International Journal of Mathematical Archive-6(11), 2015, 12-19
Available online through www.ijma.info ISSN 2229 – 5046

DISSIPATIVE FLOW AND HEAT TRANSFER OVER A SURFACE STRETCHING WITH HEAT SOURCE AND POWER LAW VELOCITY DISTRIBUTION

PRIYA MATHUR*[1], A. JHA[2], A. SHARMA[3]

[1]Poornima Institute of Engineering and Technology, Jaipur - (R.J.), India.

[2]JECRC University, Jaipur - (R.J.), India.

[3]Rajasthan University Jaipur, Jaipur - (R.J.), India.

(Received On: 14-10-15; Revised & Accepted On: 07-11-15)

ABSTRACT

The boundary layer flow and heat transfer due to plate stretching with a power law velocity distribution in the presence of a transverse magnetic field is studied. A special form of magnetic field is chosen so as to yield the similarity equation. It is assume that the induced magnetic field is negligible, which justified for flow at small magnetic Reynolds number. The external electric field is zero and the electric field due to polarization of charges is also negligible, the heat due to viscous and Joule dissipation are neglected. Numerical solution of the resulting momentum boundary layer and thermal boundary layer equations are determined using fourth order Runge-Kutta scheme together with shooting method.

Key words: *MHD, heat transfer, boundary layer, viscous dissipation, heat source.*

1. INTRODUCTION

The study of the boundary layer flow of an electrically conducting fluid through a porous media has many applications in manufacturing and natural process which include cooling of electronic devices by fans, cooling of nuclear reactors during emergency shutdown, cooling of an infinite metallic plate in a cooling bath, textile and paper industries, glass-fiber production, manufacture of plastic and rubber sheets, the utilization of geothermal energy, the boundary layer control in the field of aerodynamics, food processing, plasma studies and in the flow of biological fluids. Magnetohydrodynamics (MHD) is the study of the flow of electrically conducting fluids in a magnetic field. Many experimental and theoretical studies on conventional electrically conducting fluids indicate that magnetic field markedly changes their transport and heat transfer characteristics. The study of magnetohydrodynamics has many important applications, and may be used to deal with problems such as cooling of nuclear reactors by liquid sodium and induction flow meter, which depends on the potential difference in the fluid in the direction perpendicular to the motion and to the magnetic field [1]. Recently, the application of magnetohydrodynamics in the polymer industry and metallurgy has attracted the attention of many researchers. Several researches investigated the MHD flow [2 - 7].The study of flow over a stretching sheet has generated much interest in recent years in view of its numerous industrial applications such as the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film, condensation process of metallic plate in a cooling bath and glass, and also in polymer industries. Since the pioneering work of Sakiadis [8] which studied the stretching flow problem. Phukan[9] studied hydromagnetic flow and heat transfer over a surface stretching with a power-law velocity distribution. Mohebujjaman [10] discussed about MHD heat transfer mixed convection flow along vertical stretching sheet in presence of magnetic field with heat generation.

Gupta and Gupta [11] have investigated heat and mass transfer in Hydrodynamic fluid flow over an isothermal stretching sheet with suction/blowing effects. Chen and Char [12] have extended the works of Gupta and Gupta to that of non-isothermal stretching sheet.

Corresponding Author: Priya Mathur[1]*
[1]Poornima Institute of Engineering and Technology, Jaipur - (R.J.), India.

Viscous dissipation changes the temperature distributions by playing a role like an energy source, which leads to affected heat transfer rates. The advantage of the effect of viscous dissipation depends on whether the plate is being cooled or heated. Heat transfer analysis over porous surface is of much practical interest due to its rich applications. The previous studies are based on the constant physical properties of the fluid. However, it is known that the physical properties of the fluid may change significantly with temperature. The temperature increases leads to the increase in the transport phenomena by reducing the viscosity across the momentum boundary layer and due to which the heat transfer rate at the wall is also affected. Therefore, to accurately predict the flow and heat transfer rates, it is necessary to take into account the temperature-dependent viscosity of the fluid. The effect of temperature-dependent viscosity on heat and mass transfer laminar boundary layer flow has been discussed by many authors [13–18] in various situations. They showed that when this effect was included, the flow characteristics might change substantially compared with the constant viscosity assumption. Salem [19] investigated variable viscosity and thermal conductivity effects on MHD flow and heat transfer in visco-elastic fluid over a stretching sheet. Anjali Devi and Ganga [20] have considered the viscous dissipation effects on MHD flows

The study of heat source/sink effects on heat transfer is very important in view of several physical problems. Above mentioned studies contain only the effect of uniform heat source/sink (i.e., temperature dependent heat source/sink) on heat transfer. Eldahab and Aziz [21] have included the effect of non-uniform heat source with suction/blowing, but confined to the case of viscous fluids only. Thermal Radiation is one of the vital factors controlling the heat transfer in a non-isothermal system. Many of the researchers have considered the effect of radiation on flows involving a viscoelastic liquid. Works of Raptis [22, 23], Raptis and Perdikis [24], Siddheshwar *et al.* [25], Sujit Kumar Khan [26], address the effect of radiation in various situations. But these studies are carried under constant physical properties with uniform heat source. None of the above mentioned works discuss the combined effect of variable thermal conductivity and non-uniform heat source/sink.

Several other applications may also benefit from a better understanding of the fundamentals of mass, energy, and momentum transport in porous media, namely cooling of nuclear reactors, underground disposal of nuclear waste, petroleum reservoir operations, building insulation, food processing, and casting and welding in manufacturing processes. In certain porous media applications, working fluid heat generation (source) or absorption (sink) effects are important. Representative studies dealing with these effects have been reported by authors such as Gupta and Sridhar [27], Abel and Veena [28] and Sharma [29].

The present work deals with hydromagnetic flow and heat transfer over a stretching sheet with a power law velocity distribution of Newtonian conducting fluid in the presence of heat source.

2. MATHEMATICAL ANALYSIS

We consider a polymer sheet emerging out of slit at origin be moving with non uniform velocity in an electrically conducting incompressible viscous fluid at rest in the presence of a magnetic field. The coordinate X is in the direction of motion of the sheet and y is the coordinate normal to it,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = = v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} - \frac{\upsilon u}{k} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + \left(\frac{\partial u}{\partial y}\right)^2 + Q(T - T_\infty) \tag{3}$$

Boundary condition

$U=U(x)$, $v = 0$, $T = T_w(x)$ at $y = 0$

$$u \to 0,\ T \to T_\infty \text{ as } y \to \infty \tag{4}$$

where u and v are components of velocity respectively in x and y directions, T is the tempereture,κ is the coefficient of thermal diffusivity,Q_0 is the dimensional heat generation($Q_0>0$) or absorption($Q_0<0$) coefficient, cp is the specofic heat,ρ is the fluid density which we assume constant for the present case.μ is the coefficient of fluid viscosity which depends on temperature and k is permeability of the porous medium.

The equations (1) to (3) are not possible to make dimensionless by assuming the magnetic field to be constant. So in order to make above equation (1-3) dimensionless, applied magnetic field has to be assumed as a variable magnetic field in this system, because no similarity solution is seen to be found for the flow of arbitrarily surface stretching with power-law velocity for constant magnetic field. To establish a set of non-dimensional differential equation, we introduce following similar variables

$$u = U_0 x^m f'(\eta) \tag{5}$$

$$T - T_\infty = C\, x^n \theta(\eta) \tag{6}$$

$$\eta = y\sqrt{\frac{m+1}{2}}\sqrt{\frac{U_0 x^m}{\nu x}} = \frac{y}{x}\sqrt{\frac{m+1}{2}}\sqrt{Re_x}$$

$$v = \sqrt{\frac{2\nu U_0}{m+1}}\, x^{\frac{m-1}{2}}\left(\frac{m+1}{2} f + \frac{m-1}{2} f'\eta\right) \tag{7}$$

$$B(x) = B_0\ x^{(m-1)/2} \tag{8}$$

Where f′ and θ are the dimensionless velocity and temperature respectively, and η is similarity variable. Substitution in governing equations gives rise to the following two point boundary value problem.

3. NUMERICAL SOLUTION

The highly nonlinear coupled self-similar ODEs (11) and (12) along with the boundary conditions (13) and (14) constitute a two point boundary value problem (BVP) and is solved using shooting method, by converting it into an initial value problem (IVP). In this method, it is necessary to choose a suitable finite value of η→∞, say η∞. The following first-order system is set

$$F''' + FF'' - F'^2 - \left(\frac{1}{R1} + M^2\right)F' = 0 \tag{9}$$

$$\theta'' - \Pr\left(\frac{2n}{m+1}F'\theta - F\theta' + \frac{2n}{1+m}Q\theta\right) + EcF''^2 = 0 \tag{10}$$

Where

$R_1 = \frac{\kappa_p a}{\nu}$ is the permeability parameter.

$M^2 = \frac{2\sigma B_0^2}{\rho(1+m)}$ is magnetic parameter

$Pr = \frac{\mu c_p}{K}$ is the Prandtl number

$E_c = \frac{a^2}{c_p\left(\frac{E_0}{K}\sqrt{\frac{\gamma}{a}}\right)}$ is the Eckert number

To solve (9) and (10) with (4) as an IVP the values for $F_3(0)$ i.e. $f''(0)$ and i.e. $\theta'(0)$ are must needed but no such values are given. The initial guess values for $f''(0)$ and $\theta'(0)$ are chosen and the fourth order Runge-Kutta method is applied to obtain a solution. The most often used method of the Runge-Kutta family is the Fourth-Order one, which extends the idea of the mid-point method, by jumping 1/4th of the way first, then going half-way, then going 3/4th of the way and finally jumping all the way.

The formula for this method looks as follows

$$k_1 = f(x_n, t_n)\Delta t$$

$$k_2 = f(x_n + \frac{k_1}{2}, \frac{\Delta t_n}{2})\Delta t$$

$$k_3 = f(x_n + \frac{k_2}{2}, \frac{\Delta t_n}{2})\Delta t$$

$$k_4 = f(x_n + k_3, t_n + \Delta t)\Delta t$$

$$x_{n+1} = x_n + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} + 0(\Delta t)^5$$

In the present case the equation (10) is reduced to such system of IVP where missing value of $\theta'(0)$ and also for $\theta(0)$ for different set of values of parameters having bearing on the phenomena are chosen purely on hit and trial basis such that the boundary condition at the other end i.e. $\eta\rightarrow\infty$, $\theta(\eta)\rightarrow 0$ is satisfied as the approximate value for $\eta_{\infty.}$ The integration was then repeated with another larger value of $\eta_{\infty.}$ The values of the initial wall temperature values of $\theta(0)$ Were then compared.

If they agreed to about 6 considerable digits, the last value of n_∞ used was considered the approximate values otherwise the process was repeated until further change in n_∞ did not lead to any more change in the value of $\theta(0)$.Once the convergence is achieved we integrate the resultant ordinary differential equations using standard fourth order Runge–Kutta method with the given set of parameters to obtain the required solution.

4. RESULT AND DISCUSSION

A boundary layer problem for momentum, heat transfer over a stretching porous surface with prescribed heat flux in the presence of a transverse variable magnetic field is examined in this work. Porous medium, viscous and Joule's dissipation are taken into consideration in this study. The MHD boundary layer equations of momentum and heat transfer are solved analytically and different analytical expressions are obtained for non-dimensional velocity and temperature profiles for prescribed heat flux case. Computation results are carried out for different values of β, Magnetic parameter M^2, Prandtl number *Pr*, Eckert number *Ec* and permeability parameter *R*1.

A representative set of graphical result is given in figure1 to 9 to show the influence of different physical parameter on solution. The influences of the magnetic parameter *M* on the longitudinal velocity profile is depicted in Figure **1**. It can be seen that increasing *M* is to reduce the velocity distribution in the boundary layer which results in thinning of the boundary layer thickness, and hence induces an increase in the absolute value of the velocity gradient at the surface.figure-2 shows dimensionless velocity profiles for different values of β. The dimensionless velocity profile $f'(\eta)$ shows that due to increase of β velocity decreases. Actually, the fluid material Temperature profile become fuller and increases with the increase of Ec. in the flow is reduced with the increase in β and for this the boundary layer thickness decreases, which can be confirmed from figure.figure3 depicted that as permeability parameter R increases velocity increases .Figure 4 and figure5 shows the effect of magnetic field M and Prandtl number Pr on the temperature profile. An increase in Prandtl number Pr is associated with a decrease in the temperature distribution which is displayed in Fig.5, same effect it shows with the increase of magnetic parameter M. Figure 6 show the effect of temperature profile in the presence of different values of the Eckert number Ec.Figure-7 demonstrated, the influences magnetic parameter M on transverse velocity.Figure-8 shows the dimensionless velocity distribution F as a function of dimensionless coordinate η for various values of M with Pr=0.71 and β=0.1.it is seen from figure 1 that for fixed Pr and β. f′ decreases with increase in M.figure-9 shows the effect of heat transfer with the changes of heat source parameter Q. It is seen that hear transfer increases initially and then changes above the sheet for large values of Q.

REFERENCES

1. H.I. Andersson, An exact solution of the Navier Stokes equations for magnetohydrodynamic flow, Acta Mech. 113 (1995) 241–244.
2. P. Ganesan and G. Palani, Finite difference analysis of unsteady natural convection MHD flow past an inclined plate with variable surface heat and mass flux, International Journal of Heat and Mass Transfer, 47(19-20)(2004) 4449–4457.
3. Beg, A.Y. Bakier and V.R. Prasad, Numerical study of free convection magnetohydrodynamic heat and mass transfer from a stretching surface to a saturated porous medium with Soret and Dufour effects, Comput. Mater. Sci. 46 (2009) 57–65.
4. T. Fang and J. Zhang, Closed-form exact solutions of MHD viscous flow over a shrinking sheet, Commun. Nonlinear Sci. Numer. Simul. 14 (7) (2009) 2853– 2857.
5. K.V. Prasad, D. Pal, V. Umesh and N.S. Prasanna Rao, The effect of variable viscosity on MHD viscoelastic fluid flow and heat transfer over a stretching sheet, Commun. Nonlinear Sci. Numer. Simul.15 (2) (2010) 331–334.
6. K. Jafar, R. Nazar, A. Ishak, and I. Pop, MHD Flow and Heat Transfer vertical stretching/shrinking sheets with external magnetic field, viscous dissipation and Joule Effects, Can. J. Chem. Eng. 9999 (2011) 1–11.
7. Sakiadis, B.C. 1961. "Boundary-layer Behavior on Continuous Solid Surfaces: I Boundary Layer Equations for Two Dimensional and Axisymmetric Flow", *AIChE J* 7, pp. 26-28.
8. P.S. Gupta, A.S. Gupta, Heat and Mass Transfer on a stretching sheet with suction or blowing, Canda. J. Chem. Engng. 55 (1977) 744–746.
9. D.K.Phukan, Hydromagnetic Flow and Heat Transfer Over A Surface Stretching With A Power –Law Velocity Distribution,Ganita.(2003).Vol.54,no.2,177-188.
10. M.Mohebujjaman *et.al*, MHD Heat Transfer Mixed Convection Flow Along a Vertical Stretching Sheet in Presence of Magnetic Field With Heat Generation,(2010) IJBAS-IJENS vol:10.no-02.

11. P.S. Gupta, A.S. Gupta, Heat and Mass Transfer on a stretching sheet with suction or blowing, Canda. J. Chem. Engng. 55 (1977) 744–746.
12. C.K. Chen, M.I. Char, Heat transfer of a continuous stretching surface with suction or blowing, J. Math. Anal. Appl. 135 (1988) 568–580.
13. Mukhopadhyay S, Layek GC, Samad SA., Study of MHD boundary layer flow over a heated stretching sheet with variable viscosity. Int J Heat MassTransf 2005; 48(21-22):4460–6.
14. Mukhopadhyay S, Layek GC. Effects of thermal radiation and variable fluid viscosity on free convective flow and heat transfer past a porous stretching surface. Int J Heat Mass Transf 2008;51(9–10):2167
15. Ali ME., The effect of variable viscosity on mixed convection heat transfer along a vertical moving surface. Int. J. Therm. Sci.2006; 45(1):60–9.
16. Makinde OD, Laminar falling liquid film with variable viscosity along an inclined heated plate. Appl Math Comput2006; 175(1):80–8.
17. Alam MS, Rahman MM, Sattar MA. Transient magnetohydrodynamic free convective heat and mass transfer flow with thermophoresis past a radiate inclined permeable plate in the presence of variable chemical reaction and temperature dependent viscosity. NonlinearAnal. Modell. Control 2009; 14(1):3–20.
18. Prasad KV, Vajravelu K, Datti PS. The effects of variable fluid properties on the hydro-magnetic flow and heat transfer over a non-linearly stretching sheet. Int J Therm Sci 2010;49(3):603–10.
19. Salem AM. Variable viscosity and thermal conductivity effects on MHD flow and heat transfer in viscoelastic fluid over a stretching sheet. Phys Lett A 2007; 369(4):315–22.
20. Anjali Devil SP, Ganga B. Effects of viscous and Joules dissipation on MHD flow, heat and mass transfer past a stretching porous media. Nonlinear Anal Model Control 2009;14(3):303–14
21. Emad M. Abo-Eladahab, Mohamed A. El Aziz, Blowing/suction effect on hydromagnetic heat transfer by mixed convection from an inclined continuously stretching surface with internal heat generation/absorption, Int. J. Therm. Sci. 43 (2004) 709–719.
22. A. Raptis, Technical note: flow of a micropolar fluid past continuously moving plate by the presence of radiation, Int. J. Heat Mass Transfer 41 (1998) 2865–2866.
23. A. Raptis, C. Perdikis, Viscoelastic Flow by the Presence of Radiation, ZAMM 78 (1998) 277–279.
24. A. Raptis, Radiation and Viscoelastic Flow, Int. Comm. Heat Mass Transfer 26 (6) (1999) 889–895.
25. P.G. Siddheshwar, U.S. Mahabaleshwar, Effect of radiation and heat source on MHD flow of a viscoelastic liquid and heat transfer over a stretching sheet, Int. J. Non-Linear Mech. 40 (2005) 807–820.
26. S.K. Khan, Heat transfer in a viscoelastic fluid flow over a stretching surface with heat source/sink, suction/blowing and radiation, Int. J. Heat Mass Transfer 49 (2006) 628–639
27. Gupta RK, Sridhar T. Visco-elastic effects in non-Newtonian flow through porous media. Rheol Acta 1985;24:148–51
28. Abel S, Veena PH. Visco-elastic fluid flow and heat transfer in a porous medium over a stretching sheet. Int J Non-Linear Mech1998;33:531–8.
29. Sharrma R. Effects of viscous dissipation and heat source on unsteady boundary layer flow and heat transfer past a stretching surface embedded in a porous medium using element free Galerkin method. Appl Math Comput 2012; 219:976–87.

Figure-1

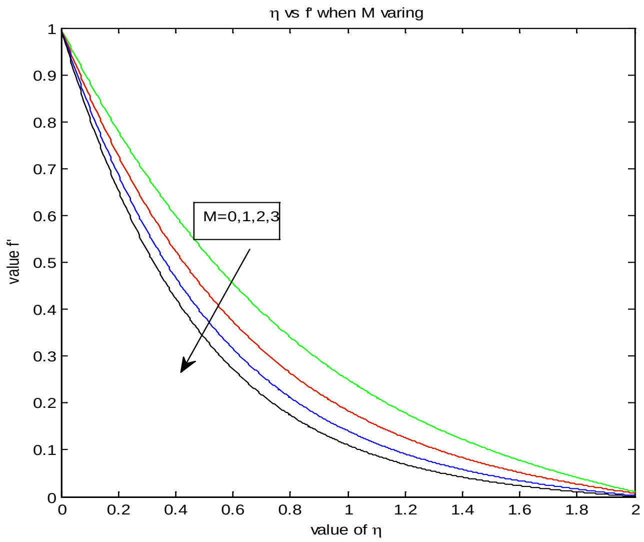


Figure-2

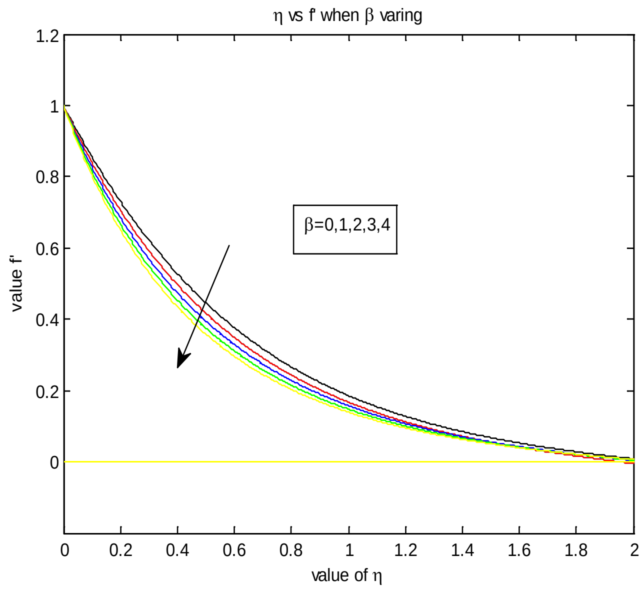


Figure-3

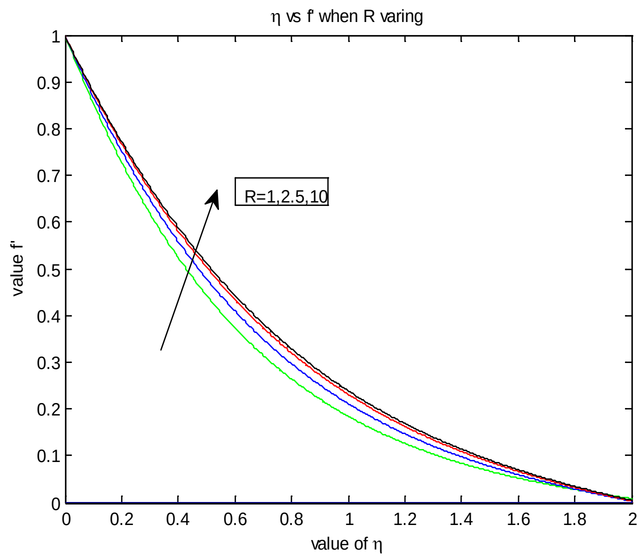


Figure-4

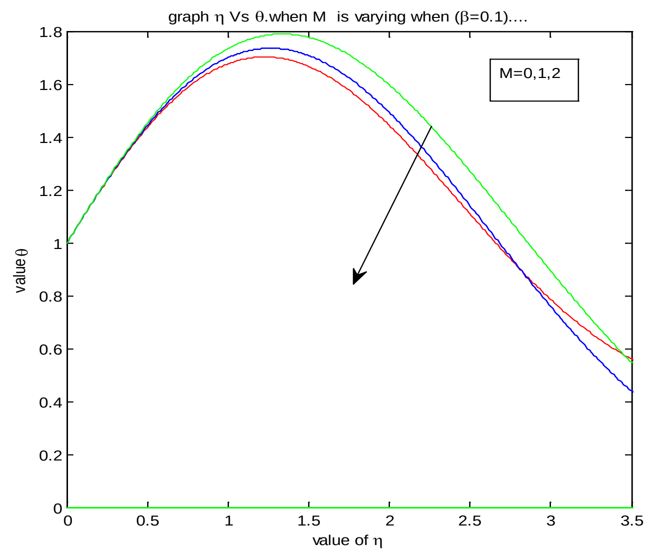


Figure-5

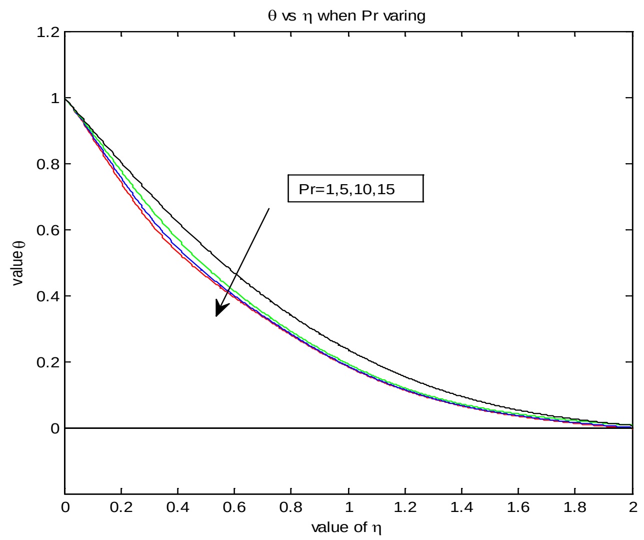


Figure-6

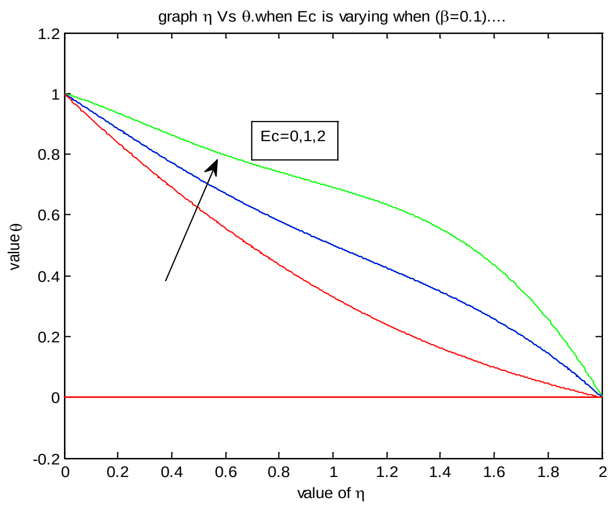


Figure-7

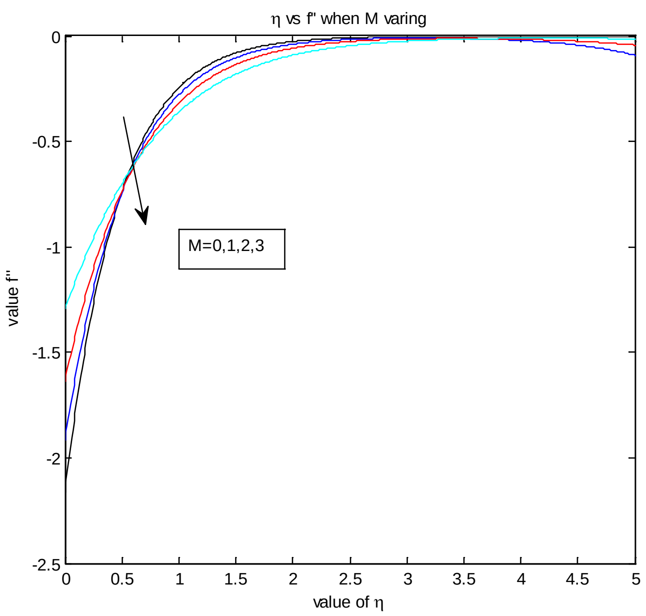


Figure-8

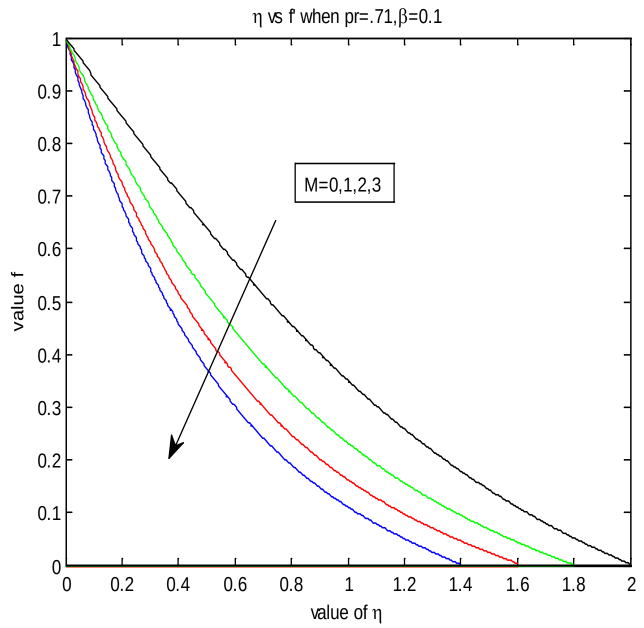
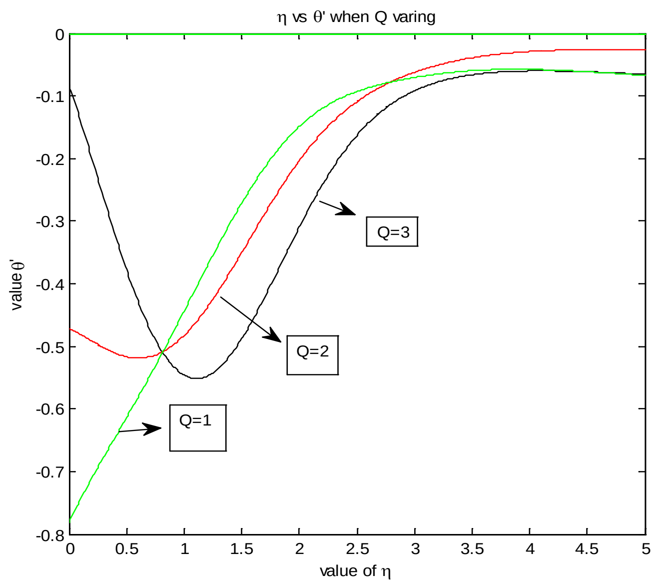


Figure-9



Source of support: Nil, Conflict of interest: None Declared