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I - $g\alpha$ - CLOSED SETS IN IDEAL BITOPOLOGICAL SPACES

Stella Irene Mary. J*[1], Vinodhini. R.[2]

[1]Associate Professor, [2]M. phil. Scholar, Department of Mathematics,
PSG college of Arts and Science, Coimbatore - 641014, India.

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ABSTRACT

In this paper a new class of closed sets namely I - $g\alpha$ - closed sets in ideal bitopological spaces (X, τ_1, τ_2, I) is introduced. Several properties and characterizations of this new class are investigated.

Keywords: *Bitopological spaces, Ideal bitopological spaces, (i, j) - $g\alpha$ - closed sets, (i, j) - I - $g\alpha$ - closed sets, (i, j) - I - $g\alpha$ - open sets.*

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1. INTRODUCTION

The concept of Bitopological Spaces was initiated by Kelly [7] in 1963. He defined a set equipped with two topologies which is called a bitopological space and denoted it by (X, τ_1, τ_2) where (X, τ_1) and (X, τ_2) are the topological spaces. In 1970, Levine [9] introduced g - closed sets in Topological space (X, τ) and studied several properties. In 1986, Fukutake [2] extended the concept of generalized closed sets to bitopological spaces. In 1965, Njasted [12] and in 1990, Jelic [6] developed the concepts of alpha open sets and α - continuous functions in bitopological spaces. As an extension of α - open sets Maki *et.al* [10] defined $g\alpha$ - closed sets in topological spaces (X, τ). In 2005, El - Tantawy and Abu.Donia [1] extended αg - closed sets induced by open sets which contains the class of $g\alpha$ - closed sets to bitopological spaces and studied several properties.

The concept of Ideal topological spaces was first introduced by Kuratowski [8] and thus opened the door for a large area of research. If I is an Ideal on X, then (X, τ_1, τ_2, I) is called an Ideal bitopological space. Vaidynathasamy [16] constructed local function in ideal topological space in 1945. Recently Tripathy and Hazarika [14], Tripathy and Mahanta [15] studied about the Ideal convergence in sequence spaces. In this article a new class of closed sets namely I - $g\alpha$ - closed sets in bitopological space is introduced. It is denoted by (i, j) - I - $g\alpha$ - closed sets where (i, j) means the topologies (τ_i, τ_j) , $i \neq j; i, j = 1,2$ this class satisfies the inclusion relation given below.
$\{(i, j)$ - $g\alpha$ - closed sets$\} \subseteq \{(i, j)$ - I - $g\alpha$ - closed sets$\}$

2. PRELIMINARIES:

Definition 2.1.1: [5] A **topology** on a set X is a collection of subsets of X having the following properties:
1) ϕ and X are in τ.
2) The union of the elements of any subcollection of τ is in τ.
3) The intersection of the elements of any finite subcollection of τ is in τ.

Definition 2.1.2: [7] A set X with topologies τ_1 and τ_2 is said to be **Bitopological space** and is denoted by (X, τ_1, τ_2).

Definition 2.1.3: [13]
(i) An **ideal I** on a non-empty set X is a collection of subsets of X which satisfies
 (a) $A \in I$ and $B \subseteq A$ implies $B \in I$ and
 (b) $A \in I$ and $B \in I$ implies $A \cup B \in I$.
 If I is an ideal on X, then (X, τ_1, τ_2, I) is called an **ideal bitopological space.**
(ii) A subset A of X is said to be **(i, j) - clopen set** if the set is both open and closed.

Corresponding Author: Stella Irene Mary. J[1]*
[1]Associate Professor, Department of Mathematics,
PSG college of Arts and Science, Coimbatore - 641014, India.

Definition 2.1.4: [4] Let (X, τ_1, τ_2) be a bitopological space. We denote the bitopologies (τ_i, τ_j) by (i, j).

($\boldsymbol{i}$) A subset A of X is said to be $\boldsymbol{(\tau_i, \tau_j)}$ **-** $\boldsymbol{\alpha}$ **- open set** or simply $\boldsymbol{(i, j)}$ **-** $\boldsymbol{\alpha}$ **- open set** if $A \subseteq \tau_i$- $int[\tau_j$ - $cl(\tau_i$ - $int(A))]$, where $int(A)$ denote the interior of (A) and $Cl(A)$ denote the closure of (A).

($\boldsymbol{ii}$) A subset A of X is said to be $\boldsymbol{(\tau_i, \tau_j)}$ **-** $\boldsymbol{\alpha}$ **- closed set** or simply $\boldsymbol{(i, j)}$ **-** $\boldsymbol{\alpha}$ **- closed set** if τ_i- cl$[\tau_j$ - int$(\tau_i$ - cl$(A))] \subseteq A$.

($\boldsymbol{iii}$) The $\boldsymbol{(i, j)}$ **-** $\boldsymbol{\alpha}$ **- interior** of A is defined as the union of all (i, j) - α - open sets contained in A. We denote (i, j) - α - interior of A by (i, j) - α - $int(A)$.

($\boldsymbol{iv}$) The $\boldsymbol{(i, j)}$ **-** $\boldsymbol{\alpha}$ **- closure** of A is defined as the intersection of all (i, j) - α - closed sets containing A. We denote (i, j) - α - closure of A by (i, j) - α - $cl(A)$.

3.1 Generalized α - closed sets in ideal bitopological spaces

Let (X, τ_1, τ_2) be a bitopological space. Throughout this chapter we denote the bitopologies (τ_i, τ_j) by (i, j).

Definition 3.1.1: [1] A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\boldsymbol{(i, j)}$**-generalized** $\boldsymbol{\alpha}$ **– closed** (In short, (i, j) - $g\alpha$ - closed) set if (j, i) - $\alpha cl(A) \subset U$ whenever $A \subset U$ and U is τ_i - α open in X.

In this section we prove several properties regarding (i, j)- α - interior and (i, j)- α - closure of sets in (X, τ_1, τ_2).

Lemma 3.1.1: Let (X, τ_1, τ_2) be a bitopological space and A be a subset of X. Then

($\boldsymbol{i}$) (i, j) - $\alpha int(A)$ is (i, j) - α - open.

($\boldsymbol{ii}$) (i, j) - $\alpha cl(A)$ is (i, j) - α - closed.

($\boldsymbol{iii}$) A is (i, j) - α - open if and only if $A = (i, j)$ - $\alpha int(A)$.

($\boldsymbol{iv}$) A is (i, j) - α - closed if and only if $A = (i, j)$ - $\alpha cl(A)$.

Proof:

(i) (i, j) - $\alpha int(A)$ is the union of all (i, j) - α - open sets contained in A. First we prove that (i, j) - α - open set is (i, j) - α - open. [Let $\{A_k\}$ where $k \in J$ be a family of (i, j) - α - open sets contained in A. Then for each $k \in J$,

$A_k \subseteq \tau_i$ - $int[\tau_j$ - $cl(\tau_i$ - $int\ A_k)]$

Now, $\bigcup_k A_k \subseteq \bigcup_k \{\tau_i$ - $int[\tau_j$ - $cl(\tau_i$ - $int\ A_k)]\}$

$\subseteq \tau_i$ - $int[\bigcup_k \tau_j$ - $cl(\tau_i$ - $int\ A_k)]$

$= \tau_i$ - $int[\tau_j$ - $cl(\bigcup_k \tau_i$ - $int\ A_k)]$

$\subseteq \tau_i$ - $int[\tau_j$ - $cl(\tau_i$ - $int\ (\bigcup_k A_k))]$

Hence $\bigcup A_k$ is a (i, j) - α - open set

Therefore, (i, j) - $\alpha int(A)$ is (i, j) - α - open.

($\boldsymbol{ii}$) (i, j) - $\alpha cl(A)$ is defined by intersection of all (i, j) - α - closed sets containing A. [Let $\{A_k\}$ where $k \epsilon J$ is a family of (i, j) - α - closed sets containing A. This implies $\{{A_k}^C\}$ is a family of (i, j) - α - open sets contained in A^C. $\bigcup {A_k}^C$ is (i, j) - α - open set [by (i)]. Consequently $(\bigcap A_k)^C$ is (i, j) - α - open set and $\bigcap A_k$ (i, j) - α - closed set]. Therefore, (i, j) - $\alpha cl(A)$ is (i, j) - α - closed.

($\boldsymbol{iii}$) Since (i, j) - $\alpha int(A)$ is the largest (i, j) - α - open set contained in A and A is (i, j) - α - open, we have $A = (i, j)$ - $\alpha int(A)$. Conversely**,** suppose $A = (i, j)$ - $\alpha int(A)$ then by (i), A is (i, j) - α - open.

($\boldsymbol{iv}$) We know that, (i, j) - $\alpha cl(A)$ is the smallest (i, j) - α - closed set containing A. Since A itself is (i, j) - α - closed set, $A = (i, j)$-$\alpha cl(A)$. Conversely**,** suppose $A = (i, j)$ - $\alpha cl(A)$ then by (ii), A is (i, j) - α - closed.

Lemma 3.1.2: Let (X, τ_1, τ_2) be a bitopological space and A be a subset of X. Then the following assertion hold.

($\boldsymbol{i}$) $x \in (i, j)$ - $\alpha cl(A)$ if and only if for every (i, j) - α - open set U containing x, $U \cap A \neq \phi$.

($\boldsymbol{ii}$) $x \in (i, j)$ - $\alpha int(A)$ if and only if there exists an (i, j) - α - open set U such that $x \in U \subseteq A$.

($\boldsymbol{iii}$) If $A \subset B$, then ($\boldsymbol{a}$) (i, j) - $\alpha int(A) \subset (i, j)$ - $\alpha int(B)$ and

($\boldsymbol{b}$) (i, j) - $\alpha cl(A) \subset (i, j)$ - $\alpha cl(B)$

Proof:

($\boldsymbol{i}$) Suppose $x \in (i, j)$ - $\alpha cl(A) = \bigcap_k B_k$, where B_k is a (i, j) - α - closed set containing A

Let U be an (i,j) - α - open set and $x \in U$ (3.1.1)

Suppose $U \cap A = \phi$ then $U^C \supset A$ and U^C is (i,j) - α - closed set containing A and hence $U^C = B_k$ for some k. Consequently, $x \in U^C$ which is a contradiction to (3.1.1). Hence $U \cap A \neq \phi$.

On the other hand, suppose for every (i,j) - α -open set U containing $x, U \cap A \neq \phi$. Suppose let $x \notin (i,j)$ - $\alpha cl(A)$ then $x \notin \bigcap_k B_k$, where B_k is a (i,j) - α - closed set containing A. That means there exists $B_k \supset A$ such that $x \notin B_k$ and $x \in {B_k}^C$ where ${B_k}^C$ is a (i,j) - α - open set contained in A and ${B_k}^C \cap A = \phi$ which is a contradiction. Therefore $x \in (i,j)$ - $\alpha cl(A)$.

$(\boldsymbol{ii})$ It is obvious by the definition of (i,j) - $\alpha int(A)$.

$(\boldsymbol{iii})$ $(\boldsymbol{a})$ (i,j) - $\alpha int(A) = \cup A_k$, where A_k is a (i,j) - α-open set contined in A (3.1.2)
Since $A_k \subset$ A $\subset$ B for all k, $\cup A_k \subset$ B, by (3.1.2), (i,j) - $\alpha int(A) \subseteq B$. The fact that (i,j) - $\alpha int(A)$ is an (i,j) - α - open set and (i,j) - $\alpha int(B)$ is the largest (i,j) - α - open set contained in B, implies (i,j) - $\alpha int(A) \subset (i,j)$ - $\alpha int(B)$.

$(\boldsymbol{b})$ $A \subset B \implies B^C \subset A^C$. By (a), (i,j) - $\alpha int(B^C) \subset (i,j)$ - $\alpha int(A^C)$ (3.1.3)
Since (i,j) - $\alpha int(B^C) = \bigcup_k {B_k}^C$ and (i,j) - $\alpha int(A^C) = \bigcup_s {A_s}^C$ where ${B_k}^C$ and ${A_s}^C$ are (i,j) - α - open sets contained in B^C and A^C respectively.

By (3.1.3), $\bigcup_k {B_k}^C \subset \bigcup_s {A_s}^C \implies (\bigcap_k B_k)^C \subset (\bigcap_s A_s)^C \Rightarrow \bigcap_s A_s \subseteq \bigcap_k B_k$

where A_s and B_k are (i,j) - α - closed sets. Therefore, (i,j) - $\alpha cl(A) \subset (i,j)$ - $\alpha cl(B)$.

Lemma 3.1.3: Let (X, τ_1, τ_2) be a bitopological space and A be a subset of X. Then
$(\boldsymbol{i})$ $X \setminus (i,j)$ - $\alpha int(A) = (i,j)$ - (i,j) - $\alpha cl(X \setminus A)$
$(\boldsymbol{ii})$ $X \setminus (i,j)$ - $\alpha cl(A) = (i,j)$ - (i,j) - $\alpha int(X \setminus A)$

Proof:
$(\boldsymbol{i})$ $X \setminus (i,j)$ - $\alpha int(A) = X \setminus \bigcup_k A_k$ where A_k is a (i,j) - α - open set contained in A

$= \bigcap_k (X \setminus A_k)$

$= \cap {A_k}^C$ where ${A_k}^C$ is (i,j) - α - closed set containing A^C
$= (i,j)$ - $\alpha cl(A^C)$

$(\boldsymbol{ii})$ $X \setminus (i,j)$ - $\alpha cl(A) = X \setminus \cap A_s$ where A_s is a (i,j) - α - closed set containing A
$= \cup (X \setminus A_s)$
$= \cup {A_s}^C$ where ${A_s}^C$ is (i,j) - α -open set contained in A^C
$= (i,j)$ - $\alpha int\ A^C$

3.2 $(\boldsymbol{i},\boldsymbol{j})$ - $\boldsymbol{I}$ - generalized $\boldsymbol{\alpha}$ - closed sets

As an extension of (i,j) - generalized α - closed sets in bitopological space to the ideal bitopological space we introduce the following definition.

Definition 3.2.1: Let (X, τ_1, τ_2, I) be an ideal bitopological space. A subset A of X is said to be $(\boldsymbol{i},\boldsymbol{j})$ **-** $\boldsymbol{I}$ **- generalized** $\boldsymbol{\alpha}$ **- closed** (in short, (i,j) - I - $g\alpha$ - closed) set if (j,i) - $\alpha cl(A) \setminus B \in I$ whenever $A \subset B$ and B is τ_i - α open in X, for i,j = 1,2 and $i \neq j$. The family of all (i,j) - I - $g\alpha$ - closed sets of X is denoted by (i,j) - I - $g\alpha C(X)$. This new class (i,j) - I - $g\alpha$ - closed set contains the class of (i,j) - $g\alpha$ - closed set.

Definition 3.2.2: Let (X, τ_1, τ_2, I) be an ideal bitopological space and $A \subseteq X$, the intersection of all (i,j) - I - $g\alpha$ - closed sets containing A is called $(\boldsymbol{i},\boldsymbol{j})$ **-** $\boldsymbol{I}$ **-** $\boldsymbol{g\alpha}$ **- closure** of A and is denoted by (i,j) - I - $g\alpha$ - $Cl(A)$.

The inclusion relation **{**(i,j) - $g\alpha$ - closed sets} $\subseteq$ {(i,j) - I - $g\alpha$ - closed sets} is proved in the following theorem.

Theorem 3.2.1: Every (i,j) - $g\alpha$ - closed is (i,j) - I - $g\alpha$ - closed.

Proof: Let A be (i,j) - $g\alpha$ - closed.

That is, (j,i) - $\alpha cl(A) \subset U$ whenever $A \subset U$ and U is τ_i - α open in X (3.2.1)

We have to show that A is (i,j) - I - $g\alpha$ - closed. That is, (j,i) - $\alpha cl(A) \setminus B \in I$ whenever $A \subset B$ and B is τ_i - α open in X, for $i,j = 1,2$ and $i \neq j$. So, let $A \subset B$ and B is τ_i - α open in X. By (3.2.1), (j,i) - $\alpha cl(A) \subset B$, and hence $(j,i) - \alpha cl(A) \setminus B = \phi \in I$. Thus A is (i,j) - I - $g\alpha$ - closed.

Remark 3.2.1: The converse of the above theorem is not necessarily true.This is proved in the following example.

Example 3.2.1: Let $X = \{a,b,c\}$. Consider the topologies $\tau_1 = \{\phi,\{a\},X\}$, $\tau_2 = \{\phi,\{a\},\{a,b\},X\}$ and $I = \{\phi,\{b\},\{c\},\{b,c\}\}$. Here $\{a\}$ is (1,2) - I - $g\alpha$ - closed set but not (1,2) - $g\alpha$ - closed.Since (2,1) - $\alpha cl(\{a\}) = X$ not a subset of $\{a\}$.

Theorem 3.2.2: Let (X,τ_1,τ_2,I) be an ideal bitopological space. If A is (i,j) - I - $g\alpha$ - closed and $A \subset B \subset (j,i)$ - $\alpha cl(A)$ in X, then B is (i,j) - I - $g\alpha$ - closed in X, where $i,j = 1,2$ and $i \neq j$.

Proof: Let $B \subset V$ and V is τ_i - α open in X. Since $A \subset B \subset (j,i)$ - $\alpha cl(A)$ and $B \subset V$, we have $A \subset V$. By hypothesis, (j,i) - $\alpha cl(A) \setminus V \in I$. Further $B \subset (j,i)$ - $\alpha cl(A)$ implies that (j,i) - $\alpha cl(B) \subset (j,i)$ - $\alpha cl(A)$
$\Rightarrow$ (j,i) - $\alpha cl(B) \setminus V \subset (j,i)$ - $\alpha cl(A) \setminus V \in I$.Consequently B is (i,j) - I - $g\alpha$ - closed.

Set operations : The following Theorem and example prove that the class of (i,j) - I - $g\alpha$ - closed sets is closed for the set operation union but not for intersection.

Theorem 3.2.3: Union of two (i,j) - I - $g\alpha$ - closed sets in an ideal bitopological space (X,τ_1,τ_2,I) is also (i,j) - I - $g\alpha$ - closed.

Proof: Let A and B be two (i,j) - I - $g\alpha$ - closed sets with $A \cup B \subset V$,where V is any is τ_i - α open set. Clearly, $A \subset V$ and $B \subset V$ as $A \cup B \subset V$. Since A and B are (i,j) - I - $g\alpha$ - closed sets, we have (j,i) - $\alpha cl(A) \setminus V \in I$ and (j,i) - $\alpha cl(B) \setminus V \in I$. Now, (j,i) - $\alpha cl(A \cup B) \setminus V = ((j,i)$ - $\alpha cl(A) \cup (j,i)$ - $\alpha cl(B))\setminus V$

$= ((j,i)$ - $\alpha cl(A) \setminus V) \cup ((j,i)$ - $\alpha cl(B) \setminus V) \in I$

[since $A \in I$ and $B \in I \Rightarrow A \cup B \in I$]. Thus (j,i) - $\alpha cl(A \cup B) \setminus V \in I$. Hence $A \cup B$ is (i,j) - I - $g\alpha$ - closed set.

Remark 3.2.2: The intersection of two (i,j) - I - $g\alpha$ - closed sets need not be (i,j) - I - $g\alpha$ - closed. This is proved in the following example.

Example 3.2.2: Let $X = \{a,b,c\}$. Consider the topologies $\tau_1 = \{\phi,\{c\},\{a,b\}, X\}$, $\tau_2 = \{\phi,\{a\},\{b,c\},X\}$ and $I = \{\phi\}$. Here $\{b,c\}$ and $\{a,c\}$ are (1,2) - I - $g\alpha$ - closed set but $\{b,c\} \cap \{a,c\} = \{c\}$ is not (1,2) - I - $g\alpha$ - closed.

Hereditary property : The following Theorem proves that the class of (i,j) - I - $g\alpha$ - closed sets admits hereditary property.

Theorem 3.2.4: Let (X,τ_1,τ_2,I) be an ideal bitopological space. Suppose A is (i,j) - I - $g\alpha$ - closed in X and $A \subset Y \subset X$. Then A is (i,j) - I - $g\alpha$ - closed relative to the subspace Y of X and with respect to the ideal $I_Y = \{P \subset Y : P \in I\}$.

Proof: Let V be τ_i - α open in X and $A \subset Y \cap V$.Therefore, we have $A \subset V$. Since A is(i,j) - I - $g\alpha$ - closed in X,we have (j,i) - $\alpha cl(A) \setminus V \in I$.Further, we see that $((j,i)$ - $\alpha cl(A) \cap Y) \setminus (Y \cap V) = ((j,i)$ - $\alpha cl(A) \setminus V) \cap Y \in I_Y$.

Thus for $A \subset Y \cap V$ and V is τ_i - α open, we have $((j,i)$ - $\alpha cl(A) \cap Y) \setminus (Y \cap V) \in I_Y$. Hence A is (i,j) - I - $g\alpha$ - closed relative to the subspace $(Y,\tau_1|Y,\tau_2|Y)$.

3.3 (i,j) - I - generalized α - open sets

We introduce the following definition namely (i,j) - I - generalized α - open set.

Definition 3.3.1: Let (X,τ_1,τ_2,I) be an ideal bitopological space. A subset A of X is said to be **(i,j) - I - generalized α - open** (in short, (i,j) - I - $g\alpha$ - open) set if $X \setminus A$ is (i,j) - I - $g\alpha$ - closed, for $i,j = 1,2$ and $i \neq j$. The family of all (i,j) - I - $g\alpha$ - open sets contained in A is denoted by (i,j) - I - $g\alpha$ - $O(X)$.

Lemma 3.3.1: Let (X,τ_1,τ_2,I) be an ideal bitopological space. Then (j,i) - $\alpha Cl(X \setminus A) = X \setminus (j,i)$ - $\alpha int(A)$.

Proof: Let $x \in (j,i)$ - $\alpha Cl(X \setminus A)$. That is for any (j,i) - α - open set U with $x \in U$, we have $U \cap (X \setminus A) \neq \phi$. This implies that there is no (j,i) - α - open set U with $x \in U$ so that $U \subseteq A$.Therefore, x is not (j,i) - α - interior point of A.

Thus $x \in X \setminus (j,i)$ - $\alpha int(A)$. On the other hand, let $x \in X \setminus (j,i)$ - $\alpha int(A)$. This implies $x \in X$ and $x \notin (j,i)$ - $\alpha int(A)$. Since $x \notin (j,i)$ - $\alpha int(A)$, there is no (j,i) - α - open set U with $x \in U$ so and $U \subseteq A$. We have $U \nsubseteq A$. Thus for any (j,i) - α - open set U with $x \in U$ we see that $U \cap (X \setminus A) \neq \phi$ which means that $x \in (j,i)$ - $\alpha Cl(X \setminus A)$.

The following Theorem gives a Characterization of (i,j) - I - $g\alpha$ - open sets in an Ideal bitopological space.

Theorem 3.3.2: Let (X,τ_1,τ_2,I) be an ideal bitopological space. A subset A of X is (i,j) - I - $g\alpha$ - open in X if and if only $(B \setminus P) \subset (j,i)$ - $\alpha int(A)$ for some $P \in I$, whenever $B \subset A$ and B is τ_i - α closed.

Proof: Suppose A is (i,j) - I - $g\alpha$ - open in X. Let $B \subset A$ and B is τ_i - α closed.Clearly $X \setminus A \subset X \setminus B$ and $X \setminus B$ is τ_i - α open. Since A is (i,j) - I - $g\alpha$ - open, we have $X \setminus A$ is (i,j) - I - $g\alpha$ - closed. By definition, (j,i) - $\alpha cl(X \setminus A) \setminus (X \setminus B) \in I$.This implies$(j,i)$ - $\alpha cl(X \setminus A) \cap (X \setminus B)^C$ = P for some $P \in I$
$[(j,i)$ - $\alpha cl(X \setminus A) \cap (X\setminus B)^C] \cup (X\setminus B) = P \cup (X\setminus B)$
$[(j,i)$ - $\alpha cl(X \setminus A) \cup (X\setminus B)] \cap [(X\setminus B)^C \cup (X\setminus B)] = P \cup (X\setminus B)$
(j,i) - $\alpha cl(X \setminus A) \cup (X\setminus B)] \cap X = P \cup (X\setminus B)$
(j,i) - $\alpha cl(X \setminus A) \cup (X\setminus B)] = P \cup (X\setminus B)$

This implies (j,i)- $\alpha cl(X\setminus A) \subset (X\setminus B) \cup P$, for some $P \in I$.Consequently $X \setminus ((X \setminus B) \cup P) \subset X \setminus (j,i)$ - $\alpha cl(X \setminus A)$. By Lemma 3.3.1, we have $B \setminus P \subset X \setminus (X \setminus (j,i)$ - $\alpha int(A))$. Hence $B \setminus P \subset (j,i)$ - $\alpha int(A)$.

Conversely, Let $X \setminus A \subset B$ where B is τ_i - α open in X, then $X \setminus B \subset A$ where B is τ_i - α closed. By hypothesis,
$(X \setminus B) \setminus P \subset (j,i)$ - $\alpha int(A)$, where $P \in I$ which implies (j,i) - $\alpha cl(X \setminus A) \subset X \setminus [(X \setminus B)\setminus P] \subset B \cup P$. Therefore, (j,i) - $\alpha cl(X \setminus A) \setminus B \subset P \in I$ which implies (j,i) - $\alpha cl(X \setminus A) \setminus B \in I$ whenever $X \setminus A \subset B$ and B is τ_i - α open $\Longrightarrow X \setminus A$ is (i,j) - I - $g\alpha$ - closed in X $\Longrightarrow A$ is (i,j) - I - $g\alpha$ - open in X.

Theorem 3.3.3: Let (X,τ_1,τ_2,I) be an ideal bitopological space. If A is (i,j) - I - $g\alpha$ - open in X and (j,i) - $\alpha int(A) \subset B \subset A$, then B is (i,j) - I - $g\alpha$ - open in X.

Proof: Assume that A is (i,j) - I - $g\alpha$ - open. Then $X \setminus A$ is (i,j) - I - $g\alpha$ - closed
We have $X \setminus A \subset X \setminus B \subset X \setminus (j,i)$ - $\alpha int(A)$ (3.2.2)
$X \setminus (j,i)$ - $\alpha int(A) = (j,i)$ - $\alpha cl(X \setminus A)$.By (3.3.1), $X \setminus A \subset X \setminus B \subset (j,i)$ - $\alpha cl(X \setminus A)$. By Theorem3.3.1, we have $X \setminus B$ is (i,j) - I - $g\alpha$ - closed. Hence B is (i,j) - I - $g\alpha$ - open.

Set operations:

Theorem 3.3.4: The intersection of two (i,j) - I - $g\alpha$ - open sets in an ideal bitopological space (X,τ_1,τ_2,I) is also (i,j) - I - $g\alpha$ - open.

Proof: Suppose A and B be two (i,j) - I - $g\alpha$ - open sets in X. Then $X \setminus A$ and $X \setminus B$ are (i,j) - I - $g\alpha$ - closed .Now, we have $X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$ is (i,j) - I - $g\alpha$ - closed, by Theorem 3.2.3. Hence $A \cap B$ is(i,j) - I - $g\alpha$ - open.

Remark 3.3.1: The union of (i,j) - I - $g\alpha$ - open sets need not be (i,j) - I - $g\alpha$ - open set. This is proved in the following example.

Example 3.3.1: Let $X = \{a,b,c\}$. Consider the topologies $\tau_1 = \{\phi,\{a\},\{b,c\},X\}$, $\tau_2 = \{\phi,\{b\},\{a,c\},X\}$ and $I = \{\phi\}$. Here $\{b\}$ and $\{c\}$ are (1,2) - I - $g\alpha$ - open but $\{b,c\}$ is not (1,2) - I - $g\alpha$ - open.

The following Theorem proves that the union of any two (i,j) - I - $g\alpha$ - open set is open with the condition.

Theorem 3.3.5: If A and B are two (i,j) - I - $g\alpha$ - open sets in X such that (j,i) - $\alpha cl(A) \cap B = \phi$ and $A \cap (j,i)$ - $\alpha cl(B) = \phi$, then $A \cup B$ is (i,j) - I - $g\alpha$ - open.

Proof: Let A and B be two (i,j) - I - $g\alpha$ - open sets in X such that (j,i) - $\alpha cl(A) \cap B = \phi$ and $A \cap (j,i)$ - $\alpha cl(B) = \phi$. Suppose V is τ_i - α closed and $V \subset A \cup B$. Then $V \cap (j,i)$ - $\alpha cl(A) \subset (A \cup B) \cap (j,i)$ - $\alpha cl(A) = (A \cap (j,i)$ - $\alpha cl(A)) \cup (B \cap (j,i)$ - $\alpha cl(A))$. $V \cap (j,i)$ - $\alpha cl(A) \subset A \cap (j,i)$ - $\alpha cl(A) = A$. Similarly,$V \cap (j,i)$ - $\alpha cl(B) \subset B \cap (j,i)$ - $\alpha cl(B) = B$. Since A and B are two (i,j) - I - $g\alpha$ - open sets in X, we have

$(V \cap (j,i)$ - $\alpha cl(A)) \setminus P \subset (j,i)$ - $\alpha int(A)$ and $(V \cap (j,i)$ - $\alpha cl(B)) \setminus Q \subset (j,i)$ - $\alpha int(B)$ for some $P, Q \in I$.
This implies that $(V \cap (j,i)$ - $\alpha cl(A)) \setminus (j,i)$ - $\alpha int(A) \subset P \in I$
$\Longrightarrow (V \cap (j,i)$ - $\alpha cl(A)) \setminus (j,i)$ - $\alpha int(A) \in I$ (3.2.3)

Similarly, $(V \cap (j,i)$ - $\alpha cl(B)) \setminus (j,i)$ - $\alpha int(B) \in I$ (3.3.3)

By (3.3.2) and (3.3.3), we have
$(V \cap (j,i)$ - $\alpha cl(A)) \setminus (j,i)$ - $\alpha int(A) \cup (V \cap (j,i)$ - $\alpha cl(B)) \setminus (j,i)$ - $\alpha int(B) \in I$. Since for any two sets $A, B \in I$ $\Longrightarrow A \cup B \in I$. Since for any sets A, B, C, D we have, $(A \cup C) \setminus (B \cup D) \subset (A \setminus B) \cup (C \setminus D)$
where $A = (V \cap (j,i)$ - $\alpha cl(A))$; $B = (j,i)$ - $\alpha int(A)$. $C = (V \cap (j,i)$ - $\alpha cl(B))$; $D = (j,i)$ - $\alpha int(B)$
$[(V \cap (j,i)$ - $\alpha cl(A)) \cup (V \cap (j,i)$ - $\alpha cl(B))] \setminus [(j,i)$- $\alpha int(A) \cup (j,i)$ - $\alpha int(B)] \in I$. Also since $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, we have $V \cap [(j,i)$ - $\alpha cl(A) \cup (j,i)$ - $\alpha cl(B)] \setminus (j,i)$ - $\alpha int(A) \cup (j,i)$– $\alpha int(B) \in I$. Thus $[V \cap ((j,i)$ - $\alpha cl(A \cup B)] \setminus (j,i)$ - $\alpha int(A) \cup (j,i)$ - $\alpha int(B) \in I$
[Since (j,i) - $\alpha cl(A) \cup \alpha cl(B) = (j,i)$ - $\alpha cl(A \cup B)$]. Further, $V = V \cap (A \cup B)$,
$V \subset V \cap ((j,i)$ - $\alpha cl(A \cup B))$. Therefore,
$V \setminus [(j,i)$ - $\alpha\mathrm{int}(A \cup B)] \subset [V \cap ((j,i)$ - $\alpha\mathrm{cl}(A \cup B))] \setminus [(j,i)$ - $\alpha\mathrm{int}(A \cup B)] \subset [V \cap ((j,i)$ - $\alpha cl(A \cup B)] \setminus ((j,i)$ - $\alpha int(A)) \cup ((j,i)$ - $\alpha int(B)) \in I$. This shows that $V \setminus R \subset (j,i)$ - $\alpha int(A \cup B)$ for some $R \in I$
$\Rightarrow A \cup B$ is (i,j) - I - $g\alpha$ - open.

Theorem 3.3.6: Let (X, τ_1, τ_2, I) be an ideal bitopological space. If A is (i,j) - I - $g\alpha$ - open set relative to B such that $A \subset B \subset X$ and B is (i,j) - I - $g\alpha$ - open relative to X, then A is (i,j) - I - $g\alpha$ - open relative to X.

Proof: Let $U \subset A$ and U is τ_i - α closed. Suppose A is (i,j) - I - $g\alpha$ - open relative to B. Then, we have $U \setminus P \subset (j,i)$ - $\alpha int_B(A)$ for some $\in I_B$, where $P \in I_B$denotes the ideal of the set B. This implies that there exists a (j,i) - α open set V_1, such that $U \setminus P \subset V_1 \cap B \subset A$. Let $U \subset B$ and U is τ_i - α closed. Suppose B is (i,j) - I - $g\alpha$ - open relative to X. Then, we have $U \setminus Q \subset (j,i)$ - $\alpha int(B)$ for some $Q \in I$ .This implies that there exists a (j,i) - α open set V_2, such that $U \setminus Q \subset V_2 \subset B$. Further $U \setminus (P \cup Q) = (U \setminus P) \cap (U \setminus Q) \subset ((V_1 \cap B) \cap V_2) \subset (V_1 \cap B) \cap B = V_1 \cap B \subset A$. This shows that $U \setminus (P \cup Q) \subset (j,i)$ - $\alpha int(A)$ for some $P \cup Q \in I$. Hence A is (i,j) - I - $g\alpha$ - open relative to X.

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