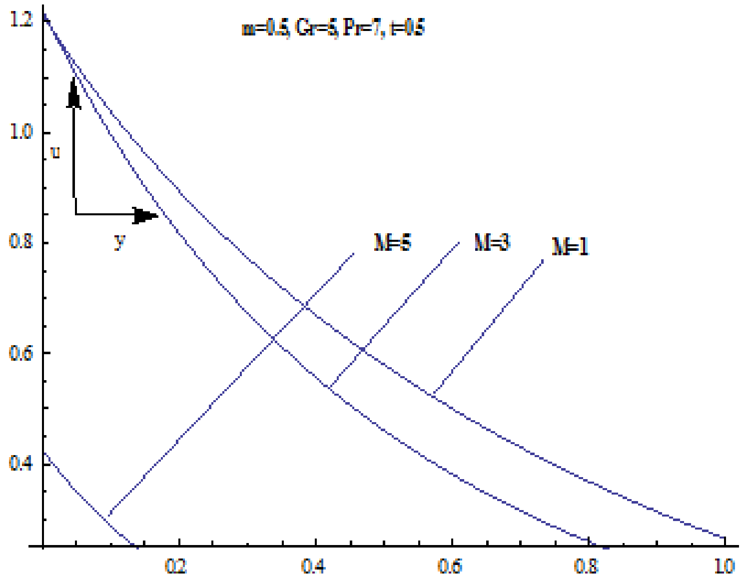

Figure-3 Velocity profiles u for different values of M.

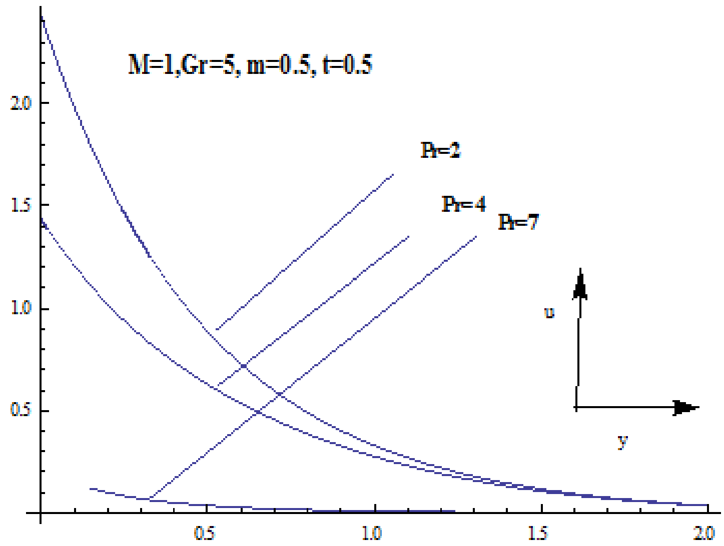

Figure-4: Velocity profiles for different values of Pr.

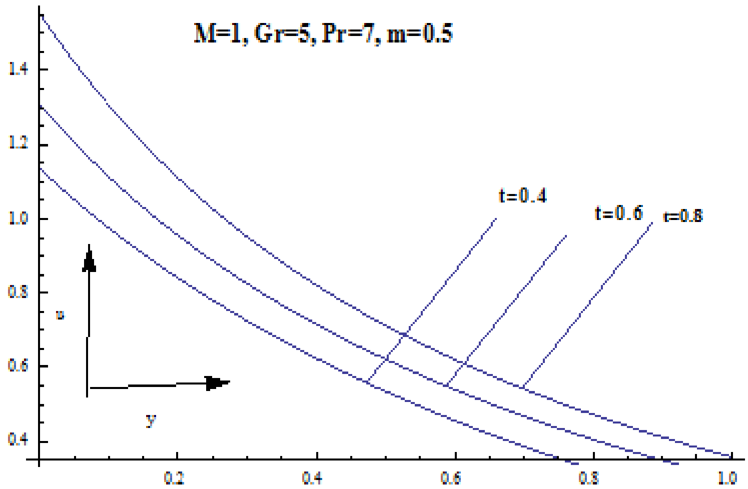

Figure-5: Velocity profile u for different values of t.

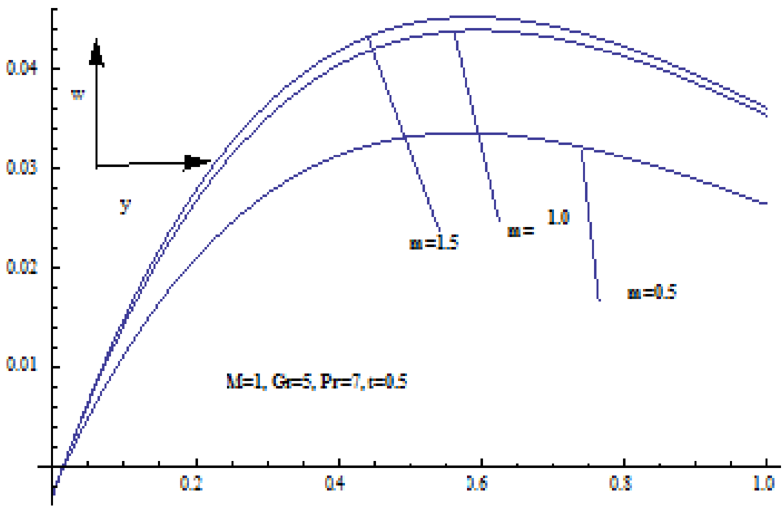

Figure-6: Velocity profile w for different values of m.

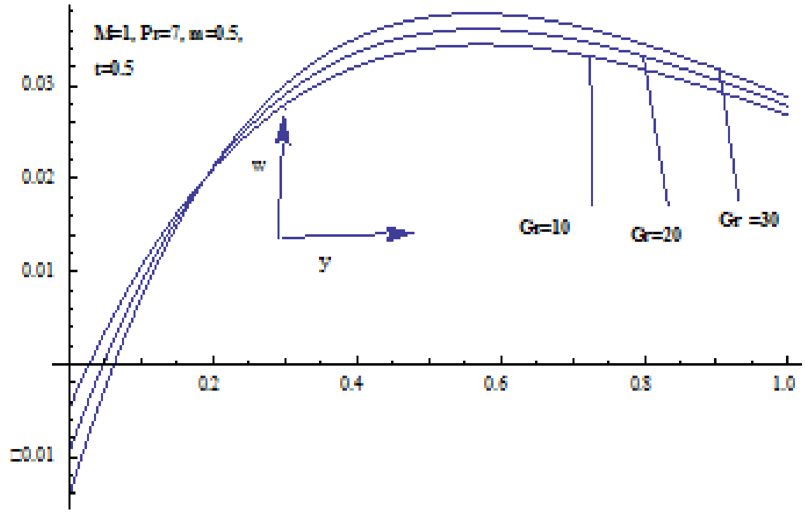

Figure-7: Velocity profile w for different values of Gr.

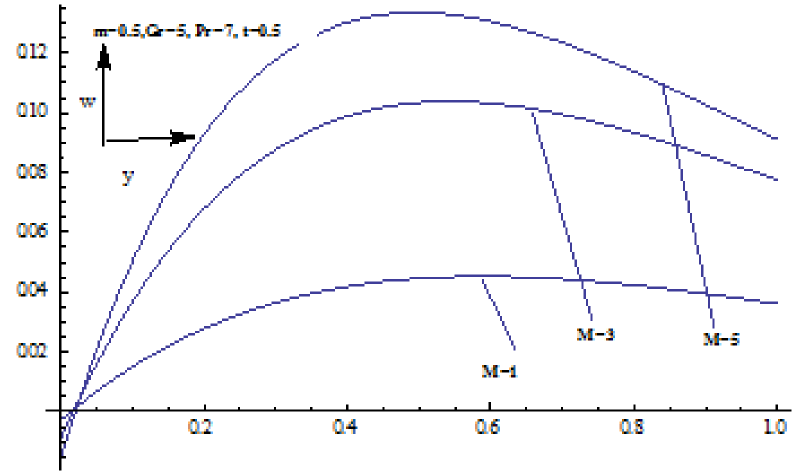

Figure-8: Velocity profile w for different values of M.

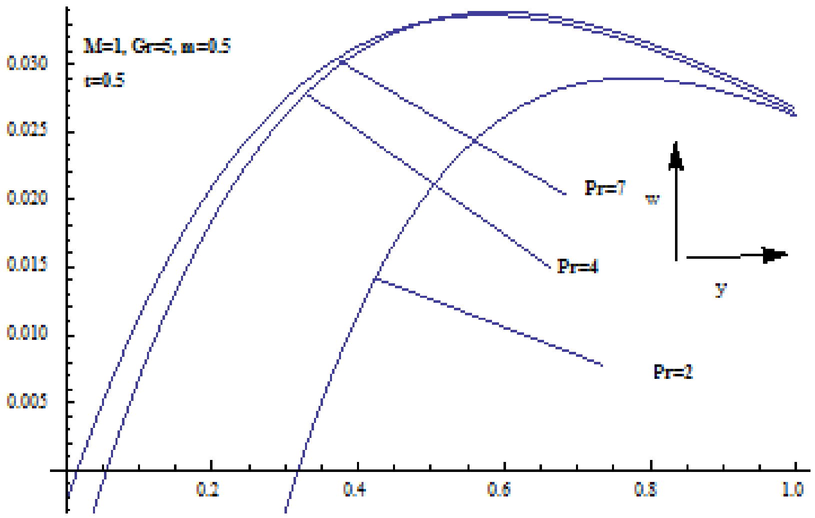

Figure-9: Velocity profile w for different values of Pr.

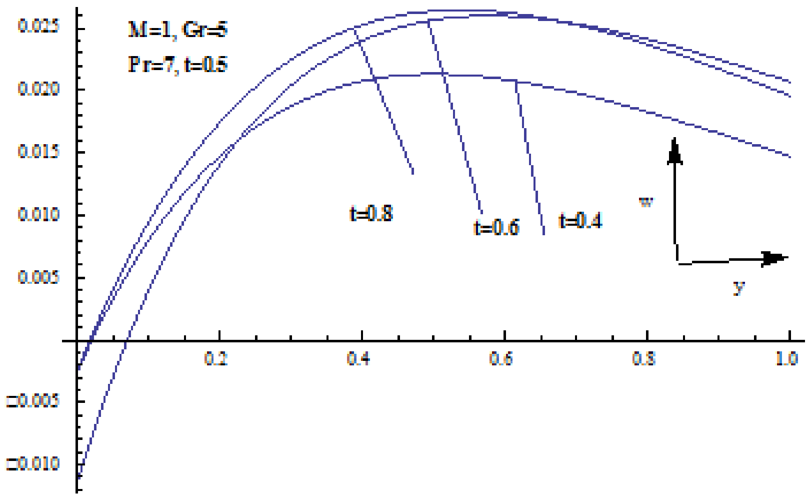

Figure-10: Velocity profile w for different values of t.

Table – 1: for the skin friction

M	m	Gr	Pr	T	τ_x	τ_z
1	0.5	5	7	0.5	-1.949	0.160
3	0.5	5	7	0.5	-2.553	0.399
5	0.5	5	7	0.5	-3.071	0.578
1	0.5	10	7	0.5	-2.796	0.180
1	0.5	20	7	0.5	-4.491	0.220
1	0.5	30	7	0.5	-6.186	0.259
1	0.5	5	2	0.5	-4.984	0.527
1	0.5	5	4	0.5	-2.540	0.196
1	0.5	5	7	0.5	-1.949	0.160
1	1.0	5	7	0.5	-1.829	0.208
1	0.5	5	7	0.4	-1.767	0.140
1	0.5	5	7	0.6	-2.182	0.181
1	0.5	5	7	0.8	-2.767	0.230

CONCLUSION

Some conclusions of the study are as under:

1. Primary Velocity increases with the increase in Grashof number, hall parameter and time.
2. Primary Velocity decreases with the increase in magnetic field parameter and Prandtl number.
3. Secondary velocity increases with increase in Grashof number, time, Hall parameter, Prandtl number and magnetic field parameter.
4. τ_x decreases with increase in Grashof number, magnetic field parameter and time, and it increases when Prandtl number and Hall parameter are increased.
5. τ_z increases with increase in Grashof number, time and magnetic, Hall parameter, field parameter, and it decreases when Prandtl number is increased.

APPENDIX

$$A=\frac{2e^{-\sqrt{a}\,y}(1+e^{2\sqrt{a}\,y}+Erf[\frac{2\sqrt{at}-y}{2\sqrt{t}}]-e^{2\sqrt{a}\,y}Erf[\frac{2\sqrt{at}+y}{2\sqrt{t}}])}{y},$$

$$B=\frac{1}{y}2e^{\frac{at}{-1+\Pr}-y\sqrt{\frac{a\Pr}{-1+\Pr}}}\left(-1-e^{2y\sqrt{\frac{a\Pr}{-1+\Pr}}}+Erf[\frac{y-2t\sqrt{\frac{a\Pr}{-1+\Pr}}}{2\sqrt{t}}]+e^{2y\sqrt{\frac{a\Pr}{-1+\Pr}}}Erf[\frac{y+2t\sqrt{\frac{a\Pr}{-1+\Pr}}}{2\sqrt{t}}]\right),$$

$$C=\sqrt{\pi}\left(-1-e^{2y\sqrt{\frac{a}{-1+\Pr}\Pr}}-Erf[\frac{2t\sqrt{\frac{a}{-1+\Pr}}-y\sqrt{\Pr}}{2t}+e^{2y\sqrt{\frac{a}{-1+\Pr}\Pr}}Erf[\frac{2t\sqrt{\frac{a}{-1+\Pr}}+y\sqrt{\Pr}}{2t}]\right),$$

$$D = \frac{2\sqrt{\pi}\left(-1 + Erf\left[\frac{y\sqrt{\Pr}}{2\sqrt{t}}\right]\right)}{y\sqrt{\Pr}},$$

$$a = \frac{M}{1+m^2}(1-im),\ \eta = \frac{y}{2\sqrt{t}}.$$

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