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EOQ MODEL WITH SHORTAGES IN BEGINNING AND PERIODIC DEMAND FOR DETERIORATING ITEMS

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ABSTRACT

Many products exists in nature which follows repetitions of demand pattern. For example, demand of water cooler is high in summer and low in winter and again increases as temperature increases and hence the demand pattern is periodic. So, we need to develop a new inventory model for such kind of products. In this paper an economic order quantity model has been presented for deteriorating item with periodic demand. Also one can start business with shortage like advance booking of products which could be fulfilled after time duration. We have incorporated the shortage at the beginning of the sale season. A new model is developed and optimal shortage duration is obtained along-with some other results. Simulation study is performed along with managerial insights. Proposed model is found useful for products having periodic demand pattern and bearing shortage at the starting.

Keywords: *Inventory, Periodic Demand, Deterioration, Shortage, Replenishment time.*

1. INTRODUCTION

In past, Harris (1915) and many researchers have been suggested inventory models with variety of demand functions. There are many products that follow periodic demand pattern and need to develop new inventory model. Also a business could be started with shortage like advance booking of LPG gas, electricity supply and pre public offer of equity share before proper functioning of a company. We incorporate two features: one is periodic demand and other is start of business with shortage in the proposed model. Few items in the market are of high need for people like sugar, wheat, oil whose shortage break the customer's faith and arrival pattern. This motivates retailers to order for excess units of item for inventory in spite of being deteriorated. Moreover, deterioration is manageable for many items by virtue of modern advanced storage technologies. Inventory model presents a real life problem (situation) which helps to run the business smoothly. Our aim is to solve the problem of the business which start with shortage and in which the demand of the products follow the periodic demand.

Burwell *et al.* (1997) solved the problem arising in business by providing freight discounts and presented an economic lot size model with price-dependent demand. Shin (1997) determined an optimal policy for retail price and lot size under day-term supplier credit policies based on constant demand where after maturing the product in market, it follows linear demand.

Matsuyama (2001) presented a general EOQ model considering holding costs, unit purchase costs, and setup costs that are time-dependent and continuous general demand functions. The problem has been solved by dynamic programming so as to find ordering point, ordering quantity, and incurred costs. A research overview by Emagharby and Keskinocak (2003) is for determining the dynamic pricing and order level. Teng and Chang (2005) presented an economic production quantity (EPQ) model for deteriorating items when the demand rate depends not only on the on-display stock, but also on the selling price per unit considering market demand. The manipulation in selling price is the best policy for the organization as well as for the customers. Wen and Chen (2005) suggested a dynamic pricing policy for selling a given stock of identical perishable products over a finite time horizon on the internet. The sale ends either when the entire stock is sold out, or when the deadline is over. Here, the objective of the seller is to find a dynamic pricing policy that maximizes the total expected revenues. Shukla and Khedlekar (2009) introduced the concept of three warehouses in which one oriented and two rented warehouses to store the deteriorating items as per requirements.

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The EOQ model designed by Hou and Lin (2006) reflects how a demand pattern which is price, time, and stock dependent affects the discount in cash. They discussed an EOQ inventory model which takes into account the inflation and time value of money of the stock-dependent selling price. Existence and uniqueness of the optimal solution has not been shown in this article. Lai *et al.* (2006) algebraically approached the optimal value of cost function rather than the traditional calculus method and modified the EPQ model earlier presented due to Chang (2004) in which he considered variable lead time with shortage. Chandel and Khedlekar (2013) also used multiple warehouses on location based and ordered quantity as centralised basis and minimized the incurred cost. Recent contribution in EPQ models is a source of esteem importance like Birbil *et al.* (2007), Hou (2007), Khedlekar and Shukla (2015), Bhaskaran *et al.* (2010), Jogelekar *et al.* (2008), Roy (2008), Kumar (2012*a, b, c*) and You (2005). Motivation is derived due to Wu (2002) and Shukla *et al.* (2010, 2014), Khedlekar *et al.* (2013) for considering the shortages in beginning of a business and henceforth developed the proposed model. Section 2 consists of the assumption and notations of the model while a section 3 deal with the mathematical formulation of model, section 4 is for numerical example and simulation.

2. ASSUMPTIONS AND NOTATIONS

Suppose demand of a product is $a\sin(bt)$. In starting suppose shortage accumulated till time t_1 so that on hand shortage is $I_1(t)$. The order receives to the company by vendor at t_1 and so the shortage ends and inventory reaches up to level $I_2(t_1)$. This inventory level is sufficient to fulfil the demand till time T. Our aim is to find the optimal time t_1 to optimize the total inventory cost. Inventory depleted due to demand and deterioration as shown in fig 1.

The followings notations are used to develop the proposed model.

$D(t)$	demand of product is $D(t) = a\sin(bt)$ where a and b >1 are positive real values.
θ	rate of deterioration of product, $\theta < 1$.
c_1	holding cost unit per unit time.
c_2	shortage cost unit per unit time.
c_3	deterioration cost.
T	cycle time.
t_1^*	optimal time for accumulated shortages.
$I_1(t)$	on hand shortage of the product ($I_1(t) > 0$).
$I_2(t)$	on hand inventory of the product ($I_2(t) > 0$).
$C(t_1)$	optimal inventory cost.
D_T	deteriorated units.
S_T	shortage units in the system.
S_C	shortage cost.
H_C	holding cost.
D_C	deterioration cost.

3. MATHEMATICAL MODEL

Suppose on hand shortage is by $I_1(t)$ and this accumulates until t_1. Management has placed the order which fulfilled at time t_1 and thus on hand inventory is $I_2(t_1)$. After time t_1, the inventory depleted due to demand and deterioration then it reduces to zero at time T (see Fig. 1).

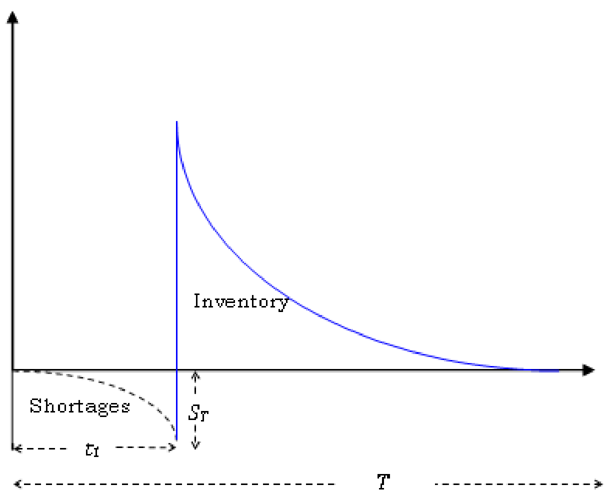

Fig. 1: Inventory Cycle

$$\frac{d}{dt}I_1(t) = -a\sin(bt)\text{, where } 0 \le t \le t_1,\ I_1(0) = 0 \tag{1}$$

$$\frac{d}{dt}I_2(t) + \theta I_2(t) = -a\sin(bt)\text{, where } t_1 \le t \le T \tag{2}$$

Boundary conditions for above two differential equations are $I_1(0) = 0$, $I_2(T) = 0$, solving equation (1) we get

$$I_1(t) = \frac{a}{b}\left(1 - \cos(bt)\right) \tag{3}$$

Solving equation (2) we get

$$I_2(t) = \frac{a}{a^2 + b^2}e^{\theta T - \theta t}\left(\theta\sin(bT) + b\cos(bT)\right) - \frac{a}{a^2 + b^2}e^{-\theta t}\left(\theta\sin(bt) + b\cos(bt)\right) \tag{4}$$

Deteriorated units in time $(t_1, T]$ is D_T

$$D_T = I_2(t_1) - a\int_{t_1}^{T}\sin(bt)$$

$$= \frac{ac_2}{b}\left(t_1 - \frac{\sin(bt_1}{b}\right) \tag{5}$$

Holding cost HC over time $(t_1, T]$ will be

$$HC = h_1\left\{\begin{array}{l}\dfrac{a}{a^2 + b^2}e^{\theta T}\left(a\sin(bT) + b\cos(bT)\right)\dfrac{e^{-\theta t} - e^{-\theta T}}{\theta} \\ -\dfrac{a}{b\left(a^2 + b^2\right)}\left(b\sin(bT) - b\sin(bt_t) - \theta\cos(bT) + \theta\cos(bt_1)\right)\end{array}\right\} \tag{6}$$

Shortages $I_1(t_1) = \frac{a}{b}\left(1 - \cos(bt_1)\right)$

$$S_C = \frac{ac_2}{b}\left(1 - \cos(bt_1)\right) \tag{7}$$

Number of units including shortage in business schedule is

$$Q = I_1(t_1) + I_2(t_1) \tag{8}$$

Total average inventory cost will be

$$C(t_1) = \left(\frac{H_C + S_C + D_C}{T}\right) \tag{9}$$

$$= \frac{1}{T}\left[\frac{ac_2}{b}\left(1 - \cos(bt_1)\right) + h_1\left\{\frac{a}{a^2 + b^2}e^{\theta T}\left(a\sin(bT) + b\cos(bT)\right)\frac{e^{-\theta t} - e^{-\theta T}}{\theta}\right\}\right]$$

$$+ \frac{h_1}{T}\left\{\frac{ac_2}{b}\left(t_1 - \frac{\sin(bt_1}{b}\right) - \frac{a}{b\left(a^2 + b^2\right)}\left(b\sin(bT) - b\sin(bt_t) - \theta\cos(bT) + \theta\cos(bt_1)\right)\right\} \tag{10}$$

To optimize the total cost function ($C(t_1)$) first derivative equating to zero

$$\frac{d}{dt_1}C(t_1) = \frac{1}{T}\left\{-\frac{ah_1e^{\theta T}}{a^2 + b^2}\left(a\sin(bT) + b\cos(bT)\right)e^{-\theta t_1} + \frac{ah_1}{b(a^2 + b^2)}\left(b^2\cos(bt_1) + b\theta\sin(bt_1)\right)\right\}$$

$$-\frac{1}{T}\left\{\frac{ac_3e^{\theta T - \theta t_1}}{a^2 + b^2}\left(a\sin(bT) + b\cos(bT)\right)\right\} - \frac{1}{T}\left\{\frac{ac_3}{a^2 + b^2}\left(\theta n\cos(bt_1) - b^2\sin(bt_1)\right)\right\}$$

$$+\frac{1}{T}\left\{ac_3\sin(bt_1) + \frac{ac_2}{b} - \frac{ac_2}{b}\cos(bt_1)\right\} \tag{11}$$

On equating $\frac{d}{dt_1}C(t_1)=0$, we get equation for optimality value of say which is $t_1=t_1^*$ Condition for optimality is

$$\frac{d^2}{dt_1^2}C(t_1)>0 \tag{12}$$

Thus Average total cost is optimum at $t_1=t_1^*$.

Table-1: Sensitivity of different parameters

Variation in Parameter	*a*	*b*	c_1	c_2	C_3	θ	*T*	t_1	*TC*	*Q*(*T*)	$I_2(t_2)$	Holding Cost	Shortage Cost	D_T
T	10	0.2	1	2	2	0.01	***14***	4.234	122.43	101	51	1378.56	49.34	4
	10	0.2	1	2	2	0.01	***12***	5.108	89.58	95	45	691.700	49.99	2
	10	0.2	1	2	2	0.01	***10***	5.713	68.47	80	30	281.140	1.000	0
	10	0.2	1	2	2	0.01	***08***	6.125	60.11	62	13	70.2400	48.88	0
	10	0.2	1	2	2	0.01	***07***	6.266	61.56	53	5	19.2000	48.59	0
Θ	10	0.2	1	2	2	**0.005**	14	4.600	120.99	104	55	1338.05	49.82	2
	10	0.2	1	2	2	**0.01**	14	4.234	122.43	101	52	1378.56	49.35	4
	10	0.2	1	2	2	**0.02**	14	3.406	125.32	76	29	1475.17	47.07	9
	10	0.2	1.4	9	***3***	0.01	14	5.855	243.66	115	66	1574.80	49.35	2
	10	0.2	1.4	9	***4***	0.01	14	5.841	243.96	115	66	1577.92	49.37	2
	10	0.2	1.4	9	***5***	0.01	14	5.826	244.26	115	66	1581.28	49.40	2
	10	0.2	1.4	9	***6***	0.01	14	5.812	244.55	115	66	1584.41	49.42	3
	10	0.2	1.4	9	***7***	0.01	14	5.798	244.85	115	66	1587.54	49.44	3
	10	0.2	1.4	9	***8***	0.01	14	5.781	245.16	115	66	1591.32	49.46	3
c_2	10	0.2	1.4	***3***	2	0.01	14	4.465	171.68	104	54	1880.65	49.68	4
	10	0.2	1.4	***4***	2	0.01	14	5.048	179.63	110	60	1753.92	50.00	3
	10	0.2	1.4	***5***	2	0.01	14	5.361	190.61	113	63	1684.85	49.89	3
	10	0.2	1.4	***6***	2	0.01	14	5.559	202.94	114	64	1640.87	49.73	3
	10	0.2	1.4	***7***	2	0.01	14	5.695	216.02	115	65	1610.55	49.58	3
	10	0.2	1.4	***8***	2	0.01	14	5.794	229.55	115	66	1588.45	49.44	2
	10	0.2	1.4	***9***	2	0.01	14	5.870	243.36	115	66	1571.44	49.33	2
c_1	10	0.2	***1.5***	2	2	0.01	14	2.520	187.58	65	23	2435.09	42.38	5
	10	0.2	***0.8***	2	2	0.01	14	4.740	100.58	107	57	1040.70	49.92	3
	10	0.2	***0.9***	2	2	0.01	14	4.496	111.26	104	55	1204.71	49.71	4
	10	0.2	***1.2***	2	2	0.01	14	3.648	146.45	91	44	1759.14	47.92	4
	10	0.2	***1.4***	2	2	0.01	14	2.957	172.96	77	32	2190.11	45.02	5
A	***15***	0.2	1.4	2	2	0.01	14	2.957	200.82	115	47	2464.51	67.53	7
	20	0.2	1.4	2	2	0.01	14	2.955	228.69	153	63	2739.45	90.03	9
	25	0.2	1.4	2	2	0.01	14	2.954	256.56	191	79	3014.24	112.5	11
	30	0.2	1.4	2	2	0.01	14	2.953	284.43	230	95	3289.10	135.0	13

4. NUMERICAL EXAMPLE AND SIMULATION

Let us assume that model parameters are a = 10 units, b = 0.2, c_1 = \$1 per unit per month, c_2 = \$2 per unit per month, C_3 = \$2 per unit per month, θ = 0.01, T = 14 days and demand of the product is $D(t)=a\sin(bt)$. Under the given parameter values and by equation (6) to (10) we get output parameters t_1 = 4.23days, average total inventory cost TC = \$122.43.93, Q = 101 units, average holding cost H_C = \$98.46.

Now, in this section, we study how the input parameters change significantly to the output parameters. We change the value of one input parameter, keeping other parameters constant. The output parameter is valuated for decision making. The data used for this purpose is in section 4.

Total inventory cost increases as the time cycle length increases (see fig 2) and same followed by economic order quantity (table 1). Increments in shortage cost provide an increment in EOQ (see fig 3). The deterioration rate (θ) uplifts the level of deterioration cost and total cost both. So they are directly proportional and have a linear trend (see fig 4 and 5).

Management needs to be aware about the deterioration cost and holding cost both and tries to keep it as low as possible. High initial demand (parameter *a*) increases the EOL and EOQ both (table 1), but optimal time interval of these two remain unchanged. From table 1, it is observed that the optimal time is highly sensitive for deterioration and holding cost.

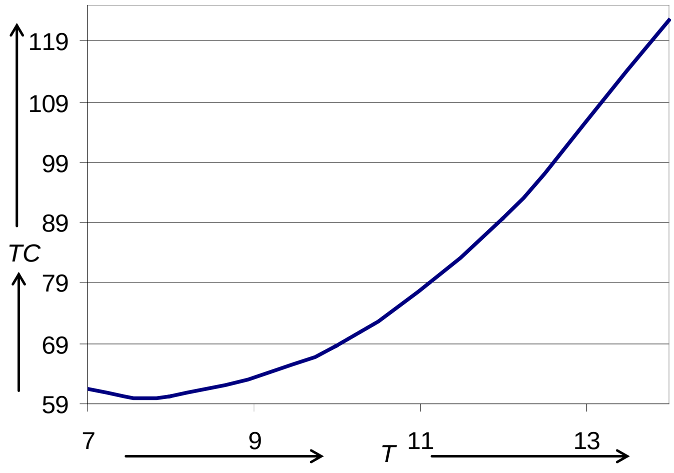

Figure 2. Effect of time cycle on total cost

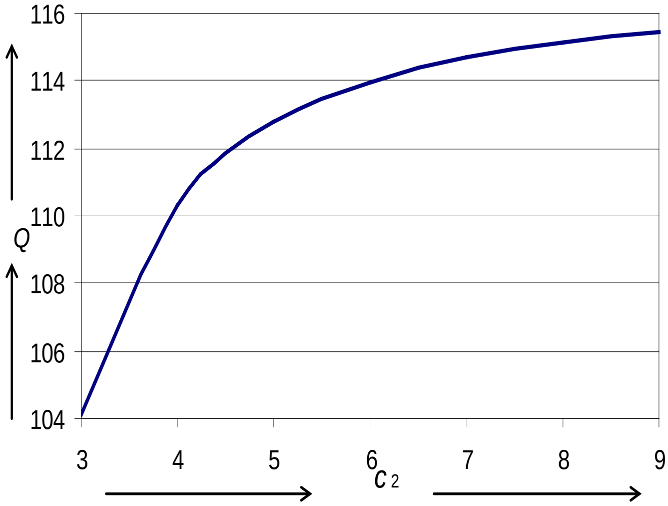

Figure 3. Effect of shortage cost on total quantity

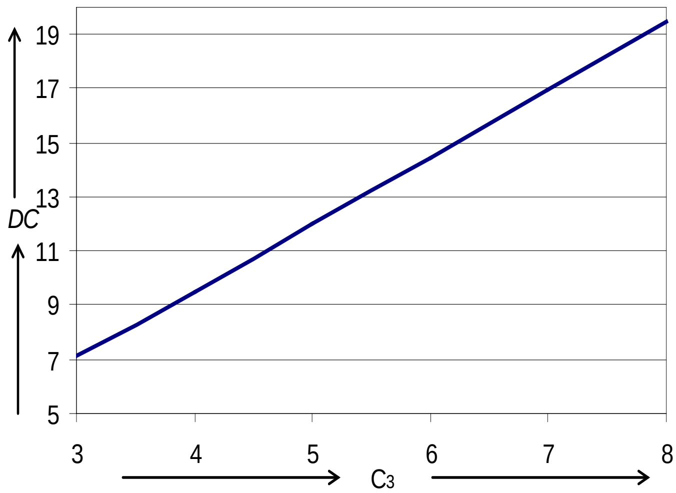

Figure 4. Effect of deterioration ratte on deterioreted cost

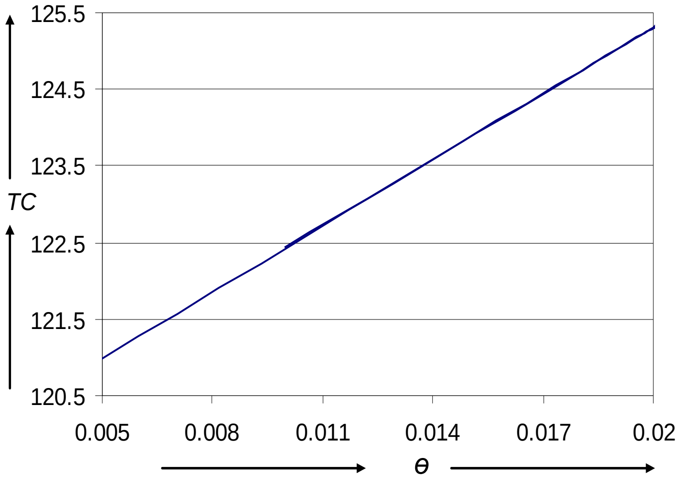

Figure 5. Effect of deterioration rate on total cost

5. CONCLUSIONS

A mathematical inventory model is suggested in the content for a business that starts with shortages and free to place the order according to the demand and customer's response. Inventory managers should keep the deterioration rate as low as possible because it increases the wastage of quantity as well as the total cost. However one can negotiate the shortage cost to customers, which may keep lower to the incurred cost. Suggested model is sensitive for deterioration and holding cost both as compared to shortage cost for a product having periodic demand. This model can be further extended for variable deterioration, ramp type demand and for finite rate of replenishment. This model may also be formulated in the fuzzy environment.

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