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ON STRONGER FORMS OF SEMI #GENERALIZED α – IRRESOLUTE FUNCTIONS

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ABSTRACT

*In this paper we introduce some stronger forms of semi #gα-irresolute functions namely strongly semi #gα-irresolute functions, strongly *semi #gα-irresolute functions and almost semi #gα-irresolute functions in topological spaces. We discuss some properties and characterizations of these functions. Moreover we examine the relationships of these functions with the other existing functions.*

***Key words**: semi #gα-closed set, semi #gα-open set, semi #gα-irresolute functions, strongly semi #gα-irresolute functions, strongly *semi #gα-irresolute functions and almost semi #gα-irresolute functions.*

1. INTRODUCTION

The concept of strongly α-irresolute functions was introduced by G. Lo Fara [10] in 1987. Later in 2003, Y. Beceren [3] introduced the notions of almost α-irresolute functions and β-preirresolute functions. R.Devi *et.al*. [5] introduced and investigated the notions of new classes of functions namely strongly $g^{\#}\alpha$-irresolute functions, strongly semi $g^{\#}\alpha$-irresolute functions and almost $g^{\#}\alpha$-irresolute functions.

In this paper we introduce some stronger forms of semi $^{\#}g\alpha$-irresolute functions namely strongly semi $^{\#}g\alpha$-irresolute functions, strongly *semi $^{\#}g\alpha$-irresolute functions and almost semi $^{\#}g\alpha$-irresolute functions in topological spaces. We discuss some properties and characterizations of these functions. Moreover we examine the relationships of these functions with the other existing functions. Throughout this paper X and Y denote the topological spaces (X,τ) and (Y,σ) on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X its closure and interior are denoted by cl(A) and int(A) respectively.

2. PRELIMINARIES

Definition 2.1: A subset A of X is said to be
- (i) semi-open [8] (resp. α-open [15], β-open [1]) if A ⊆ cl(int(A)) (resp. A ⊆ int(cl(int(A))), A ⊆ cl(int(cl(A)))).
- (ii) generalized closed (briefly g-closed) set [9] if cl(A) ⊆ U, whenever A ⊆ U and U is open in X. The complement of the g-closed set is called a g-open set.
- (iii) $g^{\#}\alpha$-closed [16] if αcl(A) ⊆ U, whenever A ⊆ U and U is g-open in X. The complement of the $g^{\#}\alpha$-closed set is called a $g^{\#}\alpha$-open set.
- (iv) $^{\#}g\alpha$-closed [4] if αcl(A) ⊆ U, whenever A ⊆ U and U is $g^{\#}\alpha$-open in X. The complement of the $^{\#}g\alpha$-closed set is called a $^{\#}g\alpha$-open set.
- (v) semi $^{\#}g\alpha$-closed [7] if scl(A) ⊆ U, whenever A ⊆ U and U is $^{\#}g\alpha$-open in X. The complement of the semi $^{\#}g\alpha$-closed set is called a semi $^{\#}g\alpha$-open set.
- (vi) gα-closed [12] if αcl(A) ⊆ U, whenever A ⊆ U and U is α-open in X. The complement of the gα-closed set is called a gα-open set.
- (vii) *gα-closed [20] if cl(A) ⊆ U, whenever A ⊆ U and U is gα-open in X. The complement of the *gα-closed set is called a *gα-open set.
- (viii) αg-closed [11] if αcl(A) ⊆ U, whenever A ⊆ U and U is open in X. The complement of the αg-closed set is called a αg-open set.
- (ix) $g^{\#}$-closed [20] if cl(A) ⊆ U, whenever A ⊆ U and U is αg-open in X. The complement of the $g^{\#}$-closed set is called a $g^{\#}$-open set.
- (x) gs-closed [2] if scl(A) ⊆ U, whenever A ⊆ U and U is open in X. The complement of the gs-closed set is called a gs-open set.

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(xi) strongly g*s-closed [17] if scl(A) ⊆ U, whenever A ⊆ U and U is gs-open in X. The complement of the strongly g*s-closed set is called a strongly g*s-open set.
(xii) gsp-closed [6] if spcl(A) ⊆ U, whenever A ⊆ U and U is open in X. The complement of the gsp-closed set is called a gsp-open set.

Definition 2.2: A function f: (X,τ) → (Y,σ) is said to be
(i) *gα-continuous [20] if f^{-1}(V) is *gα-open in X for every open set V of Y.
(ii) g#-continuous [19] if f^{-1}(V) is g#-open in X for every open set V of Y.
(iii) strongly g*s-continuous [18] if f^{-1}(V) is strongly g*s-open in X for every open set V of Y.
(iv) gs-continuous [2] if f^{-1}(V) is gs-open in X for every open set V of Y.
(v) gsp-continuous [6] if f^{-1}(V) is gsp-open in X for every open set V of Y.
(vi) semi #gα-continuous [7] if f^{-1}(V) is semi #gα-open in X for every open set V of Y.

Definition 2.3: A function f: (X,τ) → (Y,σ) is said to be semi #gα-irresolute [7] (resp. α-irresolute [13], β-irresolute[14]) if f^{-1}(V) is semi #gα-closed (resp. α-closed, β-closed) in (X,τ) for every semi #gα-closed set V (resp. α-closed, β-closed) of (Y,σ).

Definition 2.4: A function f: (X,τ) → (Y,σ) is said to be strongly α-irresolute [10] if f^{-1}(V) is open in (X,τ) for every α-open set V of (Y,σ).

Definition 2.5: A function f: (X,τ) → (Y,σ) is said to be almost α-irresolute [3] if f^{-1}(V) is β- open in (X,τ) for every α-open set V of (Y,σ).

3. STRONGLY SEMI #gα-IRRESOLUTE FUNCTIONS

Definition 3.1: A function f: (X,τ) → (Y,σ) is said to be strongly semi #gα-irresolute if f^{-1}(V) is open in X for every semi #gα-open set V of Y.

Theorem 3.2: If f: (X,τ) → (Y,σ) is a strongly semi #gα-irresolute function then it is semi #gα-irresolute.

Proof: Let V be a semi #gα-open set in Y. Since f is strongly semi #gα-irresolute function, f^{-1}(V) is open in X and hence it is semi #gα-open. Therefore f is semi #gα-irresolute.

The Converse of the above theorem need not be true by the following example.

Example 3.3: Let X = Y = {a, b, c} with τ = {ϕ, X,{a}} and σ = {ϕ, Y,{a},{a, b}}

Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b and f(c) = c.

Semi #gα-open sets in (X, τ) = {ϕ, X, {a}, {a, b}, {a, c}}.

Semi #gα-open sets in (Y, σ) = {ϕ, Y, {a}, {a, b}, {a, c}}.

Here f^{-1}(V) is semi #gα-open in X for every semi #gα-open set V of Y. Hence f is semi #gα-irresolute. But f^{-1}({a,c}) = {a,c} is semi #gα-open in Y, not open in X. Thus f is not strongly semi #gα-irresolute.

Theorem 3.4: If f: (X,τ) → (Y,σ) is a strongly semi #gα-irresolute function then it is continuous (resp. α-continuous, semi continuous, β-continuous).

Proof: Let V be an open set in Y and hence it is a semi #gα-open set in Y. Since f is strongly semi #gα-irresolute function, f^{-1}(V) is open in X (and hence it is α-open, semi open and β-open respectively). Therefore f is continuous (resp. α-continuous, semi continuous, β-continuous).

The following example shows that the converse of the above theorem need not be true.

Example 3.5: Let X = Y = {a, b, c} with τ = {ϕ, X,{a},{a, b}} and σ = {ϕ, Y,{a, b}}

Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b and f(c) = c.

α-open sets in (X, τ) = Semi-open sets in (X, τ) = β-open sets in (X, τ) = {ϕ, X, {a}, {a, b}, {a, c}}

Semi #gα-open sets in (Y, σ) = {ϕ, X, {a}, {b}, {a, b}}.

Here f^{-1}(V) is open (resp. α-open, semi open and β-open) in X for every open set V of Y.

Hence f is continuous (resp. α-continuous, semi continuous, β-continuous). But f^{-1}({b}) = {b} semi #gα-open in Y, not open in X. Thus f is not strongly semi #gα-irresolute.

Theorem 3.6: If f: (X,τ) → (Y,σ) is a strongly semi #gα-irresolute function then it is *gα-continuous (resp. g#-continuous, strongly g*s-continuous, gs-continuous, gsp-continuous).

Proof: Let V be an open set in Y and hence it is a semi #gα-open set in Y. Since f is strongly semi #gα-irresolute function, f^{-1}(V) is open in X and hence it is *gα-open (resp. g#-open, strongly g*s-open, gs-open, gsp-open). Therefore f is *gα-continuous (resp. g#-continuous, strongly g*s-continuous, gs-continuous, gsp-continuous).

The Converse of the above theorem need not be true by the following example.

Example 3.7: Let X = Y = {a, b, c} with τ = {ϕ, X,{a},{a, b}} and σ = {ϕ, Y,{a, b}}

Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b and f(c) = c.
*gα-open sets in (X, τ) = g#-open sets in (X, τ) = {ϕ, X,{a},{a, b}}.

Strongly g*s-open sets in (X, τ) = {ϕ, X,{a},{a, b},{a, c}}.
gs-open sets in (X, τ) = gsp-open sets in (X, τ) = {ϕ, X,{a},{b},{a, b},{a, c}}.
Semi #gα-open sets in (Y, σ) = {ϕ, Y,{a},{b},{a, b}}

Here f^{-1}(V) is *gα-open (resp. g#-open, strongly g*s-open, gs-open, gsp-open) in X for every open set V of Y. Hence f is *gα-continuous (resp. g#-continuous, strongly g*s-continuous, gs-continuous, gsp-continuous). But f^{-1}({b}) = {b} semi #gα-open in Y, not open in X. Thus f is not strongly semi #gα-irresolute.

Theorem 3.8: If f: (X,τ) → (Y,σ) is a strongly semi #gα-irresolute function then it is strongly α-irresolute.

Proof: Let V be α-open in Y and hence it is semi #gα-open. Since f is strongly semi #gα-irresolute, f^{-1}(V) is open in X. Thus f is strongly α-irresolute.

The reverse implication need not be true which can be seen from the following example.

Example 3.9: Let X = Y = {a, b, c} with τ = {ϕ, X,{a},{a, b}} and σ = {ϕ, Y,{a, b}}

Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b and f(c) = c.
α-open sets in (Y, σ) = {ϕ, X,{a, b}}.

Semi #gα-open sets in (Y, σ) = {ϕ, Y, {a}, {b}, {a, b}}

Here f^{-1}(V) is open in X for every α-open set V of Y. Hence f is strongly α-irresolute.

But f^{-1}({b}) = {b} semi #gα-open in Y, not open in X. Thus f is not strongly semi #gα-irresolute.

Theorem 3.10: If f: (X,τ) → (Y,σ) is a strongly semi #gα-irresolute function then for each x ∈ X and each semi #gα-open set V of Y containing f(x) there exists an open set U of X containing x such that f(U) ⊂ V.

Proof: Let x ∈ X and V be any semi #gα-open set of Y containing f(x). Since f is strongly semi #gα-irresolute, f^{-1}(V) is open in X and contains x. Let U = f^{-1}(V) then U is an open subset of X containing x and f(U) ⊂ V.

Theorem 3.11: Let f: (X,τ) → (Y,σ) and g: (Y,σ) → (Z,η) be any two functions, then their composition g∘ f: (X,τ) → (Z,η) is

(i) strongly semi #gα-irresolute if f is strongly semi #gα-irresolute and g is semi #gα-irresolute
(ii) semi #gα-irresolute if f is semi #gα-continuous and g is strongly semi #gα-irresolute.

Proof:

(i) Let V be a semi #gα-open subset of Z. Since g is semi #gα-irresolute, g^{-1}(V) is semi #gα-open in Y. Since f is strongly semi #gα-irresolute, $(g \circ f)^{-1}$ (V) = $f^{-1}(g^{-1}$(V)) is open in X and hence g∘ f is strongly semi #gα-irresolute.
(ii) Let V be a semi #gα-open subset of Z. Since g is strongly semi #gα-irresolute, g^{-1}(V) is open in Y. Since f is semi #gα-continuous, $(g \circ f)^{-1}$ (V) = $f^{-1}(g^{-1}$(V)) is semi #gα-open in X. Thus g∘ f is semi #gα-irresolute.

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4. STRONGLY *SEMI #gα-IRRESOLUTE FUNCTIONS

Definition 4.1: A function f: (X,τ) → (Y,σ) is said to be strongly *semi #gα-irresolute if f^{-1}(V) is semi open in X for every semi #gα-open set V of Y.

Theorem 4.2: If f: (X,τ) → (Y,σ) is a strongly semi #gα-irresolute function then it is strongly *semi #gα-irresolute.

Proof: Let V be a semi #gα-open set in Y. Since f is strongly semi #gα-irresolute function, f^{-1}(V) is open in X and hence it is semi open in X. Therefore f is strongly *semi #gα-irresolute.

The Converse of the above theorem need not be true by the following example.

Example 4.3: Let X = Y = {a, b, c} with τ = {ϕ, X,{a}} and σ = {ϕ, Y,{a},{a,b}}

Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b and f(c) = c.

Semi-open sets in (X, τ) = {ϕ, X,{a},{a, b},{a, c}}.

Semi #gα-open sets in (Y, σ) = {ϕ, Y,{a},{a, b},{a, c}}.

Here f^{-1}(V) is semi-open in X for every semi #gα-open set V of Y. Hence f is strongly *semi #gα-irresolute. But f^{-1}({a,c}) = {a,c} is semi #gα-open in Y, not open in X. Thus f is not strongly semi #gα-irresolute.

Theorem 4.4: Let f: (X,τ) → (Y,σ) and g: (Y,σ) → (Z,η) be any two functions, then their composition g∘ f: (X,τ) → (Z,η) is strongly *semi #gα-irresolute if f is strongly *semi #gα-irresolute and g is semi #gα-irresolute.

Proof: It is similar to the proof of Theorem 3.11.

Theorem 4.5: For a function f: (X,τ) → (Y,σ), the following are equivalent.
- (i) f is strongly *semi #gα-irresolute.
- (ii) For each x ∈ X and each semi #gα-open set V of Y containing f(x) there exists a semi open set U of X containing x such that f(U) ⊂ V.
- (iii) f^{-1}(V) ⊂ cl(int(f^{-1}(V))) for every semi #gα-open set V of Y.
- (iv) f^{-1}(M) is semi-closed in X for every semi #gα-closed set M of Y.

Proof:
(i) ⟹ (ii): Let x ∈ X and V be any semi #gα-open set of Y containing f(x). Since f is strongly *semi #gα-irresolute, f^{-1}(V) is semi open in X and contains x. Let U = f^{-1}(V). Thus there exists a semi open set U of X containing x such that f(U) ⊂ V.

(ii) ⟹ (iii): Let V be any semi #gα-open set of Y containing f(x). i.e., x ∈ f^{-1}(V). By (ii), there exists a semi open set U of X containing x such that f(U) ⊂ V. Thus we have x ∈ U ⊂ cl(int(U)) ⊂ cl(int(f^{-1}(V))) and hence f^{-1}(V) ⊂ cl(int(f^{-1}(V))).

(iii) ⟹ (iv): Let M be any semi #gα-closed subset of Y. Let V = Y \ M. Then V is semi #gα-open in Y. By (iii), we have f^{-1}(V) ⊂ cl(int(f^{-1}(V))) and hence f^{-1}(M) = X \ f^{-1}(Y \ M) = X \ f^{-1}(V) is semi-closed in X.

(iv) ⟹ (i): Let M be any semi #gα-open subset of Y. Let V = Y \ M. Then V is semi #gα-closed in Y. By (iv), we have f^{-1}(V) is semi-closed. Then f^{-1}(M) = X \ f^{-1}(Y \ M) = X \ f^{-1}(V) is semi-open in X. Therefore f is strongly *semi #gα-irresolute.

5. ALMOST SEMI #gα-IRRESOLUTE FUNCTIONS

Definition 5.1: A function f: (X,τ) → (Y,σ) is said to be almost semi #gα-irresolute if f^{-1}(V) is β-open in X for every semi #gα-open set V of Y.

Theorem 5.2: If f: (X,τ) → (Y,σ) is almost semi #gα-irresolute function then it is β-continuous.

Proof: Let V be an open set in Y and hence it is a semi #gα-open set in Y. Since f is almost semi #gα-irresolute function, f^{-1}(V) is β-open in X. Hence f is β-continuous.

The following example shows that the converse of the above theorem need not be true.

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Example 5.3: Let X = Y = {a, b, c} with τ = {ϕ, X, {a}, {a, b}} and σ = {ϕ, Y, {a, b}}

Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b and f(c) = c.

β-open sets in (X, τ) = {ϕ, X, {a}, {a, b}, {a, c}}.

Semi #gα-open sets in (Y, σ) = {ϕ, Y,{a},{b},{a, b}}.

Here f^{-1}(V) is β-open in X for every open set V of Y. Hence f is β-continuous. But f^{-1}({b}) = {b} is semi #gα-open in Y, not β-open in X. Thus f is not almost semi #gα-irresolute.

Theorem 5.4: If f: (X,τ) → (Y,σ) is almost semi #gα-irresolute function then it is almost α-irresolute.

Proof: Let V be α-open in Y and hence it is semi #gα-open. Since f is almost semi #gα-irresolute, f^{-1}(V) is β-open in X. Thus f is almost α-irresolute.

The reverse implication need not be true which can be seen from the following example.

Example 5.5: Let X = Y = {a, b, c} with τ = {ϕ, X, {a},{a, b}} and σ = {ϕ, Y,{a, b}}

Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b and f(c) = c.

β-open sets in (X, τ) = {ϕ, X,{a},{a, b},{a, c}}.

α-open sets in (Y, σ) = {ϕ, Y, {a, b}}.

Semi #gα-open sets in (Y, σ) = {ϕ, Y,{a},{b},{a, b}}.

Here f^{-1}(V) is β-open in X for every α-open set V of Y. Hence f is almost α-irresolute.

But f^{-1}({b}) = {b} is semi #gα-open in Y, not β-open in X. Thus f is not almost semi #gα-irresolute.

Theorem 5.6: Let f: (X,τ) → (Y,σ) and g: (Y,σ) → (Z,η) be any two functions, then their composition g∘ f: (X,τ) → (Z,η) is

(i) almost α-irresolute if f is almost semi #gα-irresolute and g is α-irresolute
(ii) almost semi #gα-irresolute if f is β-irresolute and g is almost semi #gα-irresolute.

Proof: The Proof is similar to that of Theorem 3.11.

Theorem 5.7: For a function f: (X,τ) → (Y,σ), the following are equivalent.

(i) f is almost semi #gα-irresolute.
(ii) For each x ∈ X and each semi #gα-open set V of Y containing f(x) there exists a β-open set U of X containing x such that f(U) ⊂ V.
(iii) f^{-1}(V) ⊂ cl(int(f^{-1}(V))) for every semi #gα-open set V of Y.
(iv) f^{-1}(M) is β-closed in X for every semi #gα-closed set M of Y.

Proof: It is similar to the proof of Theorem 4.5.

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