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NATURAL CONVECTION NEAR A VERTICAL PLATE IN A POROUS MEDIUM WITH RAMPED WALL TEMPERATURE

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ABSTRACT

Exact solution of unsteady natural convection flow past an infinite vertical plate with ramped wall temperature has been obtained. The dimensionless governing equations of the fluid flow are solved by Laplace-transform technique. The solutions for velocity, temperature are obtained and shown graphically for different physical parameters involved. Also, the variations of wall skin friction as well as Nusselt number with time have been presented graphically.

1. INTRODUCTION:

Free convection flows past an infinite vertical plate were studied extensively in the literature and play a vital role in many industrial and technological applications. Numerous investigations were performed using both analytical and numerical methods under different thermal conditions, which are continuous and well-defined at the wall. The authors [3-6, 12-14] have investigated a number of unsteady free convection flows near a vertical plate under different sets of thermal conditions at the bounding plate. Practical problems usually occupy wall conditions that are non-uniform or arbitrary. Therefore, it is very useful to investigate problems subject to step change in wall temperature. Schetz [1, 2] have investigated to develop an approximate analytical model for natural convection flow from a vertical plate with discontinuous wall temperature conditions. Different techniques were attempted on this problem. Hayday et al. [10] used a numerical approach, while Kelleher et al. [11] and Kao et al. [12] used series expansions.

Lee and Yovanovich [13] have attempted to improve the earlier results for step change in wall temperature. Chandran et al. [14] have analyzed the unsteady natural convection flow of a viscous incompressible fluid near a vertical plate with ramped wall temperature. Saha et al. [15] investigated the natural convection boundary layer adjacent to an inclined semi-infinite flat plate subjected to ramp heating. Recently, Deka et al. [16] have analyzed the unsteady free convection flow of a viscous incompressible fluid past an infinite vertical plate with ramped wall temperature in presence of heat source.

However, natural convection near an infinite vertical plate in a porous medium with ramped wall temperature has not been reported in the literature. Hence, it is now proposed to undertake this study. Here, we have considered the unsteady natural convection flow near a vertical flat plate in a porous medium and assumed that the wall temperature has a temporally ramped continuous profile. Solution for the flow problem has been obtained by Laplace–transform technique.

2. MATHEMATICAL FORMULATION:

We consider the unsteady free convection flow of a viscous incompressible fluid past an infinite vertical flat plate in a porous medium. The x' - axis is taken along the plate in the vertically upward direction, and the y' - axis is taken normal to the plate. Initially, for time $t' \le 0$, both the fluid and the plate are at rest and assumed to be at the same temperature T'_∞. At time $t' > 0$, the temperature of the plate raised to or lowered to $T'_\infty + \left(T'_w - T'_\infty\right)\dfrac{t'}{t_0}, when\ t' \le t_0$ and thereafter, for $t' > t_0$, is maintained at the constant temperature T'_w. We assume that the flow is laminar and is such that the effects of the convective and pressure gradients term in the momentum equations can be neglected. Moreover, as the plate is infinite in extent in the x' -direction, so the derivatives of all the flow variables with respect to x' vanish and as a result of the boundary layer approximations, the physical variables are functions of y', t' only.

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Thus the motion is one dimensional with only non-zero vertical velocity u' varying with y' and t' only. Due to one-dimensional nature, the equation of continuity is trivially satisfied.

Then under usual Boussinesq's approximation, the flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) - \frac{\nu}{k}u' \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} \tag{2}$$

where u' is the velocity in the x'-direction.

The initial and boundary conditions are

$$\left.\begin{array}{ll} t' \le 0: u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty & \forall y' \ge 0 \\ t' > 0: \left\{\begin{array}{ll} u' = 0, & at \quad y' = 0 \\ T' = \left\{\begin{array}{ll} T'_\infty + (T'_w - T'_\infty)\dfrac{t'}{t_0}, & for\ 0 < t' \le t_0 \\ T'_w & for\ t' > t_0 \end{array}\right\} & at \quad y' = 0 \\ u' \to 0, \quad T' \to T'_\infty, & as \quad y' \to \infty \end{array}\right\} \end{array}\right\}$$

(3)

Now, we define the following non-dimensional variables,

$$\left.\begin{array}{l} u = u'\sqrt{\dfrac{t_0}{\nu}}, \quad t = \dfrac{t'}{t_0}, \quad y = \dfrac{y'}{\sqrt{\nu t_0}}, \\ T = \dfrac{T' - T'_\infty}{T'_w - T'_\infty}, Pr = \dfrac{\mu C_p}{k}, Da = \dfrac{k}{\nu t_0} \end{array}\right\} \tag{4}$$

where t_0 is characteristic time defined by

$$t_0 = \left[\frac{\sqrt{\nu}}{g\beta(T'_w - T'_\infty)}\right]^{\frac{2}{3}}$$

Then introducing (4), the governing equations (1) - (2) reduce to,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + T - \frac{1}{Da}u \tag{5}$$

$$Pr\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2} \tag{6}$$

The initial and boundary conditions in non-dimensional form are,

$$\left.\begin{array}{ll} t \le 0: \quad u = 0, \quad T = 0, & \forall y \ge 0 \\ t > 0: \left\{\begin{array}{ll} u = 0, & at\ y = 0 \\ T = \left\{\begin{array}{ll} t & for\ 0 < t \le 1 \\ 1 & for\ t > 1 \end{array}\right\} & at\ y = 0 \\ u \to 0, \quad T \to 0 & as\ y \to \infty \end{array}\right\} \end{array}\right\} \tag{7}$$

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3. METHOD OF SOLUTION:

The dimensionless governing equations (5) and (6) subjects to the boundary condition (7) are solved by usual Laplace-transform technique and the solutions are as follows:

$$T(y,t) = F(y,t) - F(y,t-1)H(t-1) \tag{8}$$

$$u(y,t) = \frac{1}{B^2(1-Pr)}\left[\left\{u_1(y,t) - u_2(y,t)\right\} - \left\{u_1(y,t-1) - u_2(y,t-1)\right\}H(t-1)\right] \tag{9}$$

where $\mathrm{F}(y,t) = \left(t + \frac{Pry^2}{2}\right) erfc(\frac{y\sqrt{Pr}}{2\sqrt{t}}) - \sqrt{\frac{Prt}{\pi}}\; ye^{-\left(\frac{Pry^2}{4t}\right)}$

$$u_1(y,t) = B\left[\left(t + \frac{\Pr y^2}{2}\right) erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}\right) - \frac{\sqrt{\Pr t}}{\sqrt{\pi}}\, ye^{-\frac{y^2 \Pr}{4t}}\right] +$$
$$+\frac{e^{-Bt}}{2}\left[e^{iy\sqrt{\Pr B}} erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}} + i\sqrt{Bt}\right) + e^{-iy\sqrt{\Pr B}} erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}} - i\sqrt{Bt}\right)\right] - erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}\right)$$

$$u_2(y,t) = \frac{Bt}{2}\left[e^{y\sqrt{A}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{At}\right) + e^{-y\sqrt{A}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{At}\right)\right] -$$
$$-\frac{yB}{4\sqrt{A}}\left[e^{-y\sqrt{A}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{At}\right) - e^{y\sqrt{A}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{At}\right)\right] +$$
$$+\frac{e^{-Bt}}{2}\left[e^{y\sqrt{A-B}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(A-B)t}\right) + e^{-y\sqrt{A-B}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(A-B)t}\right)\right] -$$
$$-\frac{1}{2}\left[e^{y\sqrt{A}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{At}\right) + e^{-y\sqrt{A}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{At}\right)\right]$$

and

$$A = \frac{1}{Da}, \quad \mathrm{B} = \frac{1}{Da(1-\Pr)}$$

Also, $H(t-1)$ is the unit step function defined as $H(t-1) = \begin{cases} 1, & t \geq 1 \\ 0, & 0 \leq t < 1 \end{cases}$.

Now, to compare the solutions for velocity and temperature obtained above with the solutions for constant wall temperature, we have obtained the solutions for velocity and temperature for the later case as,

$$T(y,t) = erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}\right) \tag{10}$$

$$u(y,t) = \frac{1}{B(1-\Pr)}[u_3(y,t) - u_4(y,t)] \tag{11}$$

where

$$u_3(y,t) = erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}\right) - \frac{e^{-Bt}}{2}\left[e^{iy\sqrt{\Pr B}} erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}} + i\sqrt{Bt}\right) + e^{-iy\sqrt{\Pr B}} erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}} - i\sqrt{Bt}\right)\right]$$

$$u_4(y,t) = \frac{1}{2}\left[e^{y\sqrt{A}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{At}\right) + e^{-y\sqrt{A}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{At}\right)\right] -$$

$$-\frac{e^{-bt}}{2}\left[e^{y\sqrt{A-b}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(A-b)t}\right)+e^{-y\sqrt{A-b}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(A-b)t}\right)\right]$$

4. RESULTS AND DISCUSSIONS:

In order to get physical insight into the problem, we have carried out computation for *T* and *u* for different physical parameters, such as Pr , Da and *t* and analyzed graphically.

In Fig-I and Fig-II, the temperature profiles for air (Pr = 0.71) and water (Pr =7.0) are studied. It is clear that there is a fall in temperature with increase in the Prandtl number for all *t*. It is also observed that the heat transfer is more in air than in water. From the temperature profiles presented in Fig-I and Fig-II, we notice that the temperature decreases with *y* from its ramped value on the plate to its free stream value and it increases with *t*. In Fig-II, we have noticed the temperature variation of the two types of boundary conditions; namely, ramped and constant plate temperatures and observed that the fluid temperature is greater in the case of isothermal plate than in the case of ramped temperature at the plate. This should be expected since in the later case, the heating of the fluid takes place more gradually than in the isothermal case.

The velocity profiles for different values of parameters are shown in Fig-III and in Fig-IV. The effect of velocity for different time t= (0.2, 2), $Da = 0.01$, Pr =0.71 and Pr =7.0 are shown in Fig–III which demonstrates the inverse variation of temperature with Prandtl number. Again from Fig-III and Fig-IV we observed direct variation with time t. It is also seen that the velocity in the case of ramped temperature is always less than that the flow induced by a plate of constant temperature. The velocity profiles here are almost parabolic near the plate and the points of maxima on the curves get shifted to the right as t increases. Moreover, we observe that the velocity increases with the increase in Darcy number Da increases.

5. NUSSELT NUMBER AND SKIN-FRICTION:

We now study the heat transfer coefficient i.e.Nusselt number, which is given by

$$Nu = -\left.\frac{\partial T}{\partial y}\right|_{y=0}$$

$$= 2\sqrt{\frac{Pr}{\pi}}\left[\sqrt{t}-\sqrt{t-1}H(t-1)\right]$$

Nu is calculated for different Pr and t and it is presented in Fig-V. It is clear from the figure that for the ramped plate temperature, the Nusselt number increases for a range $0 < t \le 1,$ and decreases for $t > 1,$ for all Pr . But in case of constant wall temperature, the Nusselt number decreases with respect to t.

From the velocity field, we now study the skin friction. The non-dimensional form of skin-friction is given by

$$\tau = -\left.\frac{du}{dy}\right|_{y=0}$$

$$= -\frac{1}{B^2(1-\mathrm{Pr})}[\{F_1(y,t)-F_2(y,t)\}-\{F_1(y,t-1)-F_2(y,t-1)\}H(t-1)] \tag{12}$$

where

$$F_1(y,t) = -2B\sqrt{\frac{\mathrm{Pr}\,t}{\pi}} - i\sqrt{\mathrm{Pr}\,B}e^{-Bt}erf(i\sqrt{Bt})$$

$$F_2(y,t) = \left(1-Bt-\frac{B}{2A}\right)\sqrt{A}erf(\sqrt{At}) - \frac{B\sqrt{t}e^{-At}}{\sqrt{\pi}} - \sqrt{A-B}e^{-Bt}erf(\sqrt{(A-B)t})$$

Thus from (12), we obtain the numerical values of τ for different values of *Pr* (=0.71, 7.0), Da (= 0.01 and 0.05), and depicted in Fig-VI.

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From Fig-VI we conclude that due to the presence of porous medium, the skin friction is greater in case of ramped temperature of the plate than the case of isothermal plate. It is also noted that for small values of t ($t < 1$), there is a quick fall in the skin friction in the case of ramped temperature of the plate, whereas the skin friction decreases more gradually with increase in time for the case of isothermal plate. Further, we observe that the skin friction coefficient increases with the decreasing values of the Darcy number. It is also noted that the skin friction increases with increase in Pr. Lastly, we find that the skin-friction is more in water as compared to that in air.

6. CONCLUSIONS:

An exact solution is obtained to study the unsteady free convection flow past an infinite vertical plate in the presence of porous medium with ramped wall temperature. The non-dimensional forms of the governing equations of the fluid flow are solved by Laplace-transform technique. The solutions for velocity, temperature are studied graphically. Also, the Nusselt number and skin friction have been discussed. The following conclusions are being made during the study:

(1) An increase in Pr leads to a decrease in the temperature, but as the time t increases, the temperature increases.
(2) The temperature decreases with y from its ramped value at the plate to its free stream value.
(3) Fluid velocity increases with increasing values of Da and time t, but decreases when Pr increases.
(4) Nusselt number increases for $0 < t \leq 1$, and decreases for $t > 1$, for all Pr.
(5) Skin friction increases with increasing Pr and decreases with increasing values of t and Da.

NOMENCLATURE:

C_p - Specific heat of the fluid at constant pressure
Da - Darcy number
g - Acceleration due to gravity
k - Thermal conductivity of the fluid
Pr - Prandtl number
T' - Temperature of fluid
T'_w - Plate temperature
T'_∞ - Temperature of fluid far away from the plate
t′ - Time
t_0 - Characteristic time
u' - Velocity of fluid in the x′-direction
u - Dimensionless velocity component in the x′-direction
y' - Coordinate normal to the plate
y - Dimensionless coordinate normal to the plate

GREEK SYMBOLS:

β - Volumetric coefficient of thermal expansion
μ- Coefficient of viscosity
ν- Kinematic viscosity
ρ- Density of fluid
θ - Dimensionless temperature
erfc- Complementary error function
erf- error function

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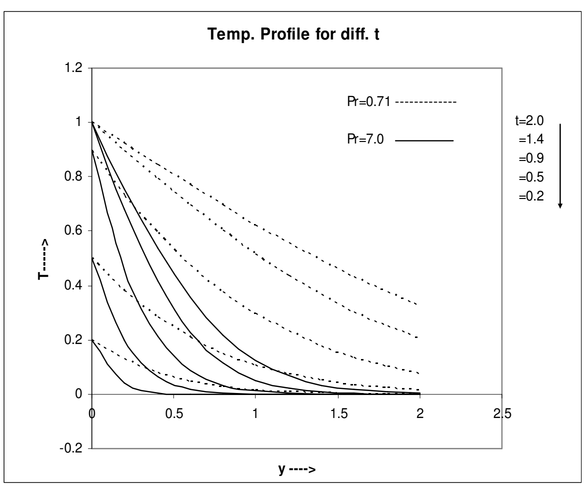

Fig-I

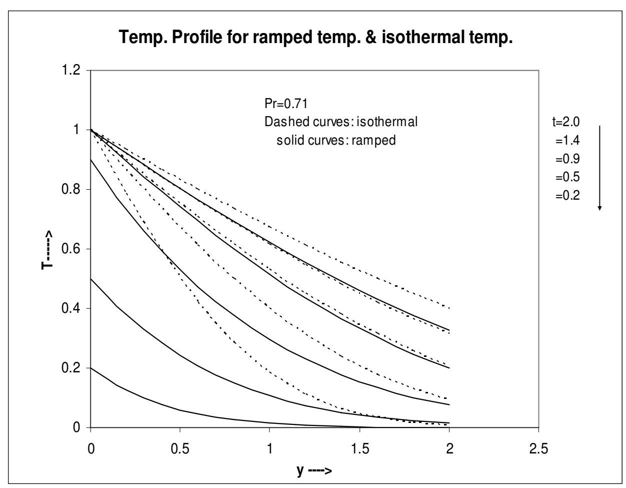

Fig-II

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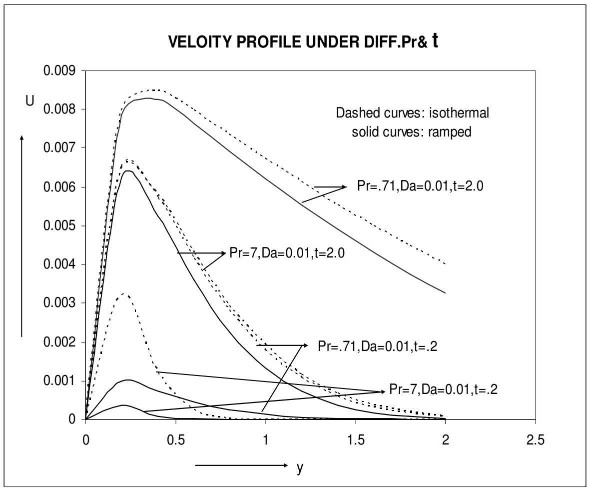

Fig-III

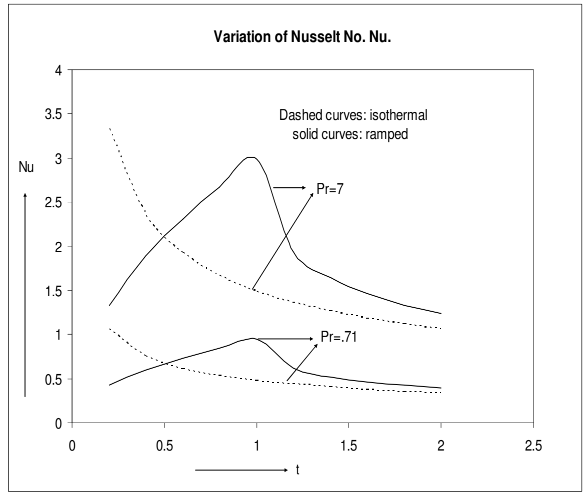

Fig-V

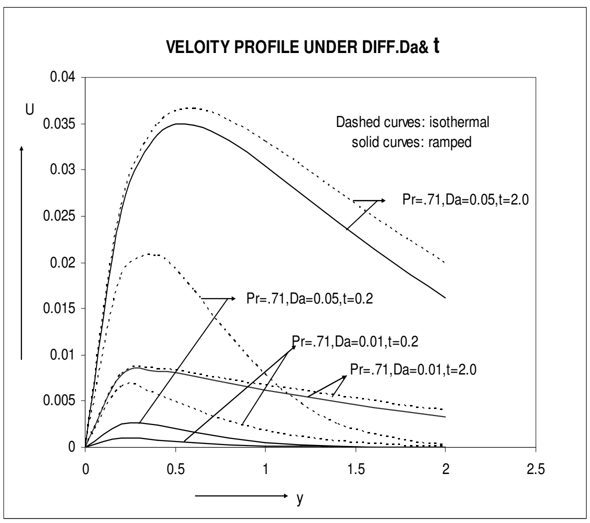

Fig-IV

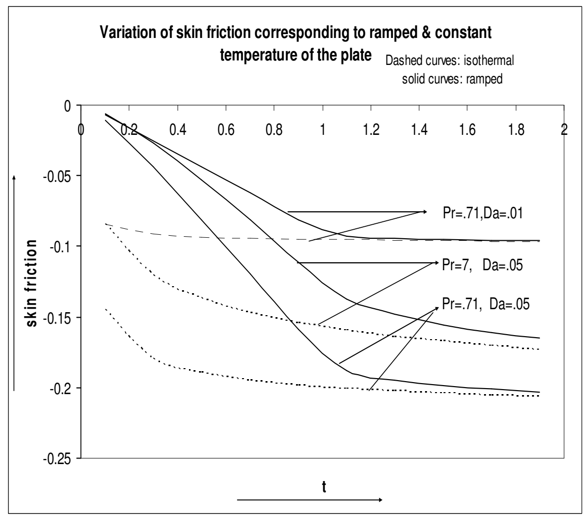

Fig-VI