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ON SOME PROPERTIES OF (1, 2)*-$\alpha b\hat{g}$-CLOSED SETS IN BITOPOLOGICAL SPACES

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ABSTRACT

A new class of closed sets called (1,2)-$\alpha b\hat{g}$-closed sets in bitopological spaces is introduced in this article. This class contains the class of all (1,2)*-α-closed sets and is contained in the class of all (1,2)*-αg-closed set. The inclusion relationships of this new class with other known classes of closed sets are investigated. Also new classes of spaces, based on the class of (1,2)*-$\alpha b\hat{g}$-closed sets are introduced and their properties are analyzed.*

Keywords: *(1,2)*-α-closed sets, (1,2)*-b-closed sets, (1,2)*-$\hat{g}$-closed sets, (1,2)*-$b\hat{g}$-closed sets, (1,2)*-$\alpha b\hat{g}$-closed sets, (1,2)*-$\alpha b\hat{g}$ continuous and (1,2)*-$T^{c}_{\alpha b\hat{g}}$-space.*

AMS subject classification: *54H05, 54C05, 54F65.*

1. INTRODUCTION

The notion of alpha open sets (briefly α-open sets) was introduced and investigated by Njastad [17]. Maki et.al [16] defined generalized alpha and alpha generalized closed sets (briefly $g\alpha$ and αg closed sets) in 1993 and 1994 respectively in topological spaces as an extension of alpha and generalized closed sets. A new class called b-open sets in a topological space was introduced by Andrijevic [2] in 1996. This class is contained in the class of semi pre-opens sets and contains all semi-open and pre-open sets. Norman Levine introduced the concept of generalized closed sets [13] (briefly g-closed set) in topological spaces in 1963. As an extension of g-closed sets, Veera Kumar [21] defined $\hat{g}$-closed sets in topological spaces in 2003. Subasree and Maria Singam [20] defined a new class namely $b\hat{g}$-closed sets in topological spaces which is a subclass of gb-closed sets and contains b-closed sets. Followed by this, Mary and Nagajothi [18] [19] defined and characterized the class $\alpha b\hat{g}$-closed sets which is a subclass of $b\alpha\hat{g}$-closed sets and contains α-closed sets.

The concept of Bitopological Spaces was introduced by Kelly [16] in 1963. A set X equipped with two topologies τ_1 and τ_2 is called Bitoplogical spaces and it is denoted by (X, τ_1, τ_2). The concept and various class of closed sets defined in topological spaces (X, τ) have been extended to bitopological spaces(X, τ_1, τ_2). Fukutake [8], [9] defined generalized closed sets and semi open sets in bitopological space in 1986 and 1989 respectively. In 1990, Jelic [11] introduced the concept of alpha open sets in bitopological space. El-Tantawy and Abu-Donia [7] extends the class of α-closed set to alpha generalized closed sets in bitopological spaces.

In this article another new class of closed sets namely (1, 2)*-$\alpha b\hat{g}$-closed sets is introduced in bitopological spaces that satisfies the inclusion relations given below:
$\{(1,2)^*\text{-}\alpha\text{-}closed\ sets\} \subset \{(1,2)^*\text{-}\alpha b\hat{g}\text{-}closed\ sets\}$ and
$\{(1,2)^*\text{-}closed\ sets\} \subset \{(1,2)^*\text{-}\alpha b\hat{g}\text{-}closed\ sets\} \subset \{(1,2) * -\alpha g - closed\ set\}$.

As an application of (1,2)*-$\alpha b\hat{g}$-$closed\ sets$, new spaces such as (1,2)*-$T^{c}_{\alpha b\hat{g}}$-space, (1,2)*-$T^{gs}_{\alpha b\hat{g}}$-space, and (1,2)*-$T^{b\alpha\hat{g}}_{\alpha b\hat{g}}$-space are defined and their relationships with other known bitopological spaces are characterized.

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2. PRELIMINARIES

Throughout this paper, (X, τ_1, τ_2) denote a bitopological space with the topologies τ_1 and τ_2.

Definition 2.1.1: [10] A $\boldsymbol{topology}$ on a set X is a collection τ of subsets of X having the following properties:

1) ϕ and X are in τ.
2) The union of the elements of any sub collection of τ is in τ.
3) The intersection of the elements of any finite sub collection of τ is in τ.

A set X for which a topology τ has been specified is called a $\boldsymbol{topological\ space.}$

Definition 2.1.2: [12] A set X with two topologies τ_1 and τ_2 is said to be a $\boldsymbol{bitopological\ space}$ and it is denoted by $(\boldsymbol{X}, \boldsymbol{\tau_1}, \boldsymbol{\tau_2})$.

Definition 2.1.3: [6] A subset A of a bitopological space (X, τ_1, τ_2) is called $\boldsymbol{\tau_{1,2}}$-$\boldsymbol{open\ set}$ if $A \in \tau_1 \cup \tau_2$. The complement of a $\tau_{1,2}$-open set is $\boldsymbol{\tau_{1,2}}$-$\boldsymbol{closed\ set.}$

Definition 2.1.4: [6] Let A be a subset of a bitopological space (X, τ_1, τ_2), then

1. The $\boldsymbol{\tau_{1,2}}$-$\boldsymbol{interior\ of\ A}$ in (X, τ_1, τ_2), denoted by $\tau_{1,2}$-$int(A)$, is defined as $\cup \{F/F \subset A\ and\ F\ is\ \tau_{1,2}$-$open\ set\}$.
2. The $\boldsymbol{\tau_{1,2}}$-$\boldsymbol{closure\ of\ A}$ in (X, τ_1, τ_2), denoted by $\tau_{1,2}$-$cl(A)$, is defined as $\cap \{G/A \subset G\ and\ G\ is\ \tau_{1,2}$-$closed\ set\}$.

Definition 2.1.5: [6] A subset A of a bitopological space (X, τ_1, τ_2) is called

1. a **(1,2)*-$\boldsymbol{semi\ open\ set}$** if $A \subseteq \tau_{1,2}$-$cl(\tau_{1,2}$-$int(A))$ and a **(1,2)*-$\boldsymbol{semi\ closed\ set}$** if $\tau_{1,2}$-$int(A)(\tau_{1,2}$-$cl(A)) \subseteq A$.
2. a **(1,2)*-$\boldsymbol{pre\ open\ set}$** if $A \subseteq \tau_{1,2}$-$int(\tau_{1,2}$-$cl(A))$ and a **(1,2)*-$\boldsymbol{pre\ closed\ set}$** if $\tau_{1,2}$-$cl(A)(\tau_{1,2}$-$int(A)) \subseteq A$.
3. a **(1,2)*-$\boldsymbol{\alpha}$- $\boldsymbol{open\ set}$** if $A \subseteq \tau_{1,2}$-$int(\tau_{1,2}$-$cl(\tau_{1,2}$-$int(A)))$ and a **(1,2)*-$\boldsymbol{\alpha}$- $\boldsymbol{closed\ set}$** if $\tau_{1,2}$-$cl(\tau_{1,2}$-$int(\tau_{1,2}$-$cl(A))) \subseteq A$.
4. a **(1,2)*-$\boldsymbol{b}$- $\boldsymbol{open\ set}$** if $A \subseteq \tau_{1,2}$-$cl(\tau_{1,2}$-$int(A)) \cup \tau_{1,2}$-$int(\tau_{1,2}$-$cl(A))$ and a **(1,2)*-$\boldsymbol{b}$- $\boldsymbol{closed\ set}$** if $\tau_{1,2}$-$int(\tau_{1,2}$-$cl(A)) \cap \tau_{1,2}$-$cl(\tau_{1,2}$-$int(A)) \subseteq A$.
5. a **(1,2)*-$\boldsymbol{semi\ pre\ open\ set}$** if $A \subseteq \tau_{1,2}$-$cl(\tau_{1,2}$-$int(\tau_{1,2}$-$cl(A)))$ and a **(1,2)*-$\boldsymbol{semi\ pre\ closed\ set}$** if $\tau_{1,2}$-$int(\tau_{1,2}$-$cl(\tau_{1,2}$-$int(A))) \subseteq A$.

Definition 2.1.6: [6]

1. The intersection of all (1,2)*-$semi\ closed\ sets$ containing A is called **(1,2)*-$\boldsymbol{semi\ closure\ of\ A}$** and it is denoted by $\boldsymbol{\tau_{1,2}}$-$\boldsymbol{scl(A)}$.
2. The intersection of all (1,2)*-α-$closed\ sets$ containing A is called **(1,2)*-$\boldsymbol{\alpha}$-$\boldsymbol{closure\ of\ A}$** and it is denoted by $\boldsymbol{\tau_{1,2}}$-$\boldsymbol{\alpha cl(A)}$.
3. The intersection of all (1,2)*-b-$closed\ sets$ containing A is called **(1,2)*-$\boldsymbol{b}$-$\boldsymbol{closure\ of\ A}$** and it is denoted by $\boldsymbol{\tau_{1,2}}$-$\boldsymbol{bcl(A)}$.
4. The intersection of all (1,2)*-$pre\ closed\ sets$ containing A is called **(1,2)*-$\boldsymbol{pre\ closure\ of\ A}$** and it is denoted by $\boldsymbol{\tau_{1,2}}$-$\boldsymbol{pcl(A)}$.
5. The intersection of all (1,2)*-$semi\ pre\ closed\ sets$ containing A is called(1,2)*-$semi\ pre\ closure\ of\ A$ and it is denoted by $\boldsymbol{\tau_{1,2}}$-$\boldsymbol{spcl(A)}$.

Definition 2.1.7: [6] The family of all (1,2)*-open sets, (1,2)*-α-open sets, (1,2)*-b-open sets, (1,2)*-$semi$ open sets in X are denoted by (1,2)*-$O(X)$, (1,2)*-$\alpha O(X)$, (1,2)*-$BO(X)$ and (1,2)*-$SO(X)$ respectively.

Definition 2.1.8: [6] A subset A of a bitopological space (X, τ_1, τ_2) or X is called

1. a **(1,2)*-$\boldsymbol{generalized\ closed\ set}$** (briefly **(1,2)*-$\boldsymbol{g}$-$\boldsymbol{closed\ set}$**) if $\tau_{1,2}$-$cl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$O(X)$.
2. a **(1,2)*-$\boldsymbol{generalized\ semi\ closed\ set}$** (briefly **(1,2)*-$\boldsymbol{gs}$-$\boldsymbol{closed\ set}$**) if $\tau_{1,2}$-$scl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$O(X)$.
3. a **(1,2)*- $\boldsymbol{semi\ generalized\ closed\ set}$** (briefly **(1,2)*-$\boldsymbol{sg}$-$\boldsymbol{closed\ set}$**) if $\tau_{1,2}$-$scl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$SO(X)$.
4. a **(1,2)*-$\boldsymbol{\alpha\ generalized\ closed\ set}$** (briefly **(1,2)*-$\boldsymbol{\alpha g}$-$\boldsymbol{closed\ set}$**) if $\tau_{1,2}$-$\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$O(X)$.
5. a **(1,2)*- $\boldsymbol{generalized\ \alpha\ closed\ set}$** (briefly **(1,2)*-$\boldsymbol{g\alpha}$-$\boldsymbol{closed\ set}$**) if $\tau_{1,2}$-$\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$\alpha O(X)$.

6. a **(1,2)*-*generalized pre closed set*** (briefly **(1,2)*-*gp-closed set***) if $\tau_{1,2}$-$pcl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$O(X)$.
7. a **(1,2)*-*generalized semi pre closed set*** (briefly **(1,2)*-*gsp-closed set***) if $\tau_{1,2}$-$spcl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$O(X)$.
8. a **(1,2)*-*strongly generalized closed set*** (briefly **(1,2)*-*strongly-g-closed set***) if $\tau_{1,2}cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$O(X)$.

Definition 2.1.9: A subset A of a bitopological space (X, τ_1, τ_2) or X is called

1. a (1,2)*-$\hat{g}$-*closed set* [21] if $\tau_{1,2}$-$cl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$SO(X)$ and the complement of (1,2)*-$\hat{g}$-*closed set* is called a (1,2)*-$\hat{g}$-*open set*.
2. a (1,2)*-gb-*closed set* if $\tau_{1,2}$-$bcl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$O(X)$ and the complement of (1,2)*-gb-*closed set* is called a (1,2)*-gb-*open set*.
3. a (1,2)*-$b\hat{g}$-*closed set* [20] if $\tau_{1,2}$-$bcl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$\hat{G}O(X)$ and the complement of (1,2)*-$b\hat{g}$-*closed set* is called a (1,2)*-$b\hat{g}$-*open set*.
4. a (1,2)*-$\alpha\hat{g}$-*closed set* [1]if $\tau_{1,2}$-$\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$\hat{G}O(X)$ and the complement of (1,2)*-$\alpha\hat{g}$-*closed set* is called a (1,2)*-$\alpha\hat{g}$-*open set*.
5. a (1,2)*-$b\alpha\hat{g}$-*closed set* [19] if $\tau_{1,2}$-$bcl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in$ (1,2)*-$\alpha\hat{G}O(X)$ and the complement of (1,2)*-$b\alpha\hat{g}$-*closed set* is called a (1,2)*-$b\alpha\hat{g}$-*open set*.

Definition 2.1.10: The family of all (1,2) *-g-open sets, (1,2) *-$\hat{g}$- open sets, (1,2) *-$\alpha\hat{g}$-open sets and (1,2) *-$b\hat{g}$-open sets in X are denoted by (1,2) *-$GO(X)$, (1,2) *-$\hat{G}O(X)$, (1,2) *-$\alpha\hat{G}O(X)$ and (1,2) *-$B\hat{G}O(X)$ respectively.

Definition 2.1.11: A function $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is called

1. a (1,2)*-*continuous function* if $f^{-1}(V)$ is (1,2)*-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).
2. a (1,2)*-g-*continuous function* if $f^{-1}(V)$ is (1,2)*-g-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).
3. a (1,2)*-α *generalized continuous function* (briefly (1,2)*-αg-continuous) if $f^{-1}(V)$ is (1,2)*-αg-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).
4. a (1,2)*- *generalized* α *continuous function* (briefly (1,2)*-$g\alpha$-continuous) if $f^{-1}(V)$ is (1,2)*-$g\alpha$-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).
5. a (1,2)*- *generalized semi continuous function* (briefly (1,2)*-gs-continuous) if $f^{-1}(V)$ is (1,2)*-gs-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).
6. a (1,2)*- *semi generalized continuous function* (briefly (1,2)*-sg-continuous) if $f^{-1}(V)$ is (1,2)*-sg-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).
7. a (1,2)*- *generalized semi-pre continuous function* (briefly (1,2)*-gsp-continuous) if $f^{-1}(V)$ is (1,2)*-gsp-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).
8. a (1,2)*- *generalized pre continuous function* (briefly (1,2)*-gp-continuous) if $f^{-1}(V)$ is (1,2)*-gp-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).
9. a (1,2)*-gb-*continuous function* if $f^{-1}(V)$ is (1,2)*-gb-closed set in (X, τ_1, τ_2) for every (1,2)*-closed set V of (Y, σ_1, σ_2).

Definition 2.1.12: [6] A bitopological space (X, τ_1, τ_2) is called

1. a (1,2)*-$T_{1/2}$-*space* if every (1,2)*-g-closed set in it is (1,2)*-closed set.
2. a (1,2)*-T_b-*space* if every (1,2)*-gs-closed set in it is (1,2)*-closed set.
3. a (1,2)*- ${}_\alpha T_b$-*space* if every (1,2)*-αg-closed set in it is (1,2)*-closed set.
4. a (1,2)*-$T^c_{b\alpha\hat{g}}$-*space* if every (1,2)*-$b\alpha\hat{g}$-closed set in it is (1,2)*-closed set.

3. (1,2)*-α$b\hat{g}$ CLOSED SETS:

In this section we introduce a new class of closed sets called (1,2)*-α$b\hat{g}$-closed sets which lie between the class of (1,2)*-α-closed sets and the class of (1,2)*-αg-closed sets.

Definition 3.1: A subset A of a bitopological space (X,τ_1,τ_2) is said to be **(1,2)*-*$\alpha b\hat{g}$-closed sets*** if $\tau_{1,2}$-αcl(A) ⊆ U, whenever A ⊆ U and U ∈ (1,2)*- b$\hat{g}$ open set in (X,τ_1,τ_2). The family of all (1,2)*- $\alpha b\hat{g}$-*open sets* in X is denoted by (1,2)*-$\alpha b\hat{g}O(X)$.

3.1 Relationship of (1,2)*- α$b\hat{g}$ closed sets with other classes of (1,2)*-closed sets:

Theorem 3.1.1: Every (1,2)*-α-closed set is (1,2)*-$\alpha b\hat{g}$-closed set.

Proof: Let A be an (1,2)*-α-closed set and U ∈ (1,2)*- $b\hat{g}$-open set such that A ⊆ U. Since A is (1,2)*-α-closed set, we have $\tau_{1,2}$-$\alpha cl(A) = A \subseteq U$. Therefore $\tau_{1,2}$-$\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U ∈ (1,2)*- $b\hat{g}$-open set. Hence A is (1,2)*-$\alpha b\hat{g}$-closed set.

Corollary 3.1.1: Every (1,2)*-α-open set is (1,2)*-$\alpha b\hat{g}$-open set.

Theorem 3.1.2:
(i) Every (1,2)*-closed set is (1,2)*-$\alpha b\hat{g}$-closed set.
(ii) Every (1,2)*-$\alpha b\hat{g}$-closed set need not be (1,2)*-closed set.
(iii) If a (1,2)*-$\alpha b\hat{g}$-closed set is (1,2)*-$b\hat{g}$-open set then it is (1,2)*-closed set.

Proof:
(i) Let A be an (1,2)*-closed set and U ∈ (1,2)*- $b\hat{g}$-open set such that A ⊆ U. Since A is (1,2)*-closed set, we have $\tau_{1,2}$-$cl(A) = A \subseteq U$. Therefore $\tau_{1,2}$-$cl(A) \subseteq U$, whenever $A \subseteq U$ and U ∈ (1,2)*- $b\hat{g}$-open set. Hence A is (1,2)*-$\alpha b\hat{g}$-closed set.
(ii) **Example 3.1.1:** Let X= {a, b, c} with τ_1= {ϕ , X, {a}} and τ_2= {ϕ , X, {a, b}}. Clearly A= {a, c} is (1,2)*-$\alpha b\hat{g}$-closed set, but not (1,2)*-closed set.
(iii) Let A be (1,2)*- $\alpha b\hat{g}$-closed set and (1,2)*- $b\hat{g}$-open set in (X, τ_1 ,τ_2). Since $A \subseteq A$, and by our assumption, we have $\tau_{1,2}$-$cl(A) \subseteq A$. It is obvious that, $A \subseteq \tau_{1,2}$-$cl(A)$. Hence A is (1,2)*-closed set in (X, τ_1 ,τ_2).

Corollary 3.1.2: Every (1,2)*-open set is (1,2)*-$\alpha b\hat{g}$-open set.

Theorem 3.1.3: Let A be a (1,2)*-$\alpha b\hat{g}$ closed sets in a bitopological space (X, τ_1, τ_2). Then A is
(i) (1,2)*-αg-closed set, (ii) (1,2)*-gs-closed set, (iii) (1,2)*-gp-closed set, (iv) (1,2)*-gsp-closed set, (v) (1,2)*-gb-closed set, (vi) (1,2)*-$b\alpha\hat{g}$-closed set.

Proof:
(i) Let A be an (1,2)*-$\alpha b\hat{g}$-closed set and U be a (1,2)*-open set such that A ⊆ U. Since every (1,2)*-open set is (1,2)*- b$\hat{g}$ open set, A ⊆ U. This implies , $\tau_{1,2}$-$\alpha cl(A) \subseteq U$. Hence A is (1,2)*-αg-closed set.
(ii) Let A be an (1,2)*-$\alpha b\hat{g}$-closed set and U be a (1,2)*-open set such that A ⊆ U. Since every (1,2)* -open set is (1,2)*- b$\hat{g}$ open set, A ⊆ U. This implies, $\tau_{1,2}$-$scl(A) \subseteq \tau_{1,2}$-$\alpha cl(A) \subseteq U$. Hence A is (1,2)*-$gs$-closed set.
(iii) Let A be an (1,2)*-$\alpha b\hat{g}$-closed set and U be a (1,2)*-open set such that A ⊆ U. Since every (1,2)* -open set is (1,2)*- b$\hat{g}$ open set, A ⊆ U. This implies, $\tau_{1,2}$-$pcl(A) \subseteq \tau_{1,2}$-$\alpha cl(A) \subseteq U$. Hence A is (1,2)*-$gp$-closed set.
(iv) Let A be an (1,2)*-$\alpha b\hat{g}$-closed set and U be a (1,2)*-open set such that A ⊆ U. Since every (1,2)* -open set is (1,2)*- b$\hat{g}$-open set, A ⊆ U. This implies, $\tau_{1,2}$-$spcl(A) \subseteq \tau_{1,2}$-$\alpha cl(A) \subseteq U$. Hence A is (1,2)*-$gsp$-closed set.
(v) Let A be an (1,2)*-$\alpha b\hat{g}$-closed set and U be a (1,2)*-open set such that A ⊆ U. Since every (1,2)* -open set is (1,2)*- b$\hat{g}$-open set, A ⊆ U. This implies, $\tau_{1,2}$-$bcl(A) \subseteq \tau_{1,2}$-$\alpha cl(A) \subseteq U$. Hence A is (1,2)*-$gb$-closed set.
(vi) Let A be an (1,2)*-$\alpha b\hat{g}$-closed set and U be a (1,2)*-$\alpha\hat{g}$-open set such that A ⊆ U. Since every (1,2)* - $\alpha\hat{g}$-open set is (1,2)*- b$\hat{g}$-open set, A ⊆ U. This implies, $\tau_{1,2}$-$bcl(A) \subseteq \tau_{1,2}$-$\alpha cl(A) \subseteq U$. Hence A is (1,2)*-$b\alpha\hat{g}$-closed set.

Remark 3.1.1: It is interesting to note that the converse part of the statements in the above Theorem need not be true. This is proved in the following examples:

Example 3.1.2: Let X= {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2={ϕ , X, {a, b}}, A= {a, c} is (1,2)*-αg-closed set, but not (1,2)*-$\alpha b\hat{g}$-closed set.

Example 3.1.3: Let X= {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ , X, {b, c}}, A= {b} is (1,2)*-gs-closed set, but not (1,2)*-$\alpha b\hat{g}$-closed set.

Example 3.1.4: Let X= {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ , X, {a, b}}, A= {a, c} is (1,2)*-gp-closed set, but not (1,2)*-$\alpha b\hat{g}$-closed set.

Example 3.1.5: Let X= {a, b, c} with τ_1= {ϕ , X, {a}} and τ_2={ϕ , X, {b, c}}, A= {c} is (1,2)*-gsp-closed set, but not (1,2)*-$\alpha b\hat{g}$-closed set.

Example 3.1.6: Let X= {a, b, c} with τ_1= {ϕ , X, {a}} and τ_2= {ϕ, X, {a, b}}, A= {a, c} is (1,2)*-gb-closed set, but not (1,2)*-$\alpha b\hat{g}$-closed set.

Example 3.1.7: Let X= {a, b, c} with τ_1= {ϕ , X, {a}} and τ_2= {ϕ X, {b, c}}, A={c} is (1,2)*-$b\alpha\hat{g}$-closed set, but not (1,2)*-$\alpha b\hat{g}$-closed set.

Remark 3.1.2: From Theorem 3.1.1 and Theorem 3.1.3 it is observed that the following inclusion relations holds: *{(1,2)*-α-closed sets} ⊆ {(1,2)*-$\alpha b\hat{g}$-closed sets} ⊆ {(1,2)*-αg-closed sets}.*

Remark 3.1.3: The following examples reveal that the class of (1,2)*-$\alpha b\hat{g}$-closed sets are $\boldsymbol{independent}$ from (1,2)*-g-closed sets, (1,2)*-$\hat{g}$-closed sets and (1,2)*-$strongly\ g$-closed sets.

Example 3.1.8:

(i) Let X= {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2={ϕ, X, {a, b}}, A= {b} is (1,2)*-$\alpha b\hat{g}$-closed set, but not (1,2)*-g-closed set.

(ii) Let X= {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {b ,c}}, A= {a ,b} is (1,2)*-g-closed set, but not (1,2)*-$\alpha b\hat{g}$-closed set.

Example 3.1.9:

(i) Let X= {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {b, c}}, A= {a, b} is (1,2)*-$\hat{g}$-closed set, but not (1,2)*-$\alpha b\hat{g}$-closed set.

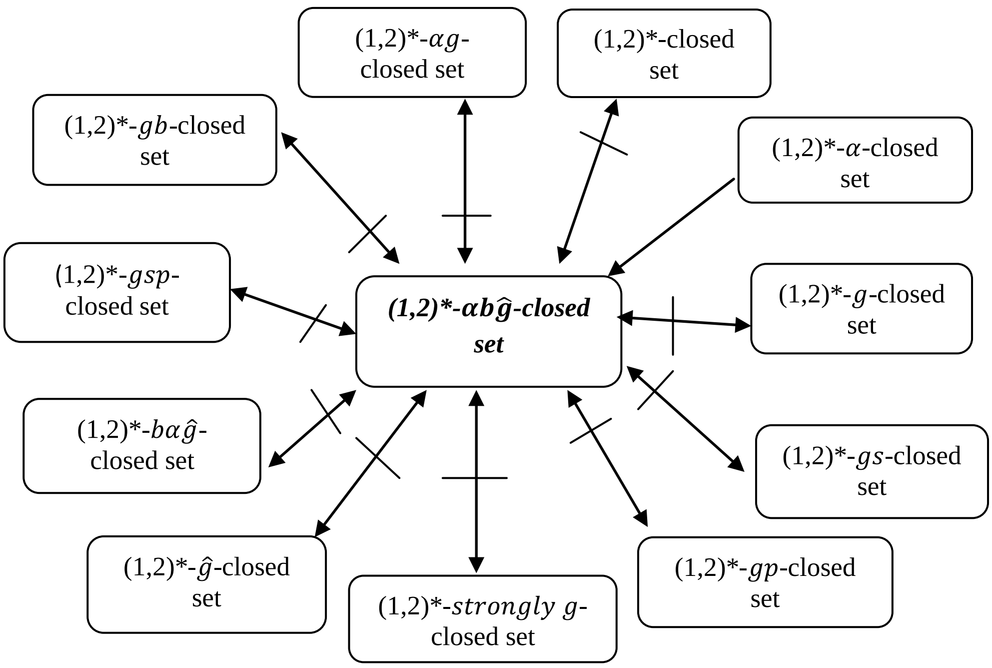
(ii) Let X= {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a, b}}, A= {b} is (1,2)*-$\alpha b\hat{g}$-closed set, but not (1,2)*-$\hat{g}$-closed set.

Example 3.1.10:

(i) Let X= {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a,b}}, A= {a, c} is (1,2)*-$strongly\ g$-closed set, but not (1,2)*-$\alpha b\hat{g}$-closed set.

(ii) Let X= {a, b, c} with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a, b}}, A={c} is (1,2)*-$\alpha b\hat{g}$-closed set, but not (1,2)*-$strongly\ g$-closed set.

Relationships of (1,2)*-$\alpha b\hat{g}$-closed sets with other closed sets are represented by the following diagram:



In the above diagram, A→B denotes A implies B, A ←+→ B denotes A implies B but B does not imply A, A←+→ B denotes B implies A but A does not imply B, A ←+→ B denotes A and B are independent.

3.2 (1,2)*-$\alpha b\hat{g}$- CONTINUOUS FUNCTION

We introduce the following definition.

Definition 3.2: A function f: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) is called (1,2)*-$\alpha b\hat{g}$-continuous if f^{-1}(V) is a (1,2)*-$\alpha b\hat{g}$-closed set of (X, τ_1 ,τ_2) for every closed set V of (Y, σ_1, σ_2).

Theorem 3.2.1: Every continuous map f: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) is (1,2)*-$\alpha b\hat{g}$-continuous.

Proof: Let V be a (1,2)*- closed set in (Y, σ_1, σ_2) , then f^{-1}(V) is a (1,2)*- closed set in (X, τ_1 ,τ_2). Since every (1,2)*-closed set is (1,2)*-$\alpha b\hat{g}$-closed set, f^{-1}(V) is a (1,2)*- $\alpha b\hat{g}$- closed set in (X, τ_1 ,τ_2). Hence f is an (1,2)*-$\alpha b\hat{g}$-continuous.

Remark 3.2.1: The converse of the above theorem need not true. This is proved in the following example:

Example 3.2.1: Let X= {a, b, c}=Y with $\tau_{1,2}$= {ϕ, X, {a}, {a, b}} and $\sigma_{1,2}$= {ϕ, Y, {a, c} }. Let *f*: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) be the identity map, then f is (1,2)*-$\alpha b\hat{g}$-continuous but not (1,2)*-continuous. For the (1,2)*-closed set {b} in (Y, σ_1, σ_2), f^{-1}({b})={b} is (1,2)*-$\alpha b\hat{g}$-closed, but not (1,2)*-closed in (X, τ_1 ,τ_2) .

The following Theorem is an application of Theorem 3.1.3.

Theorem 3.2.2: Let *f*: (X, τ_1, τ_2) → (Y, σ_1, σ_2) be a (1,2)*-$\alpha b\hat{g}$-continuous map, then f is
(a) (1,2)*-αg-continuous map, (b) (1,2)*-gs-continuous map, (c) (1,2)*-gp-continuous map, (d) (1,2)*-gsp-continuous map, (e) (1,2)*-gb-continuous map.

Proof: The proof follows from Theorem 3.1.3. We prove part *(a).*
a) Let V be a (1,2)*-closed set in (Y, σ_1, σ_2) , then f^{-1}(V) is a (1,2)*-$\alpha b\hat{g}$-closed set in(X, τ_1 ,τ_2). By Theorem 3.1.3, every (1,2)*- $\alpha b\hat{g}$-closed set is (1,2)*-αg-closed set, and hence f^{-1}(V) is a (1,2)*-αg-closed set in (X, τ_1 ,τ_2). Thus f is an (1,2)*-αg-continuous.

Similarly the other parts *b), c), d)* and *e)* can be proved.

Remark 3.2.2: The converse of the above Theorem need not true. This is proved in the following examples:

Example 3.2.2: Let X={a, b, c}=Y with $\tau_{1,2}$= {ϕ, X, {a}, {b, c}} and $\sigma_{1,2}$= {ϕ, Y, {a}, {b}, {a, b}}. Let *f*: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) be the identity map, then f is (1,2)*-αg-continuous but not (1,2)*- $\alpha b\hat{g}$-continuous. For the (1,2)*-closed set {a, c} in (Y, σ_1, σ_2), f^{-1}({a, c})= {a, c} is (1,2)*-αg-closed set, but not (1,2)*- $\alpha b\hat{g}$-closed in (X, τ_1 ,τ_2).

Example 3.2.3: Let X={a, b, c}=Y with $\tau_{1,2}$= {ϕ, X, {a}, {a, b}} and $\sigma_{1,2}$= {ϕ, Y, {a}, {b}, {a, b}}. Let *f*: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) be the identity map, then f is (1,2)*-gs-continuous but not (1,2)*- $\alpha b\hat{g}$-continuous. For the (1,2)*-closed set {a, c} in (Y, σ_1, σ_2), f^{-1}({a,c})={a,c} is (1,2)*-gs- closed set, but not (1,2)*- $\alpha b\hat{g}$-closed in (X, τ_1 ,τ_2).

Example 3.2.4: Let X={a, b, c}=Y with $\tau_{1,2}$= {ϕ, X, {a}, {b, c}} and $\sigma_{1,2}$= {ϕ, Y, {a}, {b}, {a, b}}. Define f : (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) by *f*(a)={c}, *f*(b)={b}, *f*(c)={a}, and f^{-1}(c)={a}, f^{-1}(b)={b}, f^{-1}(a)={c}, then f is (1,2)*-gp-continuous but not (1,2)*- $\alpha b\hat{g}$-continuous. For the (1,2)*-closed set {b, c} in (Y, σ_1, σ_2), f^{-1}({b, c})={a, b} is (1,2)*-gp- closed set, but not (1,2)*- $\alpha b\hat{g}$-closed in (X, τ_1 ,τ_2) .

Example 3.2.5: Let X={a, b, c}=Y with $\tau_{1,2}$= {ϕ, X, {a}, {a, b}} and $\sigma_{1,2}$= {ϕ, Y, {a}, {b}, {a, b}}. Let *f*: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) be an identity map, then f is (1,2)*-gsp-continuous but not (1,2)*- $\alpha b\hat{g}$-continuous. For the (1,2)*-closed set {a, c} in (Y, σ_1, σ_2), f^{-1}({a,c})={a,c} is (1,2)*-gsp- closed set, but not (1,2)*- $\alpha b\hat{g}$-closed in (X, τ_1 ,τ_2).

Example 3.2.6: Let X={a, b, c}=Y with $\tau_{1,2}$= {ϕ, X, {a}, {b, c}} and $\sigma_{1,2}$= {ϕ, Y, {a}, {b}, {a, b}}. Let *f*: (X, τ_1 ,τ_2) → (Y, σ_1, σ_2) be an identity map, then f is (1,2)*-gb-continuous but not (1,2)*- $\alpha b\hat{g}$-continuous. For the (1,2)*-closed set {c} in (Y, σ_1, σ_2), f^{-1}({c})={c} is (1,2)*-gb- closed set, but not (1,2)*- $\alpha b\hat{g}$-closed in (X, τ_1 ,τ_2) .

3.3 APPLICATION OF (1,2)*-α$b\hat{g}$ CLOSED SETS:

As an application of (1,2)*-α$b\hat{g}$-closed sets we introduce new spaces namely (1,2)*-$T^{c}_{\alpha b\hat{g}}$-space, (1,2)*-$T^{gs}_{\alpha b\hat{g}}$-space, and (1,2)*-$T^{b\alpha\hat{g}}_{\alpha b\hat{g}}$-space.

Definition 3.3: A bitopological space (X, τ_1 ,τ_2) is called,
(i) a (1,2)*-$T^{c}_{\alpha b\hat{g}}$-space if every (1,2)*-α$b\hat{g}$-closed in it is (1,2)*-closed set.
(ii) a (1,2)*-$T^{gs}_{\alpha b\hat{g}}$-space if every (1,2)*-gs-closed set in it is (1,2)*-α$b\hat{g}$-closed set.
(iii) a (1,2)*-$T^{b\alpha\hat{g}}_{\alpha b\hat{g}}$ -space if every (1,2)*-$b\alpha\hat{g}$-closed set in it is (1,2)*-α$b\hat{g}$-closed set.

Theorem 3.3.1: Let (X, τ_1 ,τ_2) be (1,2)*-$T^{gs}_{\alpha b\hat{g}}$-space and (1,2)*-$T^{c}_{\alpha b\hat{g}}$-space, then it is (1,2)*-$T_{1/2}$ -space.

Proof: Let A be a (1,2)*- g-closed set. Since every (1,2)*-g-closed set is a (1,2)*-gs-closed set, A is a (1,2)*-gs-closed set. Since (X, τ_1 ,τ_2) is a (1,2)*-$T^{gs}_{b\alpha\hat{g}}$-space, A is (1,2)*- $\alpha b\hat{g}$-closed set and in (1,2)*-$T^{c}_{\alpha b\hat{g}}$-space, A is (1,2)*-closed set. Hence (X, τ_1 ,τ_2) is (1,2)*-$T_{1/2}$ -space.

Remark 3.3.1: The converse of the above theorem need not be true. This is proved in the following example:

Example 3.3.1: Let X={a, b, c}=Y with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {a}, {b}, {a, b}}. In (X, τ_1 ,τ_2) every (1,2)*-g-closed set is (1,2)*-closed set. Hence (X, τ_1 ,τ_2) is a (1,2)*-$T_{1/2}$-space but not (1,2)*-$T^{gs}_{b\alpha\hat{g}}$-space, since A = {a} is (1,2)*-gs-closed set, but not (1,2)*-$\alpha b\hat{g}$-closed set.

Theorem 3.3.2:

(i) Every (1,2)*-T_b-space is (1,2)*-$T^{c}_{\alpha b\hat{g}}$-space. The converse need not be true.

(ii) Every (1,2)*-T_b-space is (1,2)*-$T^{gs}_{\alpha b\hat{g}}$-space.

Proof:

(i) Let A be a (1,2)*-α$b\hat{g}$-closed set. By theorem 3.3.1, A is (1,2)*-gs-closed set. Since (X, τ_1 ,τ_2) is (1,2)*-T_b-space, A is (1,2)*-closed set. Hence X is a (1,2)*-$T^{c}_{\alpha b\hat{g}}$-space.The converse is not true and it is proved in the following example.

Example 3.3.2: Let X={a, b, c}=Y with τ_1= {ϕ, X, {a}} and τ_2= {ϕ, X, {b, c}}. In (X, τ_1 ,τ_2) every (1,2)*-α$b\hat{g}$-closed set is (1,2)*-closed set. Hence (X, τ_1 ,τ_2) is (1,2)*-$T^{c}_{\alpha b\hat{g}}$-space but not (1,2)*-T_b-space, since A = {b} is (1,2)*-gs-closed set but not (1,2)*-closed set.

(ii) Let A be a (1,2)*-gs-closed set. Since (X, τ_1 ,τ_2) is a (1,2)*-T_b-space, A is (1,2)*-closed set. By Theorem 3.1.3, A is a (1,2)*-$\alpha b\hat{g}$-closed set. Hence (X, τ_1 ,τ_2) is a (1,2)*-$T^{gs}_{\alpha b\hat{g}}$-space.

Theorem 3.3.3: Every (1,2)*-${}_{\alpha}T_b$ -space is (1,2)*-$T^{\alpha g}_{\alpha b\hat{g}}$-space.

Proof: Let A be a (1,2)*-αg-closed set. Since (X, τ_1 ,τ_2) is a (1,2)*-${}_{\alpha}T_b$ -space, A is (1,2)*-closed set. By theorem 3.1.3, A is a (1,2)*-$\alpha b\hat{g}$-closed set. Hence (X, τ_1 ,τ_2) is a (1,2)*-$T^{\alpha g}_{\alpha b\hat{g}}$-space.

Theorem 3.3.4: Every (1,2)*-$T^{c}_{b\alpha\hat{g}}$-space is a

(i) (1,2)*-$T^{b\alpha\hat{g}}_{\alpha b\hat{g}}$-space. The converse need not be true.

(ii) (1,2)*-$T^{c}_{\alpha b\hat{g}}$-space. The converse need not be true.

Proof:

(i) Let A be a (1,2)*-$b\alpha\hat{g}$-closed set. Since (X, τ_1, τ_2) is a (1,2)*-$T^{c}_{b\alpha\hat{g}}$-space, A is (1,2)*-closed set. By Theorem3.1.2, A is a (1,2)*-$\alpha b\hat{g}$-closed set. Hence (X, τ_1, τ_2) is a (1,2)*-$T^{b\alpha\hat{g}}_{\alpha b\hat{g}}$-space.

(ii) Let A be a (1,2)*-$\alpha b\hat{g}$-closed set. By Theroem3.1.3, A is a (1,2)*-$b\alpha\hat{g}$-closed set. Since (X, τ_1, τ_2) is a (1,2)*-$T^{c}_{b\alpha\hat{g}}$-space, A is (1,2)*-closed set. By Theorem3.1.2, A is a (1,2)*-$\alpha b\hat{g}$-closed set. Hence (X, τ_1, τ_2) is a (1,2)*-$T^{c}_{\alpha b\hat{g}}$-space.

Remark 3.3.2: The converse of the above theorem need not be true. This is proved in the following examples:

Example 3.3.3: Let X={a, b, c}=Y with τ_1={ϕ , X, {a}} and τ_2={ϕ , X, {a}, {b}, {a, b}}. In (X, τ_1, τ_2) every (1,2)*-$b\alpha\hat{g}$-closed set is (1,2)*-$\alpha b\hat{g}$closed set. Hence (X, τ_1 ,τ_2) is a (1,2)*-$T^{b\alpha\hat{g}}_{\alpha b\hat{g}}$-space but not (1,2)*-$T^{c}_{b\alpha\hat{g}}$-space, since A = {b} is (1,2)*-$b\alpha\hat{g}$-closed set, but not (1,2)*-closed set.

Example 3.3.4: Let X={a, b, c}=Y with τ_1={ϕ , X, {a}} and τ_2={ϕ , X, {a}, {b}, {a, b}}. In (X, τ_1, τ_2) every (1,2)*-$\alpha b\hat{g}$-closed set is (1,2)*-closed set. Hence (X, τ_1 ,τ_2) is a (1,2)*-$T^{c}_{\alpha b\hat{g}}$-space but not (1,2)*-$T^{c}_{b\alpha\hat{g}}$-space, since A = {a} is (1,2)*-$b\alpha\hat{g}$-closed set, but not (1,2)*-closed set.

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