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ON STRONGER FORMS OF SEMI #GENERALIZED α – IRRESOLUTE FUNCTIONS

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ABSTRACT

*In this paper we introduce some stronger forms of semi #gα-irresolute functions namely strongly semi #gα-irresolute functions, strongly *semi #gα-irresolute functions and almost semi #gα-irresolute functions in topological spaces. We discuss some properties and characterizations of these functions. Moreover we examine the relationships of these functions with the other existing functions.*

***Key words**: semi #gα-closed set, semi #gα-open set, semi #gα-irresolute functions, strongly semi #gα-irresolute functions, strongly *semi #gα-irresolute functions and almost semi #gα-irresolute functions.*

1. INTRODUCTION

The concept of strongly α-irresolute functions was introduced by G. Lo Fara [10] in 1987. Later in 2003, Y. Beceren [3] introduced the notions of almost α-irresolute functions and β-preirresolute functions. R.Devi *et.al.* [5] introduced and investigated the notions of new classes of functions namely strongly $g^{\#}\alpha$-irresolute functions, strongly semi $g^{\#}\alpha$-irresolute functions and almost $g^{\#}\alpha$-irresolute functions.

In this paper we introduce some stronger forms of semi #gα-irresolute functions namely strongly semi #gα-irresolute functions, strongly *semi #gα-irresolute functions and almost semi #gα-irresolute functions in topological spaces. We discuss some properties and characterizations of these functions. Moreover we examine the relationships of these functions with the other existing functions. Throughout this paper X and Y denote the topological spaces (X,τ) and (Y,σ) on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X its closure and interior are denoted by cl(A) and int(A) respectively.

2. PRELIMINARIES

Definition 2.1: A subset A of X is said to be

(i) semi-open [8] (resp. α-open [15], β-open [1]) if A ⊆ cl(int(A)) (resp. A ⊆ int(cl(int(A))), A ⊆ cl(int(cl(A)))).

(ii) generalized closed (briefly g-closed) set [9] if cl(A) ⊆ U, whenever A ⊆ U and U is open in X. The complement of the g-closed set is called a g-open set.

(iii) $g^{\#}\alpha$-closed [16] if αcl(A) ⊆ U, whenever A ⊆ U and U is g-open in X. The complement of the $g^{\#}\alpha$-closed set is called a $g^{\#}\alpha$-open set.

(iv) #gα-closed [4] if αcl(A) ⊆ U, whenever A ⊆ U and U is $g^{\#}\alpha$-open in X. The complement of the #gα-closed set is called a #gα-open set.

(v) semi #gα-closed [7] if scl(A) ⊆ U, whenever A ⊆ U and U is #gα-open in X. The complement of the semi #gα-closed set is called a semi #gα-open set.

(vi) gα-closed [12] if αcl(A) ⊆ U, whenever A ⊆ U and U is α-open in X. The complement of the gα-closed set is called a gα-open set.

(vii) *gα-closed [20] if cl(A) ⊆ U, whenever A ⊆ U and U is gα-open in X. The complement of the *gα-closed set is called a *gα-open set.

(viii) αg-closed [11] if αcl(A) ⊆ U, whenever A ⊆ U and U is open in X. The complement of the αg-closed set is called a αg-open set.

(ix) $g^{\#}$-closed [20] if cl(A) ⊆ U, whenever A ⊆ U and U is αg-open in X. The complement of the $g^{\#}$-closed set is called a $g^{\#}$-open set.

(x) gs-closed [2] if scl(A) ⊆ U, whenever A ⊆ U and U is open in X. The complement of the gs-closed set is called a gs-open set.

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(xi) strongly g*s-closed [17] if scl(A) ⊆ U, whenever A ⊆ U and U is gs-open in X. The complement of the strongly g*s-closed set is called a strongly g*s-open set.
(xii) gsp-closed [6] if spcl(A) ⊆ U, whenever A ⊆ U and U is open in X. The complement of the gsp-closed set is called a gsp-open set.

Definition 2.2: A function f: (X,τ) → (Y,σ) is said to be
(i) *gα-continuous [20] if f^{-1}(V) is *gα-open in X for every open set V of Y.
(ii) g#-continuous [19] if f^{-1}(V) is g#-open in X for every open set V of Y.
(iii) strongly g*s-continuous [18] if f^{-1}(V) is strongly g*s-open in X for every open set V of Y.
(iv) gs-continuous [2] if f^{-1}(V) is gs-open in X for every open set V of Y.
(v) gsp-continuous [6] if f^{-1}(V) is gsp-open in X for every open set V of Y.
(vi) semi #gα-continuous [7] if f^{-1}(V) is semi #gα-open in X for every open set V of Y.

Definition 2.3: A function f: (X,τ) → (Y,σ) is said to be semi #gα-irresolute [7] (resp. α-irresolute [13], β-irresolute[14]) if f^{-1}(V) is semi #gα-closed (resp. α-closed, β-closed) in (X,τ) for every semi #gα-closed set V (resp. α-closed, β-closed) of (Y,σ).

Definition 2.4: A function f: (X,τ) → (Y,σ) is said to be strongly α-irresolute [10] if f^{-1}(V) is open in (X,τ) for every α-open set V of (Y,σ).

Definition 2.5: A function f: (X,τ) → (Y,σ) is said to be almost α-irresolute [3] if f^{-1}(V) is β- open in (X,τ) for every α-open set V of (Y,σ).

3. STRONGLY SEMI #gα-IRRESOLUTE FUNCTIONS

Definition 3.1: A function f: (X,τ) → (Y,σ) is said to be strongly semi #gα-irresolute if f^{-1}(V) is open in X for every semi #gα-open set V of Y.

Theorem 3.2: If f: (X,τ) → (Y,σ) is a strongly semi #gα-irresolute function then it is semi #gα-irresolute.

Proof: Let V be a semi #gα-open set in Y. Since f is strongly semi #gα-irresolute function, f^{-1}(V) is open in X and hence it is semi #gα-open. Therefore f is semi #gα-irresolute.

The Converse of the above theorem need not be true by the following example.

Example 3.3: Let X = Y = {a, b, c} with τ = {ϕ, X,{a}} and σ = {ϕ, Y,{a},{a, b}}

Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b and f(c) = c.

Semi #gα-open sets in (X, τ) = {ϕ, X, {a}, {a, b}, {a, c}}.

Semi #gα-open sets in (Y, σ) = {ϕ, Y, {a}, {a, b}, {a, c}}.

Here f^{-1}(V) is semi #gα-open in X for every semi #gα-open set V of Y. Hence f is semi #gα-irresolute. But f^{-1}({a,c}) = {a,c} is semi #gα-open in Y, not open in X. Thus f is not strongly semi #gα-irresolute.

Theorem 3.4: If f: (X,τ) → (Y,σ) is a strongly semi #gα-irresolute function then it is continuous (resp. α-continuous, semi continuous, β-continuous).

Proof: Let V be an open set in Y and hence it is a semi #gα-open set in Y. Since f is strongly semi #gα-irresolute function, f^{-1}(V) is open in X (and hence it is α-open, semi open and β-open respectively). Therefore f is continuous (resp. α-continuous, semi continuous, β-continuous).

The following example shows that the converse of the above theorem need not be true.

Example 3.5: Let X = Y = {a, b, c} with τ = {ϕ, X,{a},{a, b}} and σ = {ϕ, Y,{a, b}}

Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b and f(c) = c.

α-open sets in (X, τ) = Semi-open sets in (X, τ) = β-open sets in (X, τ) = {ϕ, X, {a}, {a, b}, {a, c}}

Semi #gα-open sets in (Y, σ) = {ϕ, X, {a}, {b}, {a, b}}.

Here f^{-1}(V) is open (resp. α-open, semi open and β-open) in X for every open set V of Y.

Hence f is continuous (resp. α-continuous, semi continuous, β-continuous). But f^{-1}({b}) = {b} semi $^{\#}$gα-open in Y, not open in X. Thus f is not strongly semi $^{\#}$gα-irresolute.

Theorem 3.6: If f: (X,τ) → (Y,σ) is a strongly semi $^{\#}$gα-irresolute function then it is *gα-continuous (resp. g$^{\#}$-continuous, strongly g*s-continuous, gs-continuous, gsp-continuous).

Proof: Let V be an open set in Y and hence it is a semi $^{\#}$gα-open set in Y. Since f is strongly semi $^{\#}$gα-irresolute function, f^{-1}(V) is open in X and hence it is *gα-open (resp. g$^{\#}$-open, strongly g*s-open, gs-open, gsp-open). Therefore f is *gα-continuous (resp. g$^{\#}$-continuous, strongly g*s-continuous, gs-continuous, gsp-continuous).

The Converse of the above theorem need not be true by the following example.

Example 3.7: Let X = Y = {a, b, c} with τ = {ϕ, X,{a},{a, b}} and σ = {ϕ, Y,{a, b}}

Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b and f(c) = c.
*gα-open sets in (X, τ) = g$^{\#}$-open sets in (X, τ) = {ϕ, X,{a},{a, b}}.

Strongly g*s-open sets in (X, τ) = {ϕ, X,{a},{a, b},{a, c}}.
gs-open sets in (X, τ) = gsp-open sets in (X, τ) = {ϕ, X,{a},{b},{a, b},{a, c}}.
Semi $^{\#}$gα-open sets in (Y, σ) = {ϕ, Y,{a},{b},{a, b}}

Here f^{-1}(V) is *gα-open (resp. g$^{\#}$-open, strongly g*s-open, gs-open, gsp-open) in X for every open set V of Y. Hence f is *gα-continuous (resp. g$^{\#}$-continuous, strongly g*s-continuous, gs-continuous, gsp-continuous). But f^{-1}({b}) = {b} semi $^{\#}$gα-open in Y, not open in X. Thus f is not strongly semi $^{\#}$gα-irresolute.

Theorem 3.8: If f: (X,τ) → (Y,σ) is a strongly semi $^{\#}$gα-irresolute function then it is strongly α-irresolute.

Proof: Let V be α-open in Y and hence it is semi $^{\#}$gα-open. Since f is strongly semi $^{\#}$gα-irresolute, f^{-1}(V) is open in X. Thus f is strongly α-irresolute.

The reverse implication need not be true which can be seen from the following example.

Example 3.9: Let X = Y = {a, b, c} with τ = {ϕ, X,{a},{a, b}} and σ = {ϕ, Y,{a, b}}

Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b and f(c) = c.
α-open sets in (Y, σ) = {ϕ, X,{a, b}}.

Semi $^{\#}$gα-open sets in (Y, σ) = {ϕ, Y, {a}, {b}, {a, b}}

Here f^{-1}(V) is open in X for every α-open set V of Y. Hence f is strongly α-irresolute.

But f^{-1}({b}) = {b} semi $^{\#}$gα-open in Y, not open in X. Thus f is not strongly semi $^{\#}$gα-irresolute.

Theorem 3.10: If f: (X,τ) → (Y,σ) is a strongly semi $^{\#}$gα-irresolute function then for each x ∈ X and each semi $^{\#}$gα-open set V of Y containing f(x) there exists an open set U of X containing x such that f(U) ⊂ V.

Proof: Let x ∈ X and V be any semi $^{\#}$gα-open set of Y containing f(x). Since f is strongly semi $^{\#}$gα-irresolute, f^{-1}(V) is open in X and contains x. Let U = f^{-1}(V) then U is an open subset of X containing x and f(U) ⊂ V.

Theorem 3.11: Let f: (X,τ) → (Y,σ) and g: (Y,σ) → (Z,η) be any two functions, then their composition g∘ f: (X,τ) → (Z,η) is

(i) strongly semi $^{\#}$gα-irresolute if f is strongly semi $^{\#}$gα-irresolute and g is semi $^{\#}$gα-irresolute
(ii) semi $^{\#}$gα-irresolute if f is semi $^{\#}$gα-continuous and g is strongly semi $^{\#}$gα-irresolute.

Proof:

(i) Let V be a semi $^{\#}$gα-open subset of Z. Since g is semi $^{\#}$gα-irresolute, g^{-1}(V) is semi $^{\#}$gα-open in Y. Since f is strongly semi $^{\#}$gα-irresolute, $(g \circ f)^{-1}$ (V) = $f^{-1}(g^{-1}$(V)) is open in X and hence g∘ f is strongly semi $^{\#}$gα-irresolute.
(ii) Let V be a semi $^{\#}$gα-open subset of Z. Since g is strongly semi $^{\#}$gα-irresolute, g^{-1}(V) is open in Y. Since f is semi $^{\#}$gα-continuous, $(g \circ f)^{-1}$ (V) = $f^{-1}(g^{-1}$(V)) is semi $^{\#}$gα-open in X. Thus g∘ f is semi $^{\#}$gα-irresolute.

4. STRONGLY *SEMI #gα-IRRESOLUTE FUNCTIONS

Definition 4.1: A function f: (X,τ) → (Y,σ) is said to be strongly *semi #gα-irresolute if f^{-1}(V) is semi open in X for every semi #gα-open set V of Y.

Theorem 4.2: If f: (X,τ) → (Y,σ) is a strongly semi #gα-irresolute function then it is strongly *semi #gα-irresolute.

Proof: Let V be a semi #gα-open set in Y. Since f is strongly semi #gα-irresolute function, f^{-1}(V) is open in X and hence it is semi open in X. Therefore f is strongly *semi #gα-irresolute.

The Converse of the above theorem need not be true by the following example.

Example 4.3: Let X = Y = {a, b, c} with τ = {ϕ, X,{a}} and σ = {ϕ, Y,{a},{a,b}}

Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b and f(c) = c.

Semi-open sets in (X, τ) = {ϕ, X,{a},{a, b},{a, c}}.

Semi #gα-open sets in (Y, σ) = {ϕ, Y,{a},{a, b},{a, c}}.

Here f^{-1}(V) is semi-open in X for every semi #gα-open set V of Y. Hence f is strongly *semi #gα-irresolute. But f^{-1}({a,c}) = {a,c} is semi #gα-open in Y, not open in X. Thus f is not strongly semi #gα-irresolute.

Theorem 4.4: Let f: (X,τ) → (Y,σ) and g: (Y,σ) → (Z,η) be any two functions, then their composition g∘ f: (X,τ) → (Z,η) is strongly *semi #gα-irresolute if f is strongly *semi #gα-irresolute and g is semi #gα-irresolute.

Proof: It is similar to the proof of Theorem 3.11.

Theorem 4.5: For a function f: (X,τ) → (Y,σ), the following are equivalent.

(i) f is strongly *semi #gα-irresolute.
(ii) For each x ∈ X and each semi #gα-open set V of Y containing f(x) there exists a semi open set U of X containing x such that f(U) ⊂ V.
(iii) f^{-1}(V) ⊂ cl(int(f^{-1}(V))) for every semi #gα-open set V of Y.
(iv) f^{-1}(M) is semi-closed in X for every semi #gα-closed set M of Y.

Proof:
(i) ⟹ (ii): Let x ∈ X and V be any semi #gα-open set of Y containing f(x). Since f is strongly *semi #gα-irresolute, f^{-1}(V) is semi open in X and contains x. Let U = f^{-1}(V). Thus there exists a semi open set U of X containing x such that f(U) ⊂ V.

(ii) ⟹ (iii): Let V be any semi #gα-open set of Y containing f(x). i.e., x ∈ f^{-1}(V). By (ii), there exists a semi open set U of X containing x such that f(U) ⊂ V. Thus we have x ∈ U ⊂ cl(int(U)) ⊂ cl(int(f^{-1}(V))) and hence f^{-1}(V) ⊂ cl(int(f^{-1}(V))).

(iii) ⟹ (iv): Let M be any semi #gα-closed subset of Y. Let V = Y \ M. Then V is semi #gα-open in Y. By (iii), we have f^{-1}(V) ⊂ cl(int(f^{-1}(V))) and hence f^{-1}(M) = X \ f^{-1}(Y \ M) = X \ f^{-1}(V) is semi-closed in X.

(iv) ⟹ (i): Let M be any semi #gα-open subset of Y. Let V = Y \ M. Then V is semi #gα-closed in Y. By (iv), we have f^{-1}(V) is semi-closed. Then f^{-1}(M) = X \ f^{-1}(Y \ M) = X \ f^{-1}(V) is semi-open in X. Therefore f is strongly *semi #gα-irresolute.

5. ALMOST SEMI #gα-IRRESOLUTE FUNCTIONS

Definition 5.1: A function f: (X,τ) → (Y,σ) is said to be almost semi #gα-irresolute if f^{-1}(V) is β-open in X for every semi #gα-open set V of Y.

Theorem 5.2: If f: (X,τ) → (Y,σ) is almost semi #gα-irresolute function then it is β-continuous.

Proof: Let V be an open set in Y and hence it is a semi #gα-open set in Y. Since f is almost semi #gα-irresolute function, f^{-1}(V) is β-open in X. Hence f is β-continuous.

The following example shows that the converse of the above theorem need not be true.

Example 5.3: Let X = Y = {a, b, c} with τ = {ϕ, X, {a}, {a, b}} and σ = {ϕ, Y, {a, b}}

Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b and f(c) = c.

β-open sets in (X, τ) = {ϕ, X, {a}, {a, b}, {a, c}}.

Semi #gα-open sets in (Y, σ) = {ϕ, Y,{a},{b},{a, b}}.

Here f^{-1}(V) is β-open in X for every open set V of Y. Hence f is β-continuous. But f^{-1}({b}) = {b} is semi #gα-open in Y, not β-open in X. Thus f is not almost semi #gα-irresolute.

Theorem 5.4: If f: (X,τ) → (Y,σ) is almost semi #gα-irresolute function then it is almost α-irresolute.

Proof: Let V be α-open in Y and hence it is semi #gα-open. Since f is almost semi #gα-irresolute, f^{-1}(V) is β-open in X. Thus f is almost α-irresolute.

The reverse implication need not be true which can be seen from the following example.

Example 5.5: Let X = Y = {a, b, c} with τ = {ϕ, X, {a},{a, b}} and σ = {ϕ, Y,{a, b}}

Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b and f(c) = c.

β-open sets in (X, τ) = {ϕ, X,{a},{a, b},{a, c}}.

α-open sets in (Y, σ) = {ϕ, Y, {a, b}}.

Semi #gα-open sets in (Y, σ) = {ϕ, Y,{a},{b},{a, b}}.

Here f^{-1}(V) is β-open in X for every α-open set V of Y. Hence f is almost α-irresolute.

But f^{-1}({b}) = {b} is semi #gα-open in Y, not β-open in X. Thus f is not almost semi #gα-irresolute.

Theorem 5.6: Let f: (X,τ) → (Y,σ) and g: (Y,σ) → (Z,η) be any two functions, then their composition g∘ f: (X,τ) → (Z,η) is

(i) almost α-irresolute if f is almost semi #gα-irresolute and g is α-irresolute
(ii) almost semi #gα-irresolute if f is β-irresolute and g is almost semi #gα-irresolute.

Proof: The Proof is similar to that of Theorem 3.11.

Theorem 5.7: For a function f: (X,τ) → (Y,σ), the following are equivalent.

(i) f is almost semi #gα-irresolute.
(ii) For each x ∈ X and each semi #gα-open set V of Y containing f(x) there exists a β-open set U of X containing x such that f(U) ⊂ V.
(iii) f^{-1}(V) ⊂ cl(int(f^{-1}(V))) for every semi #gα-open set V of Y.
(iv) f^{-1}(M) is β-closed in X for every semi #gα-closed set M of Y.

Proof: It is similar to the proof of Theorem 4.5.

REFERENCES

1. M.E. Abd El, Monsef, S.N. El, Deeb and R.A. Mahmoud (1983), β-open sets and β-continuous mappings, Bull.Fac.Sci. Assiut Univ. A 12 (No.1): 77 – 90.
2. S.P. Arya, T. Nour, Characterizations of S – normal spaces, Indian J. Pure Appl. Math, 21 (1990), pp. 717 – 719.
3. Y. Beceren (2000), Almost α-irresolute functions, Bull. Calcutta Math. Soc.92 (No.3), 213 - 218.
4. R. Devi, H. Maki and V. Kokilavani, The Group Structure of #Gα-Closed Sets in Topological Spaces, International Journal of General Topology, Vol.2, No.1, (June 2009), pp. 21 – 30.
5. R. Devi, K. Muthukumaraswamy and V. Kokilavani, Some Stronger Forms of G#α-Irresolute Functions, Bulletin of Pure and Applied Sciences, Vol.26E (No.1), (2007), pp. 129 – 135.
6. J. Dontchev, On Generating Semi-Preopen Sets, Mem. Fac. Sci. Kochi Univ. Ser. A. Math 16 (1995), pp.35 – 48.

7. V. Kokilavani and M. Vivek Prabu, S $^{\#}G\alpha$-Closed Sets and S $^{\#}G\alpha$-Homeomorphisms in Topological Spaces, Proceedings of National Conference on Recent Advances in Mathematical Analysis and Applications – NCRAMAA-2013, (ISBN 978-93-82338-69-7) pp. 153 – 161.
8. N. Levine, Semi-Open Sets and Semi-Continuity in Topological Spaces, Amer. Math. Monthly, 70, (1963), 36 – 41.
9. N. Levine, Generalized Closed sets in Topology, Rend. Circ. Math. Palermo, 19, (1970), 89 – 96.
10. G. Lo Fara (1987), On Strongly α-irresolute mappings, Indian J.Pure Appl. Math.18: 146 – 151.
11. H. Maki, R. Devi, and K. Balachandran, Associated Topologies of Generalized α-Closed Sets and α-Generalized Closed Sets, Mem. Fac. Sci. Kochi Univ. Ser-A Math., 14, (1994), 51-63.
12. H. Maki, R. Devi, and K. Balachandran, Generalized α-Closed Sets in Topology, Bull. Fukuoka Univ. Ed. Part III, 42, (1993), 13-21.
13. S.N. Maheswari and S.S. Thakur (1980), On α-irresolute mappings, Tamkang J.Math. 11 (No.2): 209 – 214.
14. R.A. Mahmoud and M.E. Abd El-Monsef (1990), β-irresolute and β-topological invariant, Proc. Pakistan Acad. Sci, 27 (N0.3): 285 – 296.
15. O. Njastad (1965), On Some Classes of Nearly Open Sets, Pacific J. Math. 15: 961 – 970.
16. K. Nono, R. Devi, M. Devipriya, K. Muthukumaraswamy and H. Maki (2004), On Generalized$^{\#}$ α-Closed Sets and the digital plane, Mem. Fac. Sci. Kochi Univ (Math). Vol.53, part III: 15 – 24.
17. A. Pushpalatha and K. Anitha, g*s-closed sets in Topological spaces, Int. J. Contemp. Math. Sciences, Vol .6.,March-2011,no 19,917-929.
18. A. Pushpalatha and K. Anitha, Strongly g*s-Continuous Maps and Perfectly g*s-Continuous Maps in Topological Spaces, International Mathematical Forum, Vol. 7, 2012, no. 24, 1201 – 1207.
19. M.K.R.S.Veera Kumar, g#-closed sets in topological spaces, Mem. Fac. Sci. Kochi Univ. (Japan) Ser. A, Math., 24 (2003), 1-13.
20. M. Vigneshwaran and R. Devi, On Go-kernel in the digital plane, International Journal of Mathematical Archive-3(6), (2012), 2358 – 2373.

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