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THE STUDY OF MATTER SYMMETRIES IN HIGHER DIMENSIONAL LRS BIANCHI TYPE-I SPACE-TIME WITH PERFECT FLUID

R. S. RANE[1], S. S. DABHANE*[2]

[1]Department of Mathematics, Y. C. Arts and Science College, Mangrulpir, India.

[2]Department of Mathematics,
Dr. Sau. Kamaltai Gawai Institute of Engg. & Tech., Darapur, India.

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ABSTRACT

In the present paper, the higher dimensional LRS Bianchi type-I space-time has been classified according to matter symmetries (collineations) in presence of perfect fluid. We have obtained different constraint conditions on energy-momentum tensor (T_{ab}) for perfect fluid. If we solve these constraint conditions, we obtain new exact solutions of Einstein field equations. These are studied when energy-momentum tensor is degenerate and non-degenerate by applying these constraint conditions on energy-momentum tensor (T_{ab}). We have considered only those cases which satisfy energy conditions for perfect fluid.

Keywords: *Higher dimensional LRS Bianchi type-I, perfect fluid, matter symmetries (collineations), energy-momentum tensor.*

1. INTRODUCTION

In recent years, the study of higher dimensional space-time has an active field of research in the quest of unification of gravity with other fundamental forces in physics. Higher dimensional theories proposed by Kaluza and Klein [1, 2]. They realized that a unification of gravity and electromagnetism could exist in a world with 4+1 space-time dimensions. The idea of higher dimension is particularly important in the field of cosmology to know the exact physical situation at very early stages of the formation of universe. The number of authors [3-7] have studied the higher dimensional space-time in cosmology.

In this paper, the motivation for studying matter symmetries in higher dimensional space-time, we obtain exact solutions to the Einstein's field equations which are highly non-linear partial differential equations. The Einstein's field equations are given by

$$R_{ab} - \frac{1}{2} R g_{ab} = \kappa T_{ab}, \tag{1.1}$$

where R_{ab} is Ricci tensor, $R = g^{ab} R_{ab}$ is a Ricci scalar, T_{ab} is energy-momentum tensor. Here we have assumed cosmological constant $\Lambda = 0$ and gravitational constant $\kappa = 1$.

Due to the non-linearity equations, we assume certain symmetries of the space-time metric and these symmetry assumptions are expressed in terms of isometries of metric tensor, called Killing vectors, which give rise to conservation laws [8-19]. The symmetries of energy-momentum tensor give conservation laws on matter fields. Such symmetries are called matter symmetries (collineations). The matter symmetries (collineations) can be written as [20-24]

$$L_X T_{ab} = 0 \Leftrightarrow T_{ab,c} X^c + T_{ac} X^c_{,b} + T_{bc} X^c_{,a} = 0, \quad (a, b, c = 1, 2, 3, 4, 5) \tag{1.2}$$

where L_X is Lie derivative operator along vector field $\boldsymbol{X}$ which generates the symmetry.

In this paper, the higher dimensional LRS Bianchi type-I space-time has been studied according to matter symmetries (collineations) in presence of energy-momentum tensor for a perfect fluid. We have obtained different constraint conditions on energy-momentum tensor (T_{ab}) for a perfect fluid. The plan of this paper is as follows. In the next section we characterize the matter collineations equations for higher dimensional LRS Bianchi type-I space-time in presence of perfect fluid. In section (2.1), matter symmetry equations for this space-time are solved when energy-momentum tensor is degenerate, while in section (2.2), we find a general classification for the non-degenerate case for perfect fluid. Lastly, we conclude the discussion and conclusion of the results obtained.

Corresponding Author: S. S. Dabhane[2]*

2. MATTER SYMMETRIES EQUATIONS

Here, we consider homogeneous higher dimensional LRS Bianchi type-I metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2(dy^2 + dz^2) + C^2dw^2, \tag{2.1}$$

where A, B and C are the functions of t alone.

The expression for the energy-momentum tensor with a perfect fluid has the form

$$T_{ab} = (\rho + p)u_a u_b + pg_{ab}. \tag{2.2}$$

where u^a is the five-velocity vector such that $u_a u^a = -1$, and ρ and p are the energy density and pressure of the fluid respectively.

Non-zero components of the Ricci tensor are

$$R_{11} = -A\ddot{A} - \frac{A\dot{A}\dot{C}}{C} - \frac{2A\dot{A}\dot{B}}{B} \tag{2.3}$$

$$R_{22} = R_{33} = -\dot{B}^2 - B\ddot{B} - \frac{\dot{A}\dot{B}B}{A} - \frac{\dot{B}B\dot{C}}{C} \tag{2.4}$$

$$R_{44} = -C\ddot{C} - \frac{\dot{A}\dot{C}C}{A} - \frac{2C\dot{C}\dot{B}}{B} \tag{2.5}$$

$$R_{55} = \frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}. \tag{2.6}$$

Here overhead dot denotes differentiation with respect to t. Ricci scalar is given by

$$R = -2\left[\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + 2\frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{B}\dot{C}}{BC} - \frac{\dot{B}^2}{B^2}\right]. \tag{2.7}$$

Using Einstein field equations (1.1), non-zero components of energy-momentum tensor are

$$T_{11} = A^2\left[2\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + 2\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{B}^2}{B^2}\right] \tag{2.8}$$

$$T_{22} = T_{33} = B^2\left[\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{\ddot{A}}{A} + \frac{\ddot{C}}{C}\right] \tag{2.9}$$

$$T_{44} = C^2\left[\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2}\right] \tag{2.10}$$

$$T_{55} = -\left[2\frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}^2}{B^2}\right]. \tag{2.11}$$

For the perfect fluid higher dimensional Bianchi type-I metric, using equation (2.2) we have,
$T_{11} = pA^2, T_{22} = T_{33} = pB^2, T_{44} = pC^2, T_{55} = \rho$ which yields following metric

$$ds^2_{perfect} = \rho\, dt^2 + p[A^2dx^2 + B^2(dy^2 + dz^2) + C^2dw^2]. \tag{2.12}$$

The matter tensor metric is positive-definite when $\rho > 0$ and $p > 0$.

For the perfect fluid the energy conditions are given as [25],

$$p = p(\rho), \rho > 0, 0 \le p \le \rho. \tag{2.13}$$

Using equation (1.2), we have following matter symmetries equations as,

$$T_{11,5}X^5 + 2T_{11}X^1_{,1} = 0 \tag{2.14}$$

$$T_{11}X^1_{,2} + T_{22}X^2_{,1} = 0 \tag{2.15}$$

$$T_{11}X^1_{,3} + T_{22}X^3_{,1} = 0 \tag{2.16}$$

$$T_{11}X^1_{,4} + T_{44}X^4_{,1} = 0 \tag{2.17}$$

$$T_{11}X^1_{,5} + T_{55}X^5_{,1} = 0 \tag{2.18}$$

$$T_{22,5}X^5 + 2T_{22}X^2_{,2} = 0 \tag{2.19}$$

$$T_{22}\left(X^2_{,3} + X^3_{,2}\right) = 0 \tag{2.20}$$

$$T_{22}X^2_{,4} + T_{44}X^4_{,2} = 0 \tag{2.21}$$

$$T_{22}X^2_{,5} + T_{55}X^5_{,2} = 0 \tag{2.22}$$

$$T_{22,5}X^5 + 2T_{22}X^3_{,3} = 0 \tag{2.23}$$

$$T_{22}X^3_{,4} + T_{44}X^4_{,3} = 0 \tag{2.24}$$

$$T_{22}X^3_{,5} + T_{55}X^5_{,3} = 0 \tag{2.25}$$

$$T_{44,5}X^5 + 2T_{44}X^4_{,4} = 0 \tag{2.26}$$

$$T_{44}X^4_{,5} + T_{55}X^5_{,4} = 0 \tag{2.27}$$

$$T_{55,5}X^5 + 2T_{55}X^5_{,5} = 0, \tag{2.28}$$

where comma denotes the partial derivatives and indices 1, 2, 3, 4, 5 corresponds to variables x, y, z, w and t respectively.

2.1. MATTER SYMMETRIES FOR THE DEGENERATE ENERGY-MOMENTUM TENSOR FOR PERFECT FLUID

For the degenerate energy-momentum tensor i.e. when $det(T_{ab}) = 0$, we have following single possibility when energy conditions (2.13) for perfect fluid are satisfied,

$T_{11} = T_{22} = T_{33} = T_{44} = 0$ and $T_{55} \neq 0$.

This gives, $\rho > 0$ and $p = 0$.

For this case, using (2.14)-(2.28), we have following matter symmetries equations,

$$T_{55} X^5_{,a} = 0, \quad (a = 1,2,3,4) \tag{2.1.1}$$
$$T_{55,5} X^5 + 2T_{55} X^5_{,5} = 0. \tag{2.1.2}$$

Now, equation (2.1.1) yields, X^5 is function of t alone.

On solving equation (2.1.2), we get
$X^5 = \frac{c_1}{\sqrt{T_{55}}}$ and X^1, X^2, X^3 and X^4 are arbitrary functions of x, y, z, w and t.

2.2. MATTER SYMMETRIES FOR THE NON-DEGENERATE ENERGY-MOMENTUM TENSOR FOR PERFECT FLUID

For the non-degenerate energy-momentum tensor i.e. when $det(T_{ab}) \neq 0$, we have the following single possibility when energy conditions for perfect fluid are satisfied,

$T_{aa} \neq 0 \quad (a = 1,2,3,4,5)$ *i.e.* $\rho > 0$ and $p > 0$.

Using above condition, we have following cases of non-degeneracy,
I] When one component of $\boldsymbol{X} = (X^1, X^2, X^3, X^4, X^5)$ is non-zero *i.e.*

i. $X^1 \neq 0,\ X^2 = X^3 = X^4 = X^5 = 0$
ii. $X^2 \neq 0,\ X^1 = X^3 = X^4 = X^5 = 0$
iii. $X^3 \neq 0,\ X^1 = X^2 = X^4 = X^5 = 0$
iv. $X^4 \neq 0,\ X^1 = X^2 = X^3 = X^5 = 0$
v. $X^5 \neq 0,\ X^1 = X^2 = X^3 = X^4 = 0$.

II] When two components of $\boldsymbol{X} = (X^1, X^2, X^3, X^4, X^5)$ are non-zero *i.e.*

i. $X^1 \neq 0,\ X^2 \neq 0,\ X^3 = X^4 = X^5 = 0$
ii. $X^1 \neq 0,\ X^3 \neq 0,\ X^2 = X^4 = X^5 = 0$
iii. $X^1 \neq 0,\ X^4 \neq 0,\ X^2 = X^3 = X^5 = 0$
iv. $X^1 \neq 0,\ X^5 \neq 0,\ X^2 = X^3 = X^4 = 0$
v. $X^2 \neq 0,\ X^3 \neq 0,\ X^1 = X^4 = X^5 = 0$
vi. $X^2 \neq 0,\ X^4 \neq 0,\ X^1 = X^3 = X^5 = 0$
vii. $X^2 \neq 0,\ X^5 \neq 0,\ X^1 = X^3 = X^4 = 0$
viii. $X^3 \neq 0,\ X^4 \neq 0,\ X^1 = X^2 = X^5 = 0$
ix. $X^3 \neq 0,\ X^5 \neq 0,\ X^1 = X^2 = X^4 = 0$
x. $X^4 \neq 0,\ X^5 \neq 0,\ X^1 = X^2 = X^3 = 0$.

III] When three components of $\boldsymbol{X} = (X^1, X^2, X^3, X^4, X^5)$ are non-zero *i.e.*

i. $X^1 \neq 0,\ X^2 \neq 0,\ X^3 \neq 0,\ X^4 = X^5 = 0$
ii. $X^1 \neq 0,\ X^2 \neq 0,\ X^5 \neq 0,\ X^3 = X^4 = 0$
iii. $X^1 \neq 0,\ X^4 \neq 0,\ X^5 \neq 0,\ X^2 = X^3 = 0$
iv. $X^1 \neq 0,\ X^2 \neq 0,\ X^4 \neq 0,\ X^3 = X^5 = 0$
v. $X^2 \neq 0,\ X^3 \neq 0,\ X^4 \neq 0,\ X^1 = X^5 = 0$.

For case I (i), matter symmetries equations, (2.14)-(2.28) becomes,
$T_{11} X^1_{,a} = 0 \quad (a = 1,2,3,4,5)$ which implies $X^1 = c_1$.

Similarly, for the sub-cases I-(ii), I-(iii), I-(iv), we have, solutions as $X^2 = c_2, X^3 = c_3, X^4 = c_4$ respectively. And for sub-case I-(v), we obtain the constraint equations lead to

$$T_{aa,5} X^5 = 0 \quad (a = 1,2,4) \tag{2.2.1}$$
$$T_{55} X^5_{,b} = 0 \quad (b = 1,2,3,4) \tag{2.2.2}$$
$$T_{55,5} X^5 + 2T_{55} X^5_{,5} = 0. \tag{2.2.3}$$

From equations (2.2.2) and (2.2.3), $X^5 = \frac{c_5}{\sqrt{T_{55}}}$ with constraint condition
$T_{dd} = \alpha = const. \neq 0 \ (d = 1, 2, 4)$ where c_1, c_2, c_3, c_4, c_5 are the constants.

For case II (i), substitute $X^1 \neq 0,\ X^2 \neq 0,\ X^3 = X^4 = X^5 = 0$ in equations (2.14)-(2.28), we get

$$T_{11}X^1_{,a} = 0 \quad (a = 1,3,4,5) \tag{2.2.4}$$
$$T_{22}X^2_{,b} = 0 \quad (b = 2,3,4,5) \tag{2.2.5}$$
$$T_{11}X^1_{,2} + T_{22}X^2_{,1} = 0. \tag{2.2.6}$$

Then we arrive at solutions,

$X^1 = c_1,\ X^2 = c_2.$

Similarly, in the same way, solution for sub-case II-(ii), $X^1 = c_3,\ X^3 = c_4$, for sub-case II-(iii), $X^1 = c_5,\ X^4 = c_6$ and for sub-case II-(iv), $X^1 = c_7,\ X^5 = \frac{c_8}{\sqrt{T_{55}}}$ with constraint condition $T_{dd} = \alpha = const. \neq 0 \ (d = 1, 2, 4)$.

Now, for the sub-case II-(v), from equations (2.14)-(2.28), we have

$$T_{22}X^2_{,a} = 0 \quad (a = 1,2,4,5) \tag{2.2.7}$$
$$T_{22}X^3_{,b} = 0 \quad (b = 1,3,4,5) \tag{2.2.8}$$
$$T_{22}\left(X^2_{,3} + X^3_{,2}\right) = 0. \tag{2.2.9}$$

Equation (2.2.7) and (2.2.8), gives, $X^2 = f(z)$ and $X^3 = g(y)$ then using equation (2.2.9), we have
$f(z)_{,3} = -g(y)_{,2} = \delta$, where δ is constant.
If $\delta = 0$, then $f = c_1$ and $g = c_2$.
If $\delta \neq 0$, then $f = c_1 + \delta . z$ and $g = c_2 - \delta . y$.

Now, using equations (2.14)-(2.28), for sub-cases II-(vi), II-(vii), II-(viii), II-(ix) and II-(x), we have similar type of solutions as in sub-cases II-(ii), II-(iii) and II-(iv).

For case III-(i), we put, $X^1 \neq 0,\ X^2 \neq 0,\ X^3 \neq 0,\ X^4 = X^5 = 0$ in equations (2.14)-(2.28), we get

$$T_{11}X^1_{,a} = 0 \quad (a = 1,4,5) \tag{2.2.10}$$
$$T_{22}X^2_{,b} = 0 \quad (b = 2,4,5) \tag{2.2.11}$$
$$T_{22}X^3_{,c} = 0 \quad (c = 3,4,5) \tag{2.2.12}$$
$$T_{11}X^1_{,2} + T_{22}X^2_{,1} = 0 \tag{2.2.13}$$
$$T_{11}X^1_{,3} + T_{22}X^3_{,1} = 0 \tag{2.2.14}$$
$$T_{22}\left(X^2_{,3} + X^3_{,2}\right) = 0. \tag{2.2.15}$$

Then we arrive at solutions,

$X^1 = c_4, X^2 = -c_1 z + c_3, X^3 = c_1 y + c_2.$

For case III (ii), in this sub-case, $X^1 \neq 0,\ X^2 \neq 0,\ X^5 \neq 0,\ X^3 = X^4 = 0$, in (2.14)-(2.28), we have

$$T_{11}X^1_{,a} = 0 \tag{2.2.16}$$
$$T_{22}X^2_{,a} = 0 \tag{2.2.17}$$
$$T_{55}X^5_{,a} = 0 \quad (a = 3,4) \tag{2.2.18}$$
$$T_{dd,5}X^5 = 0 \ (d = 2,4) \tag{2.2.19}$$
$$T_{11,5}X^5 + 2T_{11}X^1_{,1} = 0 \tag{2.2.20}$$
$$T_{11}X^1_{,2} + T_{22}X^2_{,1} = 0 \tag{2.2.21}$$
$$T_{11}X^1_{,5} + T_{55}X^5_{,1} = 0 \tag{2.2.22}$$
$$T_{22,5}X^5 + 2T_{22}X^2_{,2} = 0 \tag{2.2.23}$$
$$T_{22}X^2_{,5} + T_{55}X^5_{,2} = 0 \tag{2.2.24}$$
$$T_{55,5}X^5 + 2T_{55}X^5_{,5} = 0. \tag{2.2.25}$$

With constraint conditions $T_{dd} = \alpha = const. \neq 0 \ (d = 1, 2, 4)$, then we have solutions,
$X^1 = c_1,\ X^2 = c_2, X^5 = \frac{c_3}{\sqrt{T_{55}}}.$

Now, we have obtained solutions in sub-case III-(iii), using (2.14)-(2.28), as
$X^1 = c_1,\ X^4 = c_2, X^5 = \frac{c_3}{\sqrt{T_{55}}}.$

Unlike solutions of sub-case III-(i), we get, solutions for sub-case III-(iv) using (2.14)-(2.28), as
$X^2 = c_2 - c_1 w, X^3 = c_3, X^4 = c_4 + c_1 y$.

Now, for sub-case III-(v), we have, $X^2 \neq 0,\ X^3 \neq 0,\ X^4 \neq 0,\ X^1 = X^5 = 0$ then using equations (2.14)-(2.28),

$$T_{11} X^1_{,a} = 0 \quad (a = 1,3,5) \tag{2.2.26}$$

$$T_{22} X^2_{,b} = 0 \quad (b = 2,3,5) \tag{2.2.27}$$

$$T_{44} X^4_{,c} = 0 \quad (c = 3,4,5) \tag{2.2.28}$$

$$T_{11} X^1_{,2} + T_{22} X^2_{,1} = 0 \tag{2.2.29}$$

$$T_{11} X^1_{,4} + T_{44} X^4_{,1} = 0 \tag{2.2.30}$$

$$T_{22} X^2_{,4} + T_{44} X^4_{,2} = 0. \tag{2.2.31}$$

Differentiating equation (2.2.29) with respect to w and differentiating equation (2.2.30) with respect to y and subtracting these equations, we get

$$T_{22} X^2_{,14} - T_{44} X^4_{,21} = 0. \tag{2.2.32}$$

Now, differentiating equation (2.2.31) with respect to x, we get

$$T_{22} X^2_{,41} + T_{44} X^4_{,21} = 0. \tag{2.2.33}$$

Adding equations (2.2.32) and (2.2.33), we obtain $X^2_{,41} = 0$ which gives,

$$X^2 = c_1 w + c_2. \tag{2.2.34}$$

Using equation (2.2.34) in equation (2.2.31), we get,

$$X^4 = \frac{-T_{22}}{T_{44}} c_1 y + c_3 \text{ and } X^1 = c_4.$$

3. DISCUSSION AND CONCLUSION

In the classification of higher dimensional LRS Bianchi type-I space-time according to energy-momentum tensor for perfect fluid, we find fifteen matter symmetries equations. We have solved these equations for degenerate case (section 2.1) where $\det(T_{ab}) = 0$ as well as for non-degenerate case (section 2.2) whendet$(T_{ab}) = T_1 {T_2}^2 T_4 T_5 \neq 0$. From these equations we obtained different constraint conditions on energy-momentum tensor. If we solve these constraint conditions we can have new class of exact solutions of Einstein's field equations. It is observed from section (2.1) that when energy-momentum tensor is degenerate, then we obtain matter symmetries equations admit infinite dimensional.

Furthermore, it is observed from section (2.2) that when energy-momentum tensor is non-degenerate we found matter symmetries (collineations) (MC's) of one dimensional in all sub-cases of case-I (i-v) whereas two dimensional MC's in all sub-cases of case-II (i-x). Particularly, in cases-III (ii, iii) and in cases-III (i, iv, v) we have three and four MC's respectively. All these results have been summarized in the table at the end of this conclusion. (see Appendix).

In this paper, we have obtained a complete classification of matter symmetries equations for the higher dimensional LRS Bianchi type-I space-time in presence of energy-momentum tensor for perfect fluid. After solving equation (1.2) for the space-time (2.1), we got fifteen matter symmetry (collineation) equations. Using these fifteen MC equations, we have shown that if the energy-momentum tensor for perfect fluid is degenerate, then we found single case where matter symmetry is infinite dimensional. It is also observed that if energy-momentum tensor is non-degenerate, we found finite dimensional MC's for all cases, particularly one and two MC's for the cases I and II respectively, and for case III, we have three and four MC's.

APPENDIX

Table: For Non-degenerate Case

Case	Constraint	MC's
I.i	$X^1 \neq 0, \quad X^2 = X^3 = X^4 = X^5 = 0$	1
I.ii	$X^2 \neq 0, \quad X^1 = X^3 = X^4 = X^5 = 0$	1
I.iii	$X^3 \neq 0, \quad X^1 = X^2 = X^4 = X^5 = 0$	1
I.iv	$X^4 \neq 0, \quad X^1 = X^2 = X^3 = X^5 = 0$	1
I.v	$X^5 \neq 0, \quad X^1 = X^2 = X^3 = X^4 = 0$	1
II.i	$X^1 \neq 0, \quad X^2 \neq 0, \quad X^3 = X^4 = X^5 = 0$	2

II.ii	$X^1 \neq 0,$	$X^3 \neq 0,$	$X^2 = X^4 = X^5 = 0$		2
II.iii	$X^1 \neq 0,$	$X^4 \neq 0,$	$X^2 = X^3 = X^5 = 0$		2
II.iv	$X^1 \neq 0,$	$X^5 \neq 0,$	$X^2 = X^3 = X^4 = 0$		2
II.v	$X^2 \neq 0,$	$X^3 \neq 0,$	$X^1 = X^4 = X^5 = 0$		2
II.vi	$X^2 \neq 0,$	$X^4 \neq 0,$	$X^1 = X^3 = X^5 = 0$		2
II.vii	$X^2 \neq 0,$	$X^5 \neq 0,$	$X^1 = X^3 = X^4 = 0$		2
II.viii	$X^3 \neq 0,$	$X^4 \neq 0,$	$X^1 = X^2 = X^5 = 0$		2
II.ix	$X^3 \neq 0,$	$X^5 \neq 0,$	$X^1 = X^2 = X^4 = 0$		2
II.x	$X^4 \neq 0,$	$X^5 \neq 0,$	$X^1 = X^2 = X^3 = 0$		2
III.i	$X^1 \neq 0,$	$X^2 \neq 0,$	$X^3 \neq 0,$	$X^4 = X^5 = 0$	4
III.ii	$X^1 \neq 0,$	$X^2 \neq 0,$	$X^5 \neq 0,$	$X^3 = X^4 = 0$	3
III.iii	$X^1 \neq 0,$	$X^4 \neq 0,$	$X^5 \neq 0,$	$X^2 = X^3 = 0$	3
III.iv	$X^1 \neq 0,$	$X^2 \neq 0,$	$X^4 \neq 0,$	$X^3 = X^5 = 0$	4
III.v	$X^2 \neq 0,$	$X^3 \neq 0,$	$X^4 \neq 0,$	$X^1 = X^5 = 0$	4

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